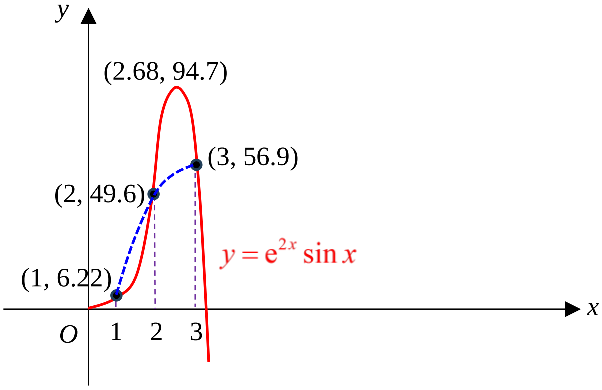
2024 H2 FM 9649 Prelim Paper 1 (ACJC_EJC_NJC_RVHS)

	Solution
P1 Q1	**(a)** $\mathbf{A}$ is a 3×4 matrix. Hence $\text{rank}(\mathbf{A}) + \text{nullity}(\mathbf{A}) = 4$. As $\text{rank}(\mathbf{A}) \le 3$, $\text{nullity}(\mathbf{A}) \ge 1$. As the dimension of the null space is at least 1, it is not the zero vector space. **(b)** $\mathbf{A} = \begin{pmatrix} 2 & 3 & 4 & -8 \\ 1 & 1 & 1 & -3 \\ 3 & 6 & 9 & -15 \end{pmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{pmatrix} 1 & 1 & 1 & -3 \\ 2 & 3 & 4 & -8 \\ 3 & 6 & 9 & -15 \end{pmatrix}$ $\xrightarrow[R_3 \to R_3 + 3R_1]{R_2 \to R_2 - 2R_1} \begin{pmatrix} 1 & 1 & 1 & -3 \\ 0 & 1 & 2 & -2 \\ 0 & 3 & 6 & -6 \end{pmatrix} \xrightarrow[R_3 \to R_3 - 3R_2]{R_1 \to R_1 - R_2} \begin{pmatrix} 1 & 0 & -1 & -1 \\ 0 & 1 & 2 & -2 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ $\begin{pmatrix} 1 & 0 & -1 & -1 \\ 0 & 1 & 2 & -2 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} x - z - w \\ y + 2z - 2w \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ $\begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = z \begin{pmatrix} 1 \\ -2 \\ 1 \\ 0 \end{pmatrix} + w \begin{pmatrix} 1 \\ 2 \\ 0 \\ 1 \end{pmatrix}$ $\ker(T) = \left\{ a \begin{pmatrix} 1 \\ -2 \\ 1 \\ 0 \end{pmatrix} + b \begin{pmatrix} 1 \\ 2 \\ 0 \\ 1 \end{pmatrix}, \quad a, b \in \mathbb{R} \right\}$

	Solution
P1 Q2	**(a)** Using GC, $I=\int_1^3 e^{2x}\sin x\, dx = 100.96$ (2 d.p) **(b)** Using Simpson's Rule, $I=\frac{1}{3}h\left[y_0+4y_1+y_2\right]$ $=\frac{1}{3}\left(\frac{3-1}{2}\right)\left[6.217676+4(49.645957)+56.931875\right]$ $=87.24446008=87.24$ (2 d.p.) Percentage error $\approx \frac{100.96-87.245}{100.96}\times 100\% = 13.6\%$ (3 s.f.) **(c)**  From diagram, we observe that there is a large discrepancy in terms of area between the actual curve $y=e^{2x}\sin x$ and the approximated quadratic curve for the interval $2\le x\le 3$, which is caused by the existence of both a turning point and a point of inflexion. Hence Simpson's Rule using 3 ordinates will not provide a good approximation in this case. Increasing the number of intervals/ordinates will give a better approximation.

	Solution
P1 Q3	**(a)** From the GC,

k	1	2	…	50	51	52
M_{2k-1}	1	2	…	2.0288	2.0288	2.0288
M_{2k}	2	2.25	…	4.1686	4.1686	4.1686

Since $M_{2k} \to 4.1686$ and $M_{2k-1} \to 2.0288$ as $k \to \infty$,

The sequence $\{M_n\}$ alternates between the 2 values of 2.0288 and 4.1686 as $n \to \infty$.

