5 Data Representation

Learning Outcome

Data Representation
- Represent data in binary and hexadecimal forms
- Write programs to perform the conversion of positive integers between different number bases: denary, binary and hexadecimal forms; and display results

Data Validation
- Understand data validation technique: check digit

5.1 Binary Representation

Computers only ‘understand’ **0**’s and **1**’s. All the data inside a computer has to be represented by patterns of **1**’s and **0**’s.

e.g.

0	1	0	1	1	0	1	0

represents character ‘Z’ in ASCII code

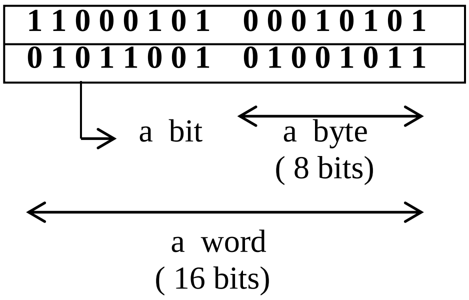
0	0	0	0	0	0	1	1

represents number 3

- A **bit** is a binary digit. It can be a **1** or **0**.
- A **byte** is taken to represent 8 bits.
- A **word** is the number of bytes that can be stored inside a memory location.

e.g. A 16-bit computer

memory location	contents
0	**1 1 0 0 0 1 0 1 0 0 0 1 0 1 0 1**
1	**0 1 0 1 1 0 0 1 0 1 0 0 1 0 1 1**



5.2 Number Systems

System	*Base*	*Digits Used*	*Used by*
Decimal (Denary)	10	0,1,2,3,4,5,6,7,8,9	humans
Binary	**2**	**0,1**	**computers**
Octal	8	0,1,2,3,4,5,6,7	Programmers as

Hexadecimal	16	0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F	‘shorthands’ for binary

A number **N** with base **r** is written as $\mathbf{N}_{\mathbf{r}}$

e.g.	101_2	71_8	$19A_{16}$	6925
	(Binary)	(Octal)	(Hex)	(Decimal)

Place Value: the value of any digit depends on its position in the number.

e.g. Decimal number **4957**

place value :	10^3	10^2	10^1	10^0
	4	**9**	**5**	**7**

e.g. Binary number $\mathbf{1101}_2$

place value :	2^3	2^2	2^1	2^0
	1	**1**	**0**	**1**

The most significant digit (**MSD**) is the digit with the highest place value in the number.
The least significant digit (**LSD**) is the digit with the lowest place value in the number.

e.g.	$1\ 0\ 0\ 1\ 0_2$		,	$4\ 9\ 7\ 5_{10}$	
	↑ MSD	↑ LSD		↑ MSD	↑ LSD

5.3 Conversion of a number from one base to another

5.3.1 From decimal to another base

Example Convert 79_{10} to binary.

2	7 9		remainder	
2	3 9	------	**1**	↑ (LSD)
2	1 9	------	**1**	
2	9	------	**1**	
2	4	------	**1**	
2	2	------	**0**	
2	1	------	**0**	
stopping condition →	0	------	**1**	(MSD)

$\mathbf{79_{10} = 1\,0\,0\,1\,1\,1\,1_{2}}$

Example Convert 79_{10} to hexadecimal.

16	7 9		remainder		
16	4	------	**F**	↑	(LSD)
	0	------	**4**	●	(MSD)

$\mathbf{79_{10} = 4\,F_{16}}$

note : F_{16} is equivalent to 15_{10}

Method :

- Divide the decimal number by the new base continuously and note the remainders until the quotient is zero.
- Arrange the remainders, with the first remainder as the least significant digit and the last remainder as the most significant digit.

5.3.2 From other base to decimal

Example Convert $1\,0\,0_{2}$ to decimal

place value:	2^{2}	2^{1}	2^{0}
	1	**0**	**0**

$1\,0\,0_{2}$

$= (1 * 2^{2}) + (0 * 2^{1}) + (0 * 2^{0})$

$= 4_{10}$

$\mathbf{1\,0\,0_{2} = 4_{10}}$

Example Convert $6\,D_{16}$ to decimal

place value:	16^{1}	16^{0}
	6	**D**

$6\,D_{16}$

$= (6 * 16^{1}) + (D * 16^{0})$

$= (6 * 16^{1}) + (13 * 16^{0})$

$= 109_{10}$

$\mathbf{6\,D_{16} = 109_{10}}$

Method: Multiply each digit by its place value and then sum up the products.

