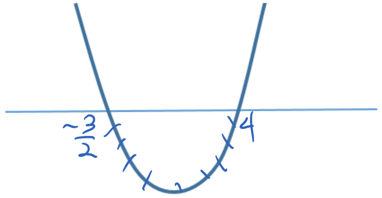
Solution Template for 2019 A-level Examination

Subject: H1 Mathematics **; Paper:** 01

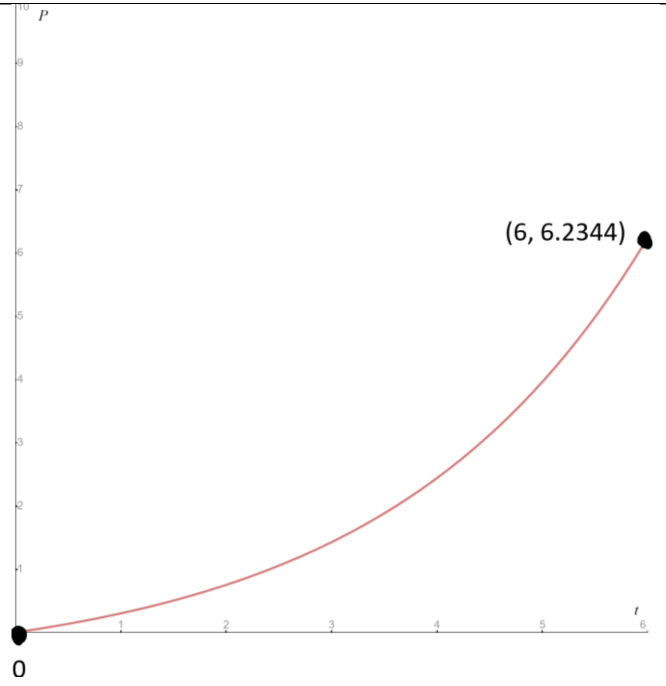
Question	Suggested Solutions
1	Let \$$A$, \$$B$ and \$$C$ be the price of Style A television, Style B television and Style C television respectively. $22A + 16B + 8C = 96480 - (1)$ $22A - 16B = 3120 - (2)$ $2A + 2B + 2C = 13260 - (3)$ Using GC, $A = 1800$ $B = 2280$ $C = 2550$ $\therefore$ The price of a Style B television is \$2280.
2(i)	Using GC, when $y = 0, x = -0.67721$ $\therefore$ the coordinates of A is $(-0.677, 0)$
2(ii)	$x = -\frac{4}{3}$

2(iii)	when $x=0, y=\ln 4$ $\therefore$ the coordinates of B is $(0,\ln 4)$ $\frac{\mathrm{d}y}{\mathrm{d}x}=1+\frac{3}{3x+4}$ when $x=0$, $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{7}{4}$, hence gradient of tangent at B is $\frac{7}{4}$. $\therefore$ Equation of tangent at B is $y=\frac{7}{4}x+\ln 4$
3(i)	$y=2x^2+6x-3\rightarrow(1)$ $y=11x+9\rightarrow(2)$ $(1)-(2)$ $2x^2-5x-12=0$ $(x-4)(2x+3)=0$ $\therefore x=4$ or $-\frac{3}{2}$ substitute $x=4$ to (2) $y=11(4)+9=53$ substitute $x=-\frac{3}{2}$ to (2) $y=11\left(-\frac{3}{2}\right)+9=-\frac{15}{2}$ $\therefore x=4, y=53$ or $x=-\frac{3}{2}, y=-\frac{15}{2}$

3(ii)	$2x^2+6x-3\le 11x+9$ $2x^2-5x-12\le 0$ $(x-4)(2x+3)\le 0$  $-\frac{3}{2}\le x\le 4$
4(i)	Let $y=\frac{1}{(2-3x)^4}=(2-3x)^{-4}$ $\frac{\mathrm{d}y}{\mathrm{d}x}=-4(2-3x)^{-5}(-3)$ $=\frac{12}{(2-3x)^5}$