(b) Observe that

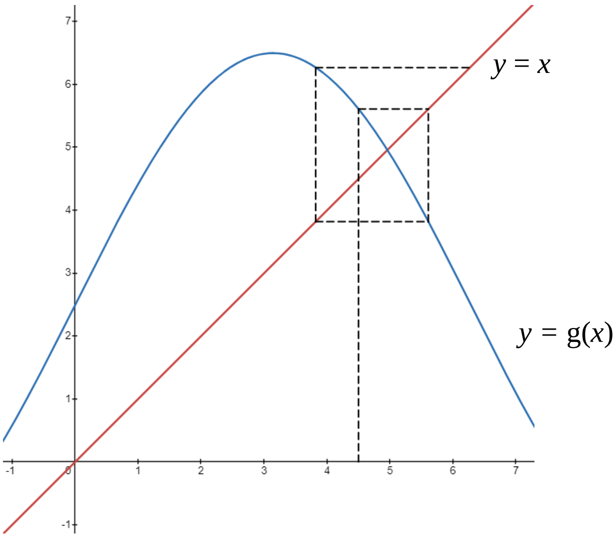
n	1	2	3	…
M_n	2	2	2.25	…

So the sequence in part (b) is the same as moving the terms in the sequence in part (a) up by 1 term, and it will have the same alternating behaviour but odd terms will approach 4.1686 and even terms will approach 2.0288 as $n \to \infty$.

(c) For a constant sequence, $M_n = M_{n+1} = M_{n+2} = \alpha$ for all $n \in \mathbb{Z}^+$. Then we have

$$M_{n+2} = \left(\frac{M_{n+1}}{2M_n}\right)^2 + \frac{2M_n}{M_{n+1}} \quad \Rightarrow \quad \alpha = \left(\frac{\alpha}{2\alpha}\right)^2 + \frac{2\alpha}{\alpha}$$

$$\Rightarrow \quad \alpha = \frac{9}{4}$$

	Solution
P1 Q4	**(a)** $\text{f}(x)=8\sin\left(\frac{x}{2}\right)-2x+5$. $\text{f}(4.5)=2.2246>0$ and $\text{f}(5)=-0.21222<0$ Since $\text{f}(4.5)\times\text{f}(5)<0$ and f is continuous in the interval $[4,5]$, the equation $\text{f}(x)=0$ has a root α in the interval $[4.5,5]$. $\Rightarrow 4.5<\alpha<5$ (Shown) **(b)** Note that $x=4\sin\left(\frac{x}{2}\right)+\frac{5}{2}$ is equivalent to $8\sin\left(\frac{x}{2}\right)-2x+5=0$. Let $\text{g}(x)=4\sin\left(\frac{x}{2}\right)+\frac{5}{2}$, then $\text{g}'(x)=2\cos\left(\frac{x}{2}\right)$ Since $4.5<\alpha<5$, we have $-1.6023<2\cos\left(\frac{\alpha}{2}\right)<-1.2563$ $\Rightarrow \lvert\text{g}'(\alpha)\rvert>1$ The sequence defined by the recurrence relation $x_{n+1}=4\sin\left(\frac{x_n}{2}\right)+\frac{5}{2}$ will not converge to α. Hence it is not suitable to find a good approximation to α. <u>Alternative:</u> Let $\text{g}(x)=4\sin(0.5x)+2.5$  From the sketch, the iterative method will spiral outwards away from the intersection. Hence it is not suitable to find a good approximation to α. **(c)** Newton-Raphson iterative formula: $x_{n+1}=x_n-\dfrac{8\sin\left(\frac{x_n}{2}\right)-2x_n+5}{4\cos\left(\frac{x_n}{2}\right)-2}$ Starting with $x_0=4.5$ and using GC, we have

x_0	4.5
x_1	4.99296
x_2	4.95916
x_3	4.95903
x_4	4.95903

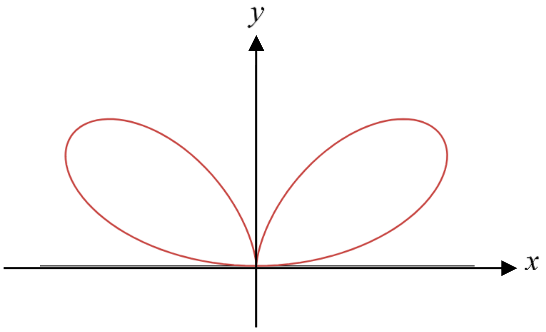
Check: $f(4.9585) = 0.00273 > 0$ and $f(4.9595) = -0.00243 < 0$
Hence $\alpha = 4.959$ (3 d.p.)

