1 Express $\frac{3x^3+13x-5}{\left(x^3+5x\right)}$ in partial fractions. [6]

2 **(a)** The variables *x* and *y* are related in such a way that, when y^2 is plotted against x^2y, a straight line is obtained which passes through (4, 6) and (6, 10).

Find the exact values of *y* for which $x=\sqrt{3}$. [4]

(b) The table shows experimental values of *x* and *y*.

x	1.5	2.0	2.5	3.0	3.5
y	35	79	176	394	890

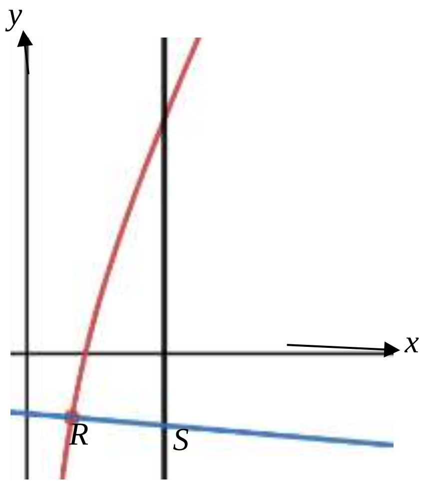
It is known that *x* and *y* are related by an equation of the form $y = Up^x$, where *U* and *p* are constants.

(i) On the grid on page 5, draw a straight line graph to illustrate this data. [2]

(ii) Use your graph to estimate the value of *U* and of *p*. [4]

(iii) Estimate the value of *x* for which *y* = 500. [2]

3 The diagram shows part of the curve $y = 3x - \frac{5}{x}$. The normal to the curve at R where $x = 1$ meets the line $x = 3$ at S.



(i) Find the equation of normal RS. [4]

(ii) Find the area bounded by the curve, the normal RS and the line $x = 3$. [5]

4 **(a)** A curve has equation $x^2 + y^2 = 20$ and a line has equation $y = 2x + m$.

(i) Find the range of values of *m* for which the line intersects the curve at 2 distinct points. [4]

(ii) Give an example of a possible value for *m* for which the line does not intersect the curve. [1]

(b) The curve with equation $y = ax^2 + 2cx + c$, where *a* and *c* are constants, lies completely above the *x*-axis.

(i) Write down the conditions which must apply to *a* and *c*. [3]

(ii) Give an example of possible values for *a* and *c* which satisfy the conditions in part **(i)**. [2]

5 **(i)** Express $\frac{6x}{3x-1}$ in the form $a+\frac{b}{3x-1}$, where *a* and *b* are constants and hence find

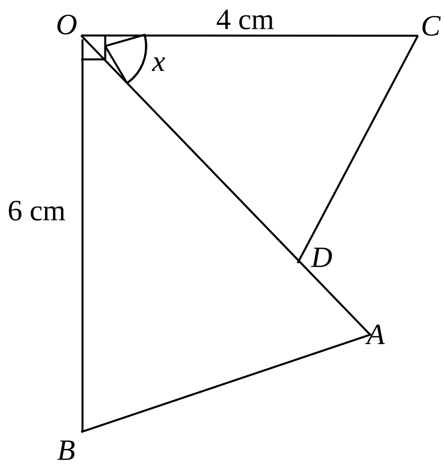
$\int \frac{6x}{3x-1}\,\mathrm{d}x$. [4]

(ii) Given that $y = x\ln(3x-1)$, find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$. [2]

(iii) Using the results from **(i)** and **(ii)**, find $\int \ln(3x-1)\,\mathrm{d}x$. [3]

6 The diagram shows two isosceles triangles *AOB* and *COD*. It is given that

$OA = OB = 6$ cm, $OC = OD = 4$ cm, $\angle COB = 90°$ and $\angle COD = x$.



The sum of the areas of ΔAOB and ΔCOD is A cm^2.

(i) Show that $A = 18\cos x + 8\sin x$. [2]

(ii) Find the value of R and α for which $A = R\cos(x-\alpha)$, where $R > 0$ and $0° < \alpha < 90°$. [3]

(iii) Given that x can vary, find the value of x if $A = 16$. Hence, find the area of triangle *AOB*. [4]

7 The equation of a curve is $y = e^{x^3 - 3x^2}$

(i) Find expressions for $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. [4]

(ii) Find the exact value of the coordinates of the stationary points. [4]

(iii) Determine the nature of each of the stationary points. [4]

8 The points $P(-2,-6)$, $Q(2,-8)$ and $R(6,0)$ lie on a circle.

(i) Show that *PR* is a diameter of the circle. [3]

(ii) Find the equation of the circle. [3]

(iii) Find the possible equations of tangent to the circle which is parallel to the x-axis. [2]

(iv) Find the equation of the perpendicular bisector of QR and show that it passes through the centre of the circle. [3]

9 **(a)** Solve the equation $8(3^{-x}) - 3^x = 2$. [4]

(b) Given that $\log_3\left(\dfrac{x^4}{y}\right) = 2 + 2\log_3 x + \log_9 y$, express y in terms of x. [4]

(c) Sketch the graph of $y = 3e^{x}$. By inserting on your sketch an additional graph, explain how a graphical solution of the equation $(3x-1)e^{-x} = 2$ may be obtained. [4]

End of paper

Answer Key

1	$3-\frac{1}{x}+\frac{x-2}{x^2+5}$
2a	$y=3\pm\sqrt{7}$
2b	ii) $p \approx 5.01$, $u \approx 2.99$ iii) $x = 3.15$
3	i) $8y=-x-15$ (o.e.) ii) 10.8 units^2
4a	i) $-10<m<10$ ii) any $m < -10$ or $m > 10$
4b	i) $a > 0$, $c > 0$, $c^2-ac < 0$ or $0 < c < a$ ii) accept any a, c such that $0 < c < a$
5	i) $2+\frac{2}{3x-1}$ ii) $\frac{3x}{3x-1}+\ln(3x-1)$ iii) $x\ln(3x-1)-x-\frac{1}{3}\ln(3x-1)+c$
6	ii) $R=\sqrt{388}$ (o.e.) $\alpha=24.0°$ iii) $x = 59.6°$, 9.10 cm^2
7	i) $\frac{dy}{dx}=(3x^2-6x)e^{x^3-3x^2}$, $\frac{d^2y}{dx^2}=[(6x-6)+(3x^2-6x)^2]e^{x^3-3x^2}$ (o.e.) ii) (0,1) and $(2, e^{-4})$
8	ii) $(x-2)^2+(y+3)^2=25$ iii) $y = 2$, $y = -8$ iv) $y=-\frac{1}{2}x-2$ (o.e.)
9a	0.631
9b	$y=\left(\frac{x}{3}\right)^{\frac{4}{3}}$ or $y=\sqrt[3]{\frac{x^4}{81}}$
9c	Insert line $y=\frac{9}{2}x-\frac{3}{2}$. The interesction of the line $y=\frac{9}{2}x-\frac{3}{2}$ and the curve $y=3e^x$ gives the solution. Since there is no intersection, there is no solution to $(3x-1)e^{-x}=2$.