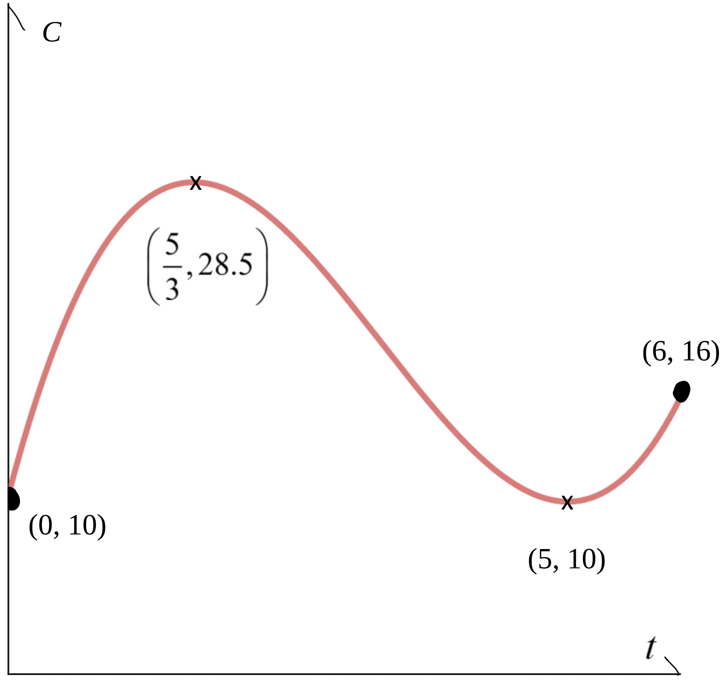
4(ii)	Let $y=\left(2\sqrt{x}-\dfrac{3}{\sqrt{x}}\right)^2$ $\therefore\ y=4x-12+\dfrac{9}{x}$ $\dfrac{\mathrm{d}y}{\mathrm{d}x}=4-\dfrac{9}{x^2}$ $\therefore\ p=4, q=-9$
4(iii)	$\int_0^1\left(x^2+2-\mathrm{e}^{-2x}\right)\,\mathrm{d}x$ $=\left[\dfrac{x^3}{3}+2x-\dfrac{\mathrm{e}^{-2x}}{-2}\right]_0^1$ $=\left[\dfrac{1}{3}+2+\dfrac{\mathrm{e}^{-2}}{2}\right]-\dfrac{1}{2}$ $=\dfrac{11}{6}+\dfrac{\mathrm{e}^{-2}}{2}$ $=\dfrac{11}{6}+\dfrac{1}{2\mathrm{e}^2}$
5(i)	Initially, i.e. when t = 0, P = 0, Hence $0=k\times1.5^0-0.6$ $\therefore k=0.6$

Appendix 2

5(ii)	
5(iii)	Using GC, when $t = 6$, $P = 6.2344 \approx 6.23$
5(iv)	Using GC, $\left.\frac{\mathrm{d}P}{\mathrm{d}t}\right\|_{t=3} = 0.821$

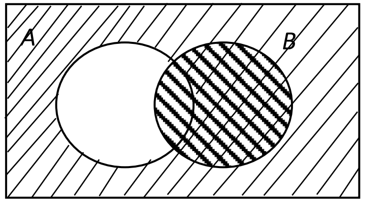
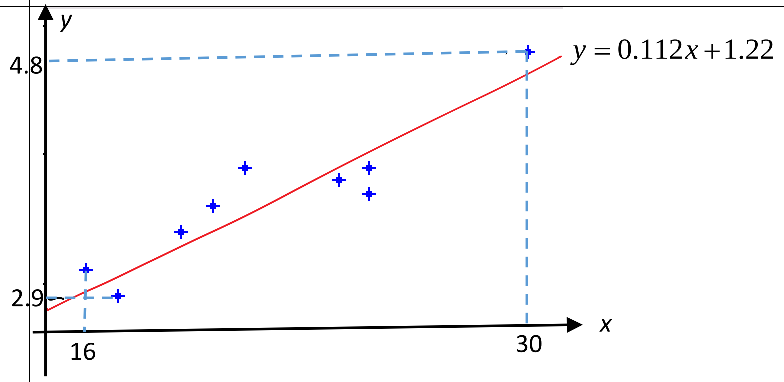
5(v)	$C = t^3 - 10t^2 + 25t + 10$ $\frac{\mathrm{d}C}{\mathrm{d}t} = 3t^2 - 20t + 25$ At stationary point on the graph, $\frac{\mathrm{d}C}{\mathrm{d}t} = 0$ $3t^2 - 20t + 25 = 0$ $\left(t-5\right)\left(3t-5\right) = 0$ $t = 5$ or $\frac{5}{3}$ $\frac{\mathrm{d}^2C}{\mathrm{d}t^2} = 6t - 20$ when $t = 5,$ $\frac{\mathrm{d}^2C}{\mathrm{d}t^2} = 6(5) - 20 = 10 > 0,$ $\therefore t = 5$ gives a minimum point on the graph . when $t = \frac{5}{3},$ $\frac{\mathrm{d}^2C}{\mathrm{d}t^2} = 6\left(\frac{5}{3}\right) - 20 = -10 < 0,$ $\therefore t = \frac{5}{3}$ gives a maximum point on the graph .