(d)

$$f'(x) = 4\cos\left(\frac{x}{2}\right) - 2 = 0 \quad \Rightarrow \quad \cos\left(\frac{x}{2}\right) = \frac{1}{2}$$

$\therefore x = \dfrac{2\pi}{3} \approx 2.0944$ is the x-coordinates of a stationary point.

Since $x_0 = 2$ is very close to the stationary point, the tangent to C at $x = 2$ is almost a horizontal line and thus will cut the x-axis far away from the initial approximation. So $x_0 = 2$ is not a suitable starting value.

	Solution
P1 Q5	**(a)** $x^4 + y^4 = 2x^2 y(1-y)$ $x^4 + 2x^2y^2 + y^4 = 2x^2 y$ $\left(x^2+y^2\right)^2 = 2x^2 y$ $r^4 = 2\left(r^2\cos^2\theta\right)\left(r\sin\theta\right)$ $r = \left(2\cos\theta\sin\theta\right)\left(\cos\theta\right) = \sin 2\theta\cos\theta$ (shown) <u>Alternative method</u> $x^4 + y^4 = 2x^2 y(1-y)$ $r^4\cos^4\theta + r^4\sin^4\theta = 2\left(r^2\cos^2\theta\right)\left(r\sin\theta\right)(1-r\sin\theta)$ $r^4\left[\left(\sin^2\theta+\cos^2\theta\right)^2 - 2\sin^2\theta\cos^2\theta\right] = 2r^3\cos^2\theta\sin\theta(1-r\sin\theta)$ $r\left(1-2\sin^2\theta\cos^2\theta\right) = 2\cos^2\theta\sin\theta - 2r\cos^2\theta\sin^2\theta$ $\therefore r = \left(2\cos\theta\sin\theta\right)\left(\cos\theta\right)$ $= \sin 2\theta\cos\theta$ (shown) **(b)**  Line of symmetry is $\theta = \frac{\pi}{2}$ or $x = 0$. **(c)** $\frac{\mathrm{d}r}{\mathrm{d}\theta} = 2\cos 2\theta\cos\theta - \sin 2\theta\sin\theta$ Length of curve C $= \int_0^{\pi}\sqrt{r^2+\left(\frac{\mathrm{d}r}{\mathrm{d}\theta}\right)^2}\,\mathrm{d}\theta$ $= \int_0^{\pi}\sqrt{\left(\sin^2 2\theta\cos^2\theta\right)+\left(2\cos 2\theta\cos\theta - \sin 2\theta\sin\theta\right)^2}\,\mathrm{d}\theta$ $= 3.58$ units (3 s.f.)

