CHAPTER 8

8.1 DEFINITION: A ***matrix*** is a set of real or complex numbers (or elements) arranged in rows and columns to form a rectangular array.

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & .... & a_{1n} \\ a_{21} & a_{22} & a_{23} & .... & a_{2n} \\ . & . & . & .... & . \\ . & . & . & .... & . \\ a_{m1} & a_{m2} & a_{m3} & .... & a_{mn} \end{bmatrix}$$

The numbers a_{11} , a_{12} , a_{13} , ... a_{mn} are called the ***elements*** of the matrix.

The ***order of a matrix*** is specified according to the number of rows and number of columns it possesses. A matrix with ***m rows*** (the horizontal lines) and ***n columns*** (the vertical lines) is called an **(m x n)** matrix or the matrix is said to be of order ***m* x *n*** .

Double - subscript notation is used for the elements in the matrix. The first subscript denotes the row and the second subscript the column containing the given element.

eg a_{ij} is the element in the ***i*** th row and ***j*** th column.

Matrices may be denoted by capital bold-faced letters.

eg **A** , **B** , $\left[a_{ij}\right]$, $\left[a_{ij}\right]_{mxn}$

8.2 EQUALITY OF MATRIX

Two matrices are said to be equal if:

i) they are the same order,

ii) their corresponding elements are equal

So, if $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} = \begin{bmatrix} 4 & 6 & 5 \\ 2 & 3 & 7 \end{bmatrix}$ then

$a_{11} = 4$; $a_{12} = 6$; $a_{13} = 5$; $a_{21} = 2$; etc

eg $\begin{bmatrix} 1 & 4 & 9 \\ 2 & 5 & 8 \end{bmatrix} = \begin{bmatrix} 1^2 & 2^2 & 3^2 \\ 1^2 + 1 & 2^2 + 1 & 3^2 + 1 \end{bmatrix}$

$\begin{bmatrix} 5x + 3y \\ 4x - 7y \end{bmatrix} = \begin{bmatrix} 13 \\ 1 \end{bmatrix}$ means $\begin{aligned} 5x + 3y &= 13 \\ 4x - 7y &= 1 \end{aligned}$

8.3 TYPES OF MATRICES

(a) Row Matrix

A matrix having only one row and is of order ***1* x *n*** .

eg $\begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}$

(b) Column Matrix

A matrix having only one column and is of order ***m*** x ***1*** .

eg $\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$

(c) Square Matrix

A matrix having the same number of rows and columns. Obviously, only one number is needed to specify its order.

eg $\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$; $\begin{bmatrix} 3 & 1 & 2 \\ 12 & 2 & 7 \\ 5 & 3 & 20 \end{bmatrix}$

8.4 ARITHMETIC OF MATRICES

Addition and Subtraction of Matrices

Two matrices A and B of the same order can be added (or subtracted) by adding (or subtracting) their corresponding elements.

For example :

(a) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 7 & 8 \\ 9 & 10 \end{bmatrix} = \begin{bmatrix} 1+7 & 2+8 \\ 3+9 & 4+10 \end{bmatrix} = \begin{bmatrix} 8 & 10 \\ 12 & 14 \end{bmatrix}$

(b) $\begin{bmatrix} 3 & 0 & -2 \\ 4 & -1 & 4 \end{bmatrix} - \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 0 \end{bmatrix} = \begin{bmatrix} 3-1 & 0-1 & -2-1 \\ 4-2 & -1-3 & 4-0 \end{bmatrix}$

$$= \begin{bmatrix} 2 & -1 & -3 \\ 2 & -4 & 4 \end{bmatrix}$$

Example 1 If $A = \begin{bmatrix} 4 & 6 & 5 & 7 \\ 3 & 1 & 9 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 8 & 3 & -1 \\ 5 & 2 & -4 & 6 \end{bmatrix}$, determine

(a) A + B; (b) B + A and (c) A - B.

Thus matrix addition is commutative. (ie A + B = B + A)

Example 2 Let $A = \begin{bmatrix} 6 & 1 \\ -2 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 7 \\ 5 & 2 \end{bmatrix}$ and $C = \begin{bmatrix} 5 & 6 \\ 8 & 12 \end{bmatrix}$.

Show that (A + B) + C = A + (B + C)

Thus matrix addition is associative. [ie (A + B) + C = A + (B + C)]

8.5 MULTIPLICATION OF MATRICES

(a) **Scalar multiplication**

To multiply a matrix by a single number (ie a scalar), each individual element of the matrix is multiplied by that factor.

eg $4 \times \begin{bmatrix} 3 & 2 & 5 \\ 6 & 1 & 7 \end{bmatrix} = \begin{bmatrix} 12 & 8 & 20 \\ 24 & 4 & 28 \end{bmatrix}$ ie In general $K\left[a_{ij} \right] = \left[K\, a_{ij} \right]$

(b) **Multiplication of two matrices**

Two matrices A and B can be multiplied together if and only if the number of columns in A is the same as the number of rows in B.