Appendix 2

5(vi)	
5(vii)	Using GC, $\int_0^6 (t^3 - 10t^2 + 25t + 10)\ \mathrm{d}t = 114$ $\therefore$ The total cost incurred by Mr Tan over 6 years is \$114000

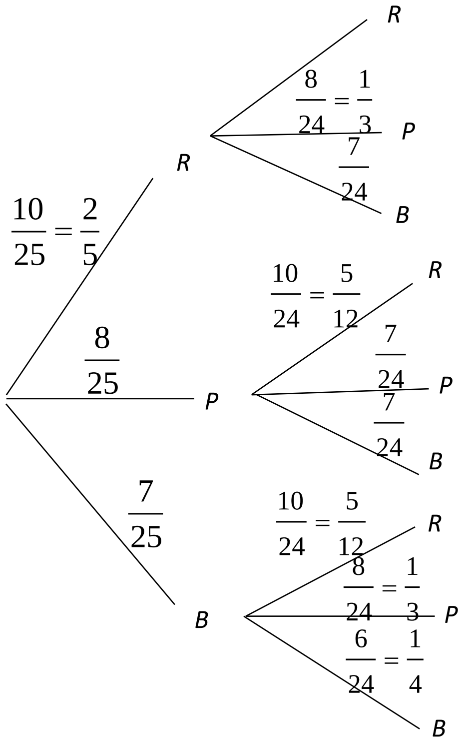
6(i)	$\boxed{\underline{E}\ \underline{I}}$ _ _ _ _ _ _ _ Group EI as 1 unit, total permutation of 8 units = 8! Permutate the unit consisting of EI = 2! $\therefore$ No. of ways = $8! \times 2! = 80640$
(ii)	No. of ways of choosing 4 letters without restriction = ${}^{9}C_{4}$ No. of ways of choosing 4 letters with no vowel = ${}^{7}C_{4}$ $\therefore$ No. of ways with at least 1 vowel chosen No. of ways of choosing 4 letters without restriction $-$No. of ways of choosing 4 letters with no vowel = ${}^{9}C_{4} - {}^{7}C_{4} = 91$ ways
7(i)	Let X be the number of members in the sales team that receive bonus out of 8. $X \sim \mathrm{B}(8, 0.32)$ $\mathrm{P}(X = 4) = 0.15694 \approx 0.157$
(ii)	$\mathrm{P}(X \geq 5) = 1 - \mathrm{P}(X \leq 4) = 0.074964 \approx 0.0750$
(iii)	Mean $= \mathrm{E}\left(X\right) = 8(0.32) = 2.56$ Variance = $\mathrm{Var}\left(X\right)$ $= 8(0.32)(1 - 0.32)$ $= 1.7408 \approx 1.74$