	Solution
P1 Q6	**(a)** $u = y^{1-n} \Rightarrow y = u^{\frac{1}{1-n}} \quad \Rightarrow \quad \frac{dy}{dx} = \frac{u^{\frac{1}{1-n}-1}}{1-n}\frac{du}{dx} = \frac{u^{\frac{n}{1-n}}}{1-n}\frac{du}{dx}$ $\frac{dy}{dx} + p(x)y = q(x)y^n \quad \Rightarrow \quad \frac{u^{\frac{n}{1-n}}}{1-n}\frac{du}{dx} + p(x)u^{\frac{1}{1-n}} = q(x)u^{\frac{n}{1-n}}$ Dividing both sides of the DE by $u^{\frac{n}{1-n}}$, $\frac{1}{1-n}\frac{du}{dx} + p(x)u = q(x)$ $\frac{du}{dx} + (1-n)p(x)u = (1-n)q(x)$ (shown) <u>Alternative:</u> $u = y^{1-n} \Rightarrow \frac{du}{dx} = (1-n)y^{-n}\frac{dy}{dx}$ $\frac{dy}{dx} + p(x)y = q(x)y^n$ Multiplying $(1-n)y^{-n}$ to both sides of the DE, $(1-n)y^{-n}\frac{dy}{dx} + p(x)(1-n)y^{1-n} = (1-n)q(x)$ $\frac{du}{dx} + (1-n)p(x)u = (1-n)q(x)$ (shown) **(b)** $\frac{dy}{dx} - xy + \frac{1}{2}xy^2 = 0, \Rightarrow \frac{dy}{dx} + (-x)y = \left(-\frac{1}{2}x\right)y^2$. Substituting $p(x) = -x$, $q(x) = -\frac{1}{2}x$ and $n = 2$ in part **(a)**, $\frac{du}{dx} + (-1)(-x)u = (-1)\left(-\frac{1}{2}x\right)$ $\frac{du}{dx} + ux = \frac{x}{2}$ Integrating factor $= e^{\int x\,dx} = e^{\frac{x^2}{2}}$ $e^{\frac{x^2}{2}}\frac{du}{dx} + xe^{\frac{x^2}{2}}u = \frac{x}{2}e^{\frac{x^2}{2}}$ $e^{\frac{x^2}{2}}u = \int \frac{x}{2}e^{\frac{x^2}{2}}\,dx = \frac{1}{2}e^{\frac{x^2}{2}} + C$ $u = \frac{1}{2} + Ce^{-\frac{x^2}{2}}$ $y = u^{\frac{1}{1-2}} = \frac{1}{u} = \frac{1}{\frac{1}{2} + Ce^{-\frac{x^2}{2}}} = \frac{2}{1 + Ae^{-\frac{x^2}{2}}}$, where $A = 2C$. Sub $x = 0$, $y = 0.1$: $0.1 = \frac{2}{1+A} \Rightarrow 1 + A = 20 \Rightarrow A = 19$. $\therefore y = \frac{2}{1+19e^{-\frac{x^2}{2}}}$.

	When $x=0.1$, $y=\dfrac{2}{1+19\mathrm{e}^{-\frac{0.1^2}{2}}}=0.100476=0.1005$ (to 4 d.p.) When $x=0.2$, $y=\dfrac{2}{1+19\mathrm{e}^{-\frac{0.2^2}{2}}}=0.101917=0.1019$ (to 4 d.p.) **(c)** $\tilde{y}_1=0.1+0(0.1\ln 1.1)=0.1$ $y_1=0.1+\dfrac{0.1}{2}\left(0(\ln 1.1)+0.1\ln 1.1\right)=0.100477$ When $x=0.1$, $y\approx 0.1005$ (to 4 d.p.). $\tilde{y}_2=0.100477+0.1(0.1\ln 1.100477)=0.101434$ $y_2=0.100477+\dfrac{0.1}{2}(0.1\ln 0.100477+0.2\ln 1.101434)=0.101921$ When $x=0.2$, $y\approx 0.1019$ (to 4 d.p.) **(d)** The first two terms of the Maclaurin series of $\ln(1+y)$ is $y-\frac{1}{2}y^2$, which is close in value to $\ln(1+y)$ for small values of y. Thus, the values found in part **(c)** are close to the values found in part **(b)**.

	Solution
P1 Q7	**(a)**

$$\frac{dx}{dt}=\frac{1}{20}x^2\left(1-\frac{x^2}{100}\right),$$

$$\int \frac{100}{x^2\left(100-x^2\right)}\,dx=\int\frac{1}{20}\,dt$$

$$\int \frac{1}{x^2}+\frac{1}{100-x^2}\,dx=\frac{1}{20}t+c$$

$$-\frac{1}{x}+\frac{1}{20}\ln\left(\frac{10+x}{10-x}\right)=\frac{1}{20}t+c \quad (\because 0<x<10)$$

$$t=\ln\left(\frac{10+x}{10-x}\right)-\frac{20}{x}+A,\ \text{ where } A=-20c.$$

Sub $t=0$, $x=2$:

$$0=\ln\frac{12}{8}-\frac{20}{2}+A$$

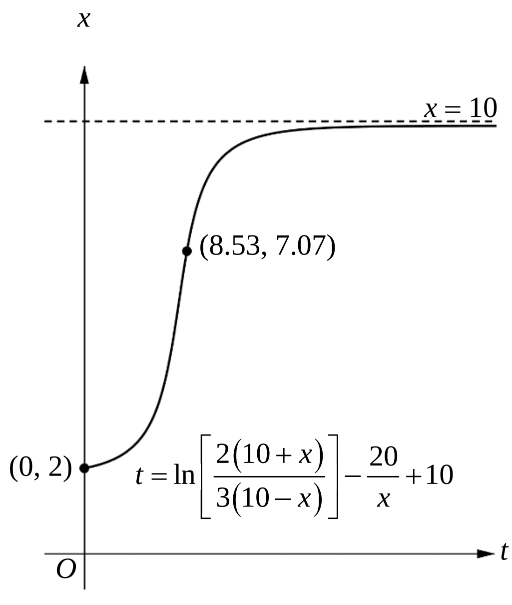
$$A=10-\ln\frac{3}{2}.$$

Thus,

$$t=\ln\left(\frac{10+x}{10-x}\right)-\frac{20}{x}+10-\ln\frac{3}{2}$$

$$t=\ln\left[\frac{2(10+x)}{3(10-x)}\right]-\frac{20}{x}+10.$$

(b)

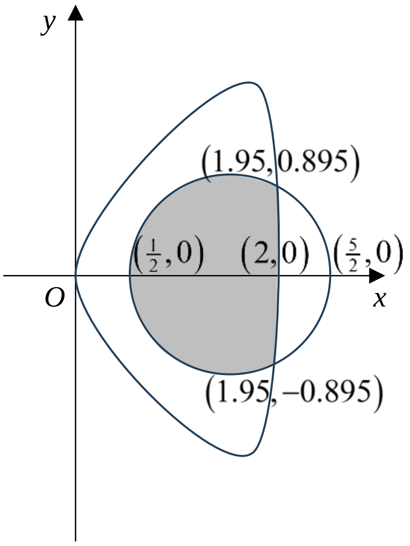


The point of inflexion corresponds to the instant when the rate of infection is the highest.

(c) Possible answers:

- Recovery rate of infected individuals
- Rate of infected individuals leaving the city
- Fatality rate of infected individuals

	Solution
P1 Q8	**(a)** The curve is symmetrical about the x-axis. When $y=0$, $\sin 2t=0 \Rightarrow t=0,\pm\frac{\pi}{2}$ Observe that $t=-\frac{\pi}{2}$ and $t=\frac{\pi}{2}$ correspond to the point $(0,0)$. Similarly, $t=0$ corresponds to the point $(2,0)$. Consider the segment of the curve C_1 in the 4th quadrant from $x=0$ to $x=2$. The corresponding range of t is from $t=-\frac{\pi}{2}$ to $t=0$ (this range will ensure that both x and $\frac{\mathrm{d}x}{\mathrm{d}t}$ are positive so that the definite integral will also be positive). $x=2\cos t \Rightarrow \frac{\mathrm{d}x}{\mathrm{d}t}=-2\sin t$, $\quad y=2\sin 2t \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}t}=4\cos 2t$ Area S $=2\times 2\pi\int_{-\frac{\pi}{2}}^{0} x\sqrt{\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2+\left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2}\ \mathrm{d}t$ (by symmetry) $=4\pi\int_{-\frac{\pi}{2}}^{0}(2\cos t)\sqrt{4\sin^2 t+16\cos^2 2t}\ \mathrm{d}t$ $=16\pi\int_{-\frac{\pi}{2}}^{0}(\cos t)\sqrt{\sin^2 t+4\cos^2 2t}\ \mathrm{d}t$ $=70.8822=70.9$ units^2 (3 s.f.) <u>Alternative:</u> Area $S=2\pi\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x\sqrt{\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2+\left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2}\ \mathrm{d}t$ $=2\pi\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}(2\cos t)\sqrt{4\sin^2 t+16\cos^2 2t}\ \mathrm{d}t$ $=8\pi\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}(\cos t)\sqrt{\sin^2 t+4\cos^2 2t}\ \mathrm{d}t$ $=70.8822=70.9$ units^2 (3 s.f.) **(b)** Sub. $x=2\cos t$ and $y=2\sin 2t$ into the equation $\left(x-\frac{3}{2}\right)^2+y^2=1$: $\left(2\cos t-\frac{3}{2}\right)^2+(2\sin 2t)^2=1$ Using GC, $t=\pm 0.231931$ since $-\frac{1}{2}\pi\le t\le\frac{1}{2}\pi$. When $t=0.231931$, $x=1.94645$ and $y=0.89481$ When $t=-0.231931$, $x=1.94645$ and $y=-0.89481$ (Can also be deduced by symmetry) Hence the coordinates are $(1.95, 0.895)$ and $(1.95, -0.895)$ (3 s.f.) **(c)** In the diagram, R is represented by the shaded region.