5.3.3 From binary to hexadecimal and vice versa

Example Convert 1 0 1 0 1 0 0 1 0 1 0 1 1 1 $_2$ to hexadecimal

By sectoring in group of four

0 0 **1 0**	**1 0 1 0**	**0 1 0 1**	**0 1 1 1**	(base 2)
				↓
2	A	5	7	(base 16)

1 0 1 0 1 0 0 1 0 1 0 1 1 1 $_2$ = 2 A 5 7 $_{16}$

Example Convert 1 1 1 0 0 $_2$ to hexadecimal

By sectoring in group of four

0 0 0 **1**	**1 1 0 0**	(base 2)
1	C	(base 16)

↓

1 1 1 0 0 $_2$ = 1 C $_{16}$

Method:

- Starting from LSD, sector in group of four. Insert dummy 0's when there is not enough
- For each group of four, calculate its decimal value, and it is be the digit for hexadecimal

Example Convert 2 A 5 7 $_{16}$ to binary

2	A	5	7	(base 16)
↓	↓	↓	↓	↓
0 0 1 0	**1 0 1 0**	**0 1 0 1**	**0 1 1 1**	(base 2)

2 A 5 7 $_{16}$ = 0 0 1 0 1 0 1 0 0 1 0 1 0 1 1 1 $_2$

Method: Convert each digit into the binary representation of four digit

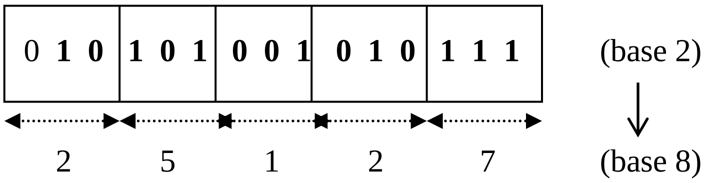
5.3.4 Conversion between octal and other bases

Although octal bases are excluded from the new syllabus, it is an interesting part to learn.

Conversion between octal and binary is similar to the conversion between hexadecimal and binary.

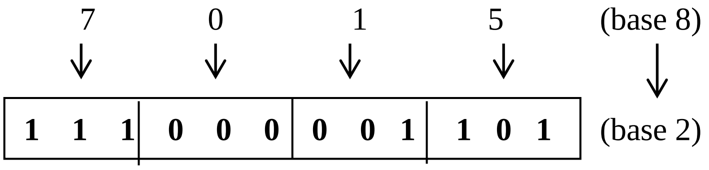
Example Convert $1\ 0\ 1\ 0\ 1\ 0\ 0\ 1\ 0\ 1\ 0\ 1\ 1\ 1_2$ to octal

By sectoring in group of three



$\mathbf{1\ 0\ 1\ 0\ 1\ 0\ 0\ 1\ 0\ 1\ 0\ 1\ 1\ 1_2\ =\ 2\,5\,1\,2\,7_8}$

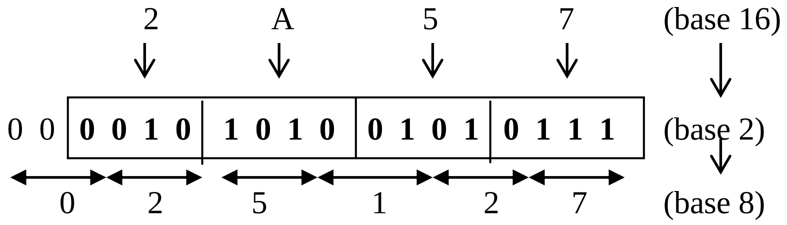
Example Convert $7\,0\,1\,5_8$ to binary



$\mathbf{7\,0\,1\,5_8\ =\ 1\,1\,1\,0\,0\,0\ 0\,0\,1\,1\,0\,1_2}$

However, the conversion between octal and hexadecimal uses binary as an intermediate step.

