JC1 H2 Mathematics (9758)
Term 4 Revision Topical Quick Check
Chapter 10 Integration Techniques

1 **HCI Promo 9758/2022/Q8**

(a) Find $\int 3t \tan^{-1}(3t)\,\mathrm{d}t$. [4]

(b) Using the substitution $u = x^2 + 1$, show that $\int_0^{\sqrt{7}} x^3 \left(x^2+1\right)^{\frac{1}{3}}\,\mathrm{d}x$ can be expressed as

$$\frac{1}{2}\int_a^b u^{\frac{4}{3}} - u^{\frac{1}{3}}\,\mathrm{d}u,$$

where a and b are constants to be determined.

Hence find the exact value of $\int_0^{\sqrt{7}} x^3 \left(x^2+1\right)^{\frac{1}{3}}\,\mathrm{d}x$. [5]

1 HCI Promo 9758/2022/Q8

(a) Find $\int 3t\tan^{-1}(3t)\,\mathrm{d}t$. [4]

1 (a)	$\int 3t\tan^{-1}(3t)\,\mathrm{d}t$ $=\frac{3t^2}{2}\tan^{-1}(3t)-\int\frac{3t^2}{2}\frac{3}{1+(3t)^2}\,\mathrm{d}t$ $=\frac{3t^2}{2}\tan^{-1}(3t)-\frac{9}{2}\int\frac{t^2}{1+9t^2}\,\mathrm{d}t$ $=\frac{t^2}{2}\tan^{-1}(3t)-\frac{1}{2}\int 1-\frac{1}{(1+9t^2)}\,\mathrm{d}t$ $=\frac{3t^2}{2}\tan^{-1}(3t)-\frac{1}{2}t+\frac{1}{2(3)}\tan^{-1}(3t)+C$ $=\frac{3}{2}t^2\tan^{-1}(3t)-\frac{1}{2}t+\frac{1}{6}\tan^{-1}(3t)+C$

(b) Using the substitution $u=x^2+1$, show that $\int_0^{\sqrt{7}} x^3\left(x^2+1\right)^{\frac{1}{3}}\,\mathrm{d}x$ can be expressed as

$$\frac{1}{2}\int_a^b u^{\frac{4}{3}}-u^{\frac{1}{3}}\,\mathrm{d}u,$$

where a and b are constants to be determined.

Hence find the exact value of $\int_0^{\sqrt{7}} x^3\left(x^2+1\right)^{\frac{1}{3}}\,\mathrm{d}x$. [5]

(b)	Let $u=x^2+1$, then $\frac{\mathrm{d}u}{\mathrm{d}x}=2x$. When $x=0,\ u=1$. When $x=\sqrt{7},\ u=8$. $\int_0^{\sqrt{7}} x^3\left(x^2+1\right)^{\frac{1}{3}}\,\mathrm{d}x=\frac{1}{2}\int_0^{\sqrt{7}} x^2\left(x^2+1\right)^{\frac{1}{3}}(2x)\,\mathrm{d}x$ $=\frac{1}{2}\int_1^8 (u-1)(u)^{\frac{1}{3}}\,\mathrm{d}u$ $=\frac{1}{2}\int_1^8 u^{\frac{4}{3}}-u^{\frac{1}{3}}\,\mathrm{d}u$ (Shown) $=\frac{1}{2}\left[\frac{3}{7}u^{\frac{7}{3}}-\frac{3}{4}u^{\frac{4}{3}}\right]_1^8$ $=\frac{3}{2}\left[\frac{1}{7}(2)^7-\frac{1}{4}(2)^4-\frac{1}{7}+\frac{1}{4}\right]$ $=\frac{1209}{56}$

2 EJC Promo 9758/2022/Q6

(a) Find $\int x\mathrm{e}^{3x^2+1}\,\mathrm{d}x$. [1]

(b) Find $\int \sin^2(5x)\,\mathrm{d}x$. [3]

(c) Find $\int \frac{x}{4x^2-4x+17}\,\mathrm{d}x$. [5]

2 EJC Promo 9758/2022/Q6

(a) Find $\int x\mathrm{e}^{3x^2+1}\,\mathrm{d}x$. [1]

2 (a)	$\int x\mathrm{e}^{3x^2+1}\,\mathrm{d}x = \frac{1}{6}\int (6x)\mathrm{e}^{3x^2+1}\,\mathrm{d}x$ $= \frac{1}{6}\mathrm{e}^{3x^2+1} + c$

(b) Find $\int \sin^2(5x)\,\mathrm{d}x$. [3]