Equation of semicircle from $\left(x-\frac{3}{2}\right)^2+y^2=1 \Rightarrow y=\sqrt{1-\left(x-\frac{3}{2}\right)^2}$

By symmetry, volume of solid

$$=2\times\left[\underbrace{2\pi\int_{0.5}^{1.94645} xy\ \mathrm{d}x}_{C_2}+\underbrace{2\pi\int_{1.94645}^{2} xy\ \mathrm{d}x}_{C_1}\right]=4\pi\times\left[\int_{0.5}^{1.94645} x\sqrt{1-\left(x-\frac{3}{2}\right)^2}\ \mathrm{d}x+\int_{0.231931}^{0} xy\frac{\mathrm{d}x}{\mathrm{d}t}\ \mathrm{d}t\right]$$

$$=4\pi\times\left[\int_{0.5}^{1.94645} x\sqrt{1-\left(x-\frac{3}{2}\right)^2}\ \mathrm{d}x+\int_{0.231931}^{0}(2\cos t)(2\sin 2t)(-2\sin t)\ \mathrm{d}t\right]$$

$=20.731$

$=20.7 \quad (3\text{ sf})$

Alternative (disc) method for 8(iii)

Equation of circle: $\left(x-\frac{3}{2}\right)^2+y^2=1 \Rightarrow x=\frac{3}{2}\pm\sqrt{1-y^2}$

Volume of solid

$$=2\times\left[\underbrace{\pi\int_{0}^{0.89481} x^2\ \mathrm{d}y}_{C_1}+\underbrace{\pi\int_{0.89481}^{1} x^2\ \mathrm{d}y}_{C_2\text{ for }x\geq\frac{3}{2}}-\underbrace{\pi\int_{0}^{1} x^2\ \mathrm{d}y}_{C_2\text{ for }x\leq\frac{3}{2}}\right]$$

$$=2\pi\times\left[\int_{0}^{0.231931} x^2\frac{\mathrm{d}y}{\mathrm{d}t}\ \mathrm{d}t+\int_{0.89481}^{1}\left(\frac{3}{2}+\sqrt{1-y^2}\right)^2\ \mathrm{d}y-\int_{0}^{1}\left(\frac{3}{2}+\sqrt{1-y^2}\right)^2\ \mathrm{d}y\right]$$

$$=2\pi\times\left[\int_{0}^{0.231931}(2\cos t)^2(4\cos 2t)\ \mathrm{d}t+\int_{0.89481}^{1}\left(\frac{3}{2}+\sqrt{1-y^2}\right)^2\ \mathrm{d}y-\int_{0}^{1}\left(\frac{3}{2}+\sqrt{1-y^2}\right)^2\ \mathrm{d}y\right]$$

$=20.731$

$=20.7 \quad (3\text{ sf})$

	Solution
P1 Q9	**(a)** Since the recurrence relation fits the form $\frac{P_{n+1}}{P_n}=1+k$, there is a constant common ratio of $(1+k)$. This model predicts that the population grows infinitely as $n\to\infty$, which is not practical. **(b)(i)** When $P_n>b$, $\left(1-\frac{P_n}{b}\right)<0\Rightarrow P_{n+1}-P_n<0$. So the population should decrease. **(b)(ii)** When $0<P_n<b$, $\left(1-\frac{P_n}{b}\right)>0\Rightarrow P_{n+1}-P_n>0$. So the population should increase. **(c)** For $\Delta P=a\left(1-\frac{P_n}{b}\right)P_n=0$, $P_n=0$ or b. The possible limits are 0 and *b*. The constant *b* represents the carrying capacity of the environment for the population of the cockroaches. **(d)** Auxiliary equation is $\lambda^2-\sqrt{2}\lambda+1=0$. Solving, $\lambda=\frac{1}{\sqrt{2}}\pm\frac{1}{\sqrt{2}}\mathrm{i}=\mathrm{e}^{\pm\mathrm{i}\frac{\pi}{4}}$, where $r=1,\theta=\frac{\pi}{4}$. Then a general solution is $Q_n=\left(A\cos\frac{n\pi}{4}+B\sin\frac{n\pi}{4}\right)$. This model is not suitable as Q_n can be negative for certain values of *n*. [Specifically, $Q_0=A>0$ and $Q_4=-A<0$.] **(e)** Auxiliary equation is $\lambda^2-1.9\lambda+0.9=0$. Solving, $\lambda=0.9$ or 1. Then a general solution is $Q_n=C(0.9)^n+D(1)^n=C(0.9)^n+D$. **(f)** [Since $0<(0.9)^n\le 1$ for all $n\in\mathbb{Z}_0^+$, so Q_n lies between $C+D$ and *D*. Since $Q_0=C+D$ is already given to be positive, we just need $D\ge 0$.] At $n=0$, $Q_0=50=C+D$. At $n=1$, $Q_1=0.9C+D$. Solving for *D*, $D=10Q_1-450$. For $D\ge 0$, $Q_1\ge 45$. <u>Alternatively:</u> Solving for Q_1, $Q_1=0.9(C+D)+0.1D=45+0.1D$. For $D\ge 0$, $Q_1\ge 45$.