Appendix 2

8(i)	$\mathrm{P}(A\mid B)=0.1$ $\dfrac{\mathrm{P}(A\cap B)}{\mathrm{P}(B)}=0.1$ $\mathrm{P}(A\cap B)=0.1\times 0.4=0.04$
8(ii)	It is the probability that event *B* and (i.e. intersect) the complement of event *A* will occur at the same time.
8(iii)	$\mathrm{P}(A\cap B')=\mathrm{P}(A)-\mathrm{P}(A\cap B)$ $=0.6-0.04$ $=0.56$ $\mathrm{P}(A'\cup B)=1-\mathrm{P}(A\cap B')$ $=1-0.56$ $=0.44$ 
9(i)	
9(ii)	Using GC, $r=0.92083\approx 0.921$

	It shows that there is a strong positive linear correlation between x (the marks of the students for the written test) and y (the marks of the students for the practical test).
9(iii)	Using GC, $y = 0.11236x + 1.2183$ $y = 0.112x + 1.22$, where $a = 0.112$ and $b = 1.22$ *[For the sketch of the regression line on the scatter diagram, please refer to the scatter diagram in (i)]*
9(iv)	When $x = 22$, $y = 0.11236\left(22\right) + 1.2183$ $= 3.69022$ $= 3.7\ \left(\text{to 1 decimal place}\right)$ Since $r = 0.921$ is close to 1 and $x = 22$ is within the range of x $\left(16 \le x \le 30\right)$ given in the table, the estimate is an interpolation, therefore the estimate is reliable.
10(i)	**1st sock** **2nd sock**

$\frac{9}{24} = \frac{3}{8}$

	 Event *R*: red sock chosen Event *P*: pink sock chosen Event *B*: blue sock chosen
10(ii)	Probability that both socks are red = P(*RR*) $= \frac{2}{5}\times\frac{3}{8}$ $= \frac{3}{20}$
10(iii)	Probability that the 2 socks are of different colours

	$=1-\mathrm{P}\left(2\text{ socks are of same colour}\right)$ $=1-\left[\mathrm{P}\left(RR\right)+\mathrm{P}\left(PP\right)+\mathrm{P}\left(BB\right)\right]$ $= =1-\left(\frac{3}{20}+\frac{8}{25}\times\frac{7}{24}+\frac{7}{25}\times\frac{1}{4}\right)$ $=\frac{103}{150}$
10(iv)	$=\mathrm{P}\left(\text{exactly 2 red socks}\right)+\mathrm{P}\left(\text{exactly 3 red socks}\right)$ $=\mathrm{P}\left(RRP\right)+\mathrm{P}\left(RRB\right)+\mathrm{P}\left(RPR\right)+\mathrm{P}\left(RBR\right)+\mathrm{P}\left(PRR\right)+\mathrm{P}\left(BRR\right)+\mathrm{P}\left(RRR\right)$ $\text{Required probability}=\frac{10}{25}\times\frac{9}{24}\times\frac{8}{23}+\frac{10}{25}\times\frac{9}{24}\times\frac{7}{23}+\frac{10}{25}\times\frac{8}{24}\times\frac{9}{23}+\frac{10}{25}\times\frac{7}{24}\times\frac{9}{23}+\frac{8}{25}\times\frac{10}{24}\times\frac{9}{23}$ $+\frac{7}{25}\times\frac{10}{24}\times\frac{9}{23}+\frac{10}{25}\times\frac{9}{24}\times\frac{8}{23}$ $=\frac{159}{460}$
11(i)	Let A be the time taken by a runner from the Arrows to run 5000m (in minutes). $A\sim\mathrm{N}\left(14.8,0.55^2\right)$ $P\left(A>15.0\right)$ $=0.35806$ ≈ 0.358 (3 s.f.)
11(ii)	Let B be the time taken by a runner from the Beavers to run 5000m (in minutes). $B\sim\mathrm{N}\left(15.2,0.65^2\right)$ Let $T=A_1+A_2+B_1+B_2+B_3$