(b)	$\int \sin^2(5x)\,\mathrm{d}x = \int \frac{1-\cos 10x}{2}\,\mathrm{d}x$ $= \frac{1}{2}x - \frac{1}{2}\left(\frac{\sin 10x}{10}\right) + C$ $= \frac{1}{2}x - \frac{1}{20}\sin 10x + C$

(c) Find $\int \frac{x}{4x^2-4x+17}\,\mathrm{d}x$. [5]

(c)	$\int \frac{x}{4x^2-4x+17}\,\mathrm{d}x = \int \frac{\frac{1}{8}(8x-4)+\frac{1}{2}}{4x^2-4x+17}\,\mathrm{d}x$ $= \frac{1}{8}\int \frac{8x-4}{4x^2-4x+17}\,\mathrm{d}x + \frac{1}{2}\int \frac{1}{4x^2-4x+17}\,\mathrm{d}x$ $= \frac{1}{8}\ln\left(4x^2-4x+17\right) + \frac{1}{2}\int \frac{1}{(2x-1)^2+4^2}\,\mathrm{d}x$ $= \frac{1}{8}\ln\left(4x^2-4x+17\right) + \frac{1}{2}\left(\frac{1}{2}\cdot\frac{1}{4}\tan^{-1}\left(\frac{2x-1}{4}\right)\right) + C$ $= \frac{1}{8}\ln\left(4x^2-4x+17\right) + \frac{1}{16}\tan^{-1}\left(\frac{2x-1}{4}\right) + C$

3 MI PU2 P1 Promo 9758/2022/Q4

(i) Find $\int \cos 2x \sin x \, \mathrm{d}x$. [3]

(ii) Find $\int \frac{\mathrm{e}^{\sin^{-1} 2x}}{\sqrt{1-4x^2}} \, \mathrm{d}x$. [2]

(iii) Find $\int \frac{5}{x^2+6x+13} \, \mathrm{d}x$. [3]

3(i)	$\int \cos 2x \sin x \, \mathrm{d}x = \int \left(2\cos^2 x - 1\right)\sin x \, \mathrm{d}x$ $= \int \left(2\sin x \cos^2 x - \sin x\right)\mathrm{d}x$ $= -\frac{2}{3}\cos^3 x + \cos x + C$ **Alternative Method** $\int \cos 2x \sin x \, \mathrm{d}x = \int \frac{1}{2}\left(\sin 3x - \sin x\right)\mathrm{d}x$ $= \frac{1}{2}\left(\frac{-\cos 3x}{3} + \cos x\right) + c$ $= -\frac{1}{6}\cos 3x + \frac{1}{2}\cos x + c$
3(ii)	$\int \frac{1}{\sqrt{1-4x^2}} \mathrm{e}^{\sin^{-1} 2x} \, \mathrm{d}x = \frac{1}{2}\int \frac{2}{\sqrt{1-(2x)^2}} \mathrm{e}^{\sin^{-1} 2x} \mathrm{d}x$ $= \frac{1}{2}\mathrm{e}^{\sin^{-1} 2x} + c$
3(iii)	$\int \frac{5}{x^2+6x+13} \mathrm{d}x = \int \frac{5}{(x+3)^2 - 3^2 + 13} \mathrm{d}x$ $= \int \frac{5}{(x+3)^2 + 2^2} \mathrm{d}x$ $= \frac{5}{2}\tan^{-1}\left(\frac{x+3}{2}\right) + c$

Answer Key

No.	Year	JC	Answers
1	2022	HCI	(a) $\frac{3}{2}t^2\tan^{-1}(3t)-\frac{1}{2}t+\frac{1}{6}\tan^{-1}(3t)+C$ (b) $\frac{1209}{56}$
2	2022	EJC	(a) $\frac{1}{6}e^{3x^2+1}+c$ (b) $\frac{1}{2}x-\frac{1}{20}\sin 10x+c$ (c) $\frac{1}{8}\ln\left(4x^2-4x+17\right)+\frac{1}{16}\tan^{-1}\left(\frac{2x-1}{4}\right)+c$
3	2022	MI	(i) $-\frac{2}{3}\cos^3 x+\cos x+C$ $\left(\text{or } -\frac{1}{6}\cos 3x+\frac{1}{2}\cos x+c\right)$ (ii) $\frac{1}{2}e^{\sin^{-1}2x}+c$ (iii) $\frac{5}{2}\tan^{-1}\left(\frac{x+3}{2}\right)+c$