<table>
<tr><td></td><td>Solution</td></tr>
<tr><td>P1
Q10</td><td>

(a) α is the number of units of grain required to produce one unit of grain

β is the number of units of grain required to produce one unit of cattle

(b)(i) $\mathbf{I}-\mathbf{P}=\begin{pmatrix}-0.02 & -0.04\\ -0.01 & 0.94\end{pmatrix}$

$(\mathbf{I}-\mathbf{P})^{-1}=\begin{pmatrix}-48.958 & -2.0833\\ -0.52083 & 1.0417\end{pmatrix}$

$\begin{pmatrix}g\\ c\end{pmatrix}=\mathbf{P}\begin{pmatrix}g\\ c\end{pmatrix}+\begin{pmatrix}300\\ 100\end{pmatrix} \Rightarrow (\mathbf{I}-\mathbf{P})\begin{pmatrix}g\\ c\end{pmatrix}=\begin{pmatrix}300\\ 100\end{pmatrix}$

$\begin{pmatrix}g\\ c\end{pmatrix}=(\mathbf{I}-\mathbf{P})^{-1}\begin{pmatrix}300\\ 100\end{pmatrix}$

$=\begin{pmatrix}-14896\\ -52.083\end{pmatrix}(5\text{ sf})=\begin{pmatrix}-14900\\ -52.1\end{pmatrix}(3\text{ sf})$

(b)(ii) $\mathbf{I}-\mathbf{P}=\begin{pmatrix}0.96 & -1.02\\ -0.01 & 0.94\end{pmatrix}$

$(\mathbf{I}-\mathbf{P})^{-1}=\begin{pmatrix}1.0536 & 1.1432\\ 0.011208 & 1.0760\end{pmatrix}$

$\begin{pmatrix}g\\ c\end{pmatrix}=(\mathbf{I}-\mathbf{P})^{-1}\begin{pmatrix}300\\ 100\end{pmatrix}$

$=\begin{pmatrix}430.40\\ 110.96\end{pmatrix}(5\text{sf})=\begin{pmatrix}430\\ 111\end{pmatrix}(3\text{sf})$

(c) The economy will not be able to cope with the demand for external consumption in the first case where $\alpha=1.02$, $\beta=0.04$, as the negative solutions for g and c are not feasible.

The economy will be able to cope with the demand for external consumption in the second case where $\alpha=0.04$, $\beta=1.02$, as there are feasible positive solutions for g and c.

(d) As α is the number of units of grain required to produce one unit of grain, $\alpha<1$ is necessary for production of grain to be sustainable.

On the other hand, $\beta<1$ is not necessary as it refers to the number of units of grain required to produce one unit of cattle, and it is possible that more grain can be produced to contribute to the production of cattle.

(e) The equation becomes $(\mathbf{I}-\mathbf{P})\begin{pmatrix}g\\ c\end{pmatrix}=\begin{pmatrix}0\\ 0\end{pmatrix}$.

For non-zero solutions, $\det(\mathbf{P}-\mathbf{I})=0$

$\det\begin{pmatrix}\alpha-1 & \alpha\\ 0.01 & -0.94\end{pmatrix}=0$

$-0.94\alpha+0.94-0.01\alpha=0$

$\alpha=\frac{94}{95}\approx 0.989\ (3\text{ s.f.})$

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