	$\mathrm{E}(T)=2(14.8)+3(15.2)=75.2$ $\mathrm{Var}(T)=2(0.55^2)+3(0.65^2)=1.8725$ $\therefore T \sim \mathrm{N}(75.2, 1.8725)$ $P(T<75)$ $=0.44189$ ≈ 0.442 (2 s.f.)
11(iii)	Let $X = A_1 + A_2 + A_3 + A_4 + A_5 + A_6$ and let $Y = B_1 + B_2 + B_3 + B_4 + B_5 + B_6$ $\mathrm{E}(X)=6\mathrm{E}(A)=88.8$ $\mathrm{Var}(X)=6\mathrm{Var}(A)=1.815$ $\therefore X \sim \mathrm{N}(88.8, 1.815)$ $\mathrm{E}(Y)=6\mathrm{E}(B)=91.2$ $\mathrm{Var}(Y)=6\mathrm{Var}(B)=2.535$ $\therefore Y \sim \mathrm{N}(91.2, 2.535)$ $X-Y \sim N(-2.4, 4.35)$ $\mathrm{P}(\lvert X-Y\rvert \le 5)$ $=\mathrm{P}(-5 \le X-Y \le 5)$ $=0.89353$ ≈ 0.894

11(iv)	Let M be the event that the Arrows win a medal, and N be the event that the Beavers win a medal. $\mathrm{P}(M)=\mathrm{P}(X<90)$ $=0.81346$ ≈ 0.813 $\mathrm{P}(N)=\mathrm{P}(Y<90)$ $=0.22552$ ≈ 0.226 $\mathrm{P}(M'\cap N')$ $=\mathrm{P}((M\cup N)')$ $=1-\mathrm{P}(M\cup N)$ $=1-[\mathrm{P}(M)+\mathrm{P}(N)-\mathrm{P}(M\cap N)]$ $=1-[\mathrm{P}(M)+\mathrm{P}(N)-\mathrm{P}(M)\times\mathrm{P}(N)]$ $=0.14447$ ≈ 0.144
12(i)	Unbiased estimate of the population mean is $\bar{x}$ $=\frac{\sum x}{n}=\frac{5928}{60}=98.8$ Unbiased estimate of the population variance is s^2

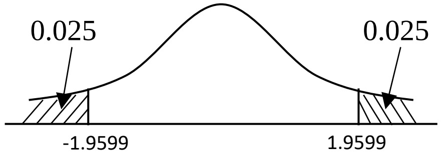
	$= \frac{1}{n-1}\left[\sum x^2 - \frac{\left(\sum x\right)^2}{n}\right]$ $= \frac{1}{60-1}\left[587000 - \frac{5928^2}{60}\right]$ $= 22.264$ ≈ 22.3
12(ii)	Let X be the mass of a cake (in g) produced by the baker, and μ be the population mean mass of the cakes. $\mathrm{H}_0 : \mu = 100$ $\mathrm{H}_1 : \mu < 100$ Test at 5% significance level Under H_0, since $n = 60$ is large, by Central Limit Theorem, $\bar{X} \sim \mathrm{N}\left(100, \frac{22.264}{60}\right)$ approximately Using GC, $\bar{x}$ gives $z_{\text{calc}} = -1.9699$ and p-value $= 0.0244$ Since p-value $= 0.0244 < 0.05$, we reject H_0 and conclude that there is sufficient evidence, at 5% significance level, that the mean mass of these cakes is less than 100g.
12(iii)	$\mathrm{H}_0 : \mu = 100$ $\mathrm{H}_1 : \mu \neq 100$ Test at 5% significance level Under H_0, since $n = 60$ is large, by Central Limit Theorem, test statistic $Z = \frac{\bar{X} - 100}{\sqrt{\frac{19.0}{60}}} \sim \mathrm{N}(0,1)$ approximately Since there is sufficient evidence to reject the baker's statement,

there is sufficient evidence to reject H_0 .

Therefore, test statistic Z will lie in the critical region.

$$Z \le -1.9599 \quad \text{or} \quad Z \ge 1.9599$$

$$\frac{\bar{x}-100}{\sqrt{\frac{19.0}{60}}} \le -1.9599 \quad \text{or} \quad \frac{\bar{x}-100}{\sqrt{\frac{19.0}{60}}} \ge 1.9599$$



$$\bar{x} \le 98.897 \quad \text{or} \quad \bar{x} \ge 101.10$$

$$\therefore \left\{\bar{x} \in \mathbb{R} : \bar{x} \le 98.9 \text{ or } \bar{x} \ge 101\right\}$$