Let $A = \left[a_{ij} \right]_{m \times n}$; and $B = \left[b_{ij} \right]_{n \times q}$

Then the product AB is an $\boldsymbol{m} \times \boldsymbol{q}$ matrix.

Example 3 Find the product of $\begin{bmatrix} 2 & 3 \\ 5 & 6 \\ 7 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 7 \end{bmatrix}$.

Example 4 Find the product of $\begin{bmatrix} 5 & 6 & 7 & 8 \end{bmatrix} \begin{bmatrix} -5 \\ 6 \\ 8 \\ 10 \end{bmatrix}$.

Example 5 Find the product of $\begin{bmatrix} 2 & 0 & 0 \\ 4 & 3 & 0 \\ 5 & 6 & 1 \end{bmatrix}\begin{bmatrix} 5 \\ 6 \\ 7 \end{bmatrix}$.

Example 6 If $A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ -1 & -3 \end{bmatrix}$, find AB and B A .

Example 7 If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 3 & 2 & 1 \end{bmatrix}$, find A B .

AB is not defined since the number of columns of A does not agree with the number of rows of B.

8.6 PROPERTIES OF MATRIX MULTIPLICATION

1. $(KA)B = A(KB) = K(AB)$
2. $A(BC) = (AB)C$ - Associative
3. $AB \neq BA$ - Not Commutative
4. $(A+B)C = AC + BC$ - Is Distributive wrt addition
5. $C(A+B) = CA + CB$
6. If $AB = 0$,
   It **does not** necessarily imply that $A = 0$ or $B = 0$.
7. If $AB = AC$
   It **does not** necessarily imply that $B = C$.

TUTORIAL 8

1 If $A = \begin{bmatrix} 6 & 2 \\ -1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & -5 \\ -6 & -7 \end{bmatrix}$, find:

(a) $A + B$; (b) $B + A$ and (c) $A - B$

2 If $A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & -2 & -3 \\ 4 & 5 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -1 & 2 \\ 0 & 3 & -1 \\ 0 & 1 & 3 \end{bmatrix}$, find:

(a) $A + B$ and (b) $2A - 2B$.

3 If $A = \begin{bmatrix} 3 & -5 \\ 1 & -2 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 0 \\ 3 & 1 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$,

(a) verify the associative law $(A + B) + C = A + (B + C)$;

(b) find $3A - B - C$

4 If $A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, find $(I - 2A + 4B)$.

5 Determine the values of the variables for which the following main equations are true :

(a) $\begin{bmatrix} 2 & -1 \\ x & -2 \end{bmatrix} = \begin{bmatrix} y-2 & z \\ 3 & t-1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 4 & y \\ 2 & 3 & 3 \\ 3 & x & 4 \end{bmatrix} = \begin{bmatrix} 1 & 3-t & 5 \\ 2 & 3 & 3 \\ z-1 & 4 & z \end{bmatrix}$

6 Perform the following matrix multiplications, if possible :

(a) $\begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 2 & -2 \end{bmatrix}$ (b) $\begin{bmatrix} 4 & 3 \\ 5 & 6 \end{bmatrix}\begin{bmatrix} -1 & -3 \\ 2 & 4 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 & 1 \\ -1 & 0 & 2 \\ 3 & 4 & 5 \end{bmatrix}\begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & -1 \\ 2 & 0 & 3 \end{bmatrix}$

CHALLENGING QUESTION

1. A computer retailer sells three types of computers: the personal computer (PC), the business computer (BC) and the business computer (IC). The retailer has two stores. The matrix below shows the number of each computer model in stock at each store.

$$\begin{matrix} & \text{PC} & \text{BC} & \text{IC} \end{matrix}$$
$$\begin{pmatrix} StoreA \\ StoreB \end{pmatrix}\begin{bmatrix} 6 & 18 & 12 \\ 10 & 22 & 28 \end{bmatrix}$$

The store plans to have a sale next month. In order to have enough computers in stock at each store when the sale begins, it plans to order 1.5 times the current number. How many of each computer should be ordered for each store?

2. A drug company tested 400 patients to see if a new medicine is effective. Half of the patients received the new drug and half received a placebo. The results for the first 200 patients are shown in the following matrix.

$$\begin{matrix} & \text{New drug} & \text{Placebo} \end{matrix}$$
$$\begin{pmatrix} Effective \\ Not\_effective \end{pmatrix}\begin{bmatrix} 70 & 40 \\ 30 & 60 \end{bmatrix}$$

Using the same matrix format, the results for a second 200 patients were $\begin{bmatrix} 65 & 42 \\ 35 & 58 \end{bmatrix}$.

What were the results for the entire test group?