Example Convert $2\,A\,5\,7_{16}$ to octal



$\mathbf{2\,A\,5\,7_{16}\ =\ 2\,5\,1\,2\,7_8}$

Tutorial 5A

1. Perform the following conversion between different bases:

Binary 1010	Denary:
Denary 27	Binary:
Denary 63	Hexadecimal:
Hexadecimal B48	Denary:
Binary 10110	Hexadecimal:
Hexadecimal FF60	Binary:

2. Write a non-recursive function, for each part, that converts
   (a) Binary numbers to decimal numbers
   (b) Decimal numbers to hexadecimal numbers
   (c) Hexadecimal numbers to binary numbers

3. Write a recursive function for decimal to binary conversion.

5.4 Check Digit

Recall in Section 1.5, we discussed many ways of data validation to ensure the correct input of data. Code numbers such as customer number, employee number or product number are often lengthy and prone to errors when being keyed in. One way of preventing these errors is to add a check digit to the end of the code number. The check digit is derived by applying some algorithm to the digits of the code number. In this way, the code number with its check digit is self-checking.

Modulus 11 System

- Each digit of the code is assigned with a weight. The right hand digit is given a weight of 2, the next digit to the left is 3, and so on. (the check digit to be appended will have a weight of 1)

- Each digit is multiplied by its weight and the products are added together.

- The sum of the product is divided by 11 and the remainder is subtracted from 11 to give the check digit.

- As the check digit is a single digit, we have two special cases.

  If the remainder is 0 ,the check digit 11 (11 – 0 = 11) is converted to 0,
  If the remainder is 1 , the check digit 10 (11 – 1 = 10) is replaced by X.

Using the above algorithm, the weighted sum, including the check digit, will become exactly divisible by 11.

Example Calculate the check digit for the code number 1587.

Original number : 1 5 8 7
Weights 5 4 3 2
Multiply digits by its weight : 5 20 24 14
Add products together : 5 + 20 + 24 + 14 = 63
Divide by 11 : 5 remainder 8
Subtract remainder from 11 : 11 – 8 = 3
Check digit : 3

The complete code number is therefore 1587**3**

Example The Singapore NRIC number consists of 7 digits and an alphabet appended behind. This alphabet is calculated from the 7 digits using the modulus 11 system.

The weight for the Singapore NRIC number is shown below:

2	7	6	5	4	3	2

The following conversion table is to convert the check digit to the corresponding alphabet

Check Digit	1	2	3	4	5	6	7	8	9	10	11
Alphabet	A	B	C	D	E	F	G	H	I	Z	J

Calculate the check digit for the NRIC number 0123456

Original number :	0	1	2	3	4	5	6
Weights	2	7	6	5	4	3	2
Multiply digits by its weight :	0	7	12	15	16	15	12
Add products together :	0 + 7 + 12 + 15 + 16 + 15 + 12 = 77						
Divide by 11 :	7 remainder 0						
Subtract remainder from 11 :	11 – 0 = 11						
Check digit :	11						

The completer NRIC number is therefore S0123456J

Tutorial 5B

1 A manufacturer produces many different types of product, and each type is identified by a code number consisting of up to six digits. Whenever a code number is typed into a computer, a check digit is calculated and appended to the number.

(a) Explain carefully the purpose of the check digit.

[1]

(b) One way of calculating the check digit would be to add together the digits of the code number, and use the units digit of the total as the check digit. As an example, suppose the code number of one particular product is 508795. Since 5 + 0 + 8 + 7 + 9 + 5 = 34, the check digit would be 4, and the code would be entered as 5087954. Explain why this method is inadequate.

[2]

(c) Design a method of calculating the check digit which overcomes the limitations you identified in part (b).

[4]

2. A department store operates its own computerized credit system by issuing privileged customers with credit cards against which purchases can be charged, up to the customer's credit limit.

Each credit card has a five-digit account number, for example
3475D
where D is a modulus-eleven check-digit for the account number.

(a) Describe an algorithm which is suitable for a routine which checks whether the credit card number is valid.

[6]

(b) Using your algorithm, calculate the value of D.

[1]