2024 4E5N AM Prelim P2 MS

Qn	Solution
1a	$$\begin{array}{r} 3x-1 \\ x^2+1\overline{)3x^3-x^2+4x+2} \\ 3x^3 \qquad +3x \\ -x^2+x+2 \\ -x^2 \qquad -1 \\ x+3 \end{array}$$
1b	$(-2)^3+a(-2)=1+a$ $-8-2a=1+a$ $-9=3a$ $a=-3$
2	$\frac{19-3\sqrt{5}}{2+2\sqrt{5}}$ $=\frac{19-3\sqrt{5}}{2+2\sqrt{5}}\times\frac{2-2\sqrt{5}}{2-2\sqrt{5}}$ $=\frac{38-38\sqrt{5}-6\sqrt{5}+30}{4-4(5)}$ $=\frac{68-44\sqrt{5}}{-16}$ $=-\frac{17}{4}+\frac{11}{4}\sqrt{5}$
3i	$T_{r+1}=\binom{9}{r}\left(x^2\right)^{9-r}\left(-\frac{2}{x}\right)^r$ $=\binom{9}{r}(-2)^r x^{18-3r}$ $0=18-3r$ $r=6$ $T_7=5376$ $-3=18-3r$ $r=7$ $T_8=-4608\left(\frac{1}{x^3}\right)$
3ii	Term indep of $x=3(5376)+(-1)(-4608)$ $=20736$

4i	$\tan A = \frac{3}{4}$ $\tan B = \frac{15}{8}$ $\tan(A+B) = \frac{\frac{3}{4}+\frac{15}{8}}{1-\frac{3}{4}\left(\frac{15}{8}\right)}$ $= -\frac{84}{13}$ or $-6\frac{6}{13}$
4ii	$\cos B = 2\cos^2\frac{B}{2} - 1$ $\cos^2\frac{B}{2} = \frac{1}{2}(\cos B + 1)$ $= \frac{1}{2}\left(-\frac{8}{17}+1\right)$ $= \frac{9}{34}$ $\cos\frac{B}{2} = -\frac{3}{\sqrt{34}}$ $180^\circ < B < 270^\circ$ $90^\circ < \frac{B}{2} < 135^\circ$
5a	$y = \frac{2x-3}{3x+4}$ $\frac{dy}{dx} = \frac{(3x+4)2-(2x-3)3}{(3x+4)^2}$ $= \frac{17}{(3x+4)^2}$ $\frac{17}{25} = \frac{17}{(3x+4)^2}$ $(3x+4)^2 = 25$ $x = \frac{1}{3}$ $y = -\frac{7}{15}$ $P(\frac{1}{3}, -\frac{7}{15})$

5b	$\frac{dy}{dx} = 4x^2 - 2x + 3$ $= 4\left(x^2 - \frac{x}{2}\right) + 3$ $= 4\left(\left(x - \frac{1}{4}\right)^2 - \left(\frac{1}{4}\right)^2\right) + 3$ $= 4\left(x - \frac{1}{4}\right)^2 + \frac{11}{4}$ $\left(x - \frac{1}{4}\right)^2 \geq 0$ $4\left(x - \frac{1}{4}\right)^2 + \frac{11}{4} \geq \frac{11}{4}$ > 0 $\frac{dy}{dx} > 0$ Therefore, y is always increasing for all real values of x
6i	$12 - 2x = 2x^2 - 6x - 4$ $0 = 2x^2 - 4x - 16$ $0 = x^2 - 2x - 8$ $0 = (x+2)(x-4)$ $x = -2$ or 4 $y = 16$ or 4 (−2, 16) and (4, 4)
6ii	midpt $= \left(\frac{-2+4}{2}, \frac{16+4}{2}\right)$ $= (1, 10)$ grad $= \frac{16-4}{-2-4}$ $= -2$ grad of perpen bisector $= \frac{1}{2}$ eqn of perpen bisector: $y - 10 = \frac{1}{2}(x - 1)$ $y = \frac{1}{2}x + \frac{19}{2}$

7a	$5x-3<2x(5-x)$ $5x-3<10x-2x^2$ $2x^2-5x-3<0$ $(2x+1)(x-3)<0$ $-\frac{1}{2}<x<3$
7b	$y=a(x-50)^2+10$ $40=a(-50)^2+10$ $a=\frac{3}{250}$ $y=\frac{3}{250}(x-50)^2+10$
8i	<table><tr><td>$x=0.9$</td><td>$x=1$</td><td>$x=1.1$</td></tr><tr><td>$\frac{dy}{dx}<0$</td><td>$\frac{dy}{dx}=0$</td><td>$\frac{dy}{dx}<0$</td></tr></table>*A* is a point of inflexion <table><tr><td>$x=3.9$</td><td>$x=4$</td><td>$x=4.1$</td></tr><tr><td>$\frac{dy}{dx}<0$</td><td>$\frac{dy}{dx}=0$</td><td>$\frac{dy}{dx}>0$</td></tr></table>*B* is a minimum point

8ii	$\frac{dy}{dx} = a(x-1)^2(x-4)$ $= a(x^2 - 2x + 1)(x-4)$ $= a(x^3 - 6x^2 + 9x - 4)$ $y = a(\frac{1}{4}x^4 - 2x^3 + \frac{9}{2}x^2 - 4x) + c$ $0 = a(64 - 128 + 72 - 16) + c$ $8a = c$ $5 = a(\frac{1}{4} - 2 + \frac{9}{2} - 4) + c$ $5 = -\frac{5}{4}a + 8a$ $a = \frac{20}{27}$ $c = \frac{160}{27}$ $y = \frac{20}{27}(\frac{1}{4}x^4 - 2x^3 + \frac{7}{2}x^2 - 4x) + \frac{160}{27}$ ⟶ $= \frac{5}{27}x^4 - \frac{40}{27}x^3 + \frac{70}{27}x^2 - \frac{80}{27}x + \frac{160}{29}$
9i	$\frac{12-h}{12} = \frac{r}{5}$ $12 - h = \frac{12r}{5}$ $h = 12 - \frac{12r}{5}$
9ii	$V = \pi r^2 h$ $= \pi r^2(12 - \frac{12r}{5})$ $= 12\pi r^2 - \frac{12}{5}\pi r^3$

9iii	$V = 12\pi r^2 - \frac{12}{5}\pi r^3$ $\frac{dV}{dr} = 24\pi r - \frac{36}{5}\pi r^2$ $0 = 24\pi r - \frac{36}{5}\pi r^2$ $0 = 12\pi r(2 - \frac{3}{5}r)$ $r = 0$ (rej) or $\frac{10}{3}$ $\frac{d^2V}{dr^2} = 24\pi - \frac{72}{5}\pi r$ $\left.\frac{d^2V}{dr^2}\right\|_{r=\frac{10}{3}} = -24\pi$ < 0 $\therefore V$ is max when $r = \frac{10}{3}$ $V = \frac{400}{9}\pi$ or $139.62\ldots\ldots$
10a	$2\sin 2x + 3\cos x = 0$ $4\sin x\cos x + 3\cos x = 0$ $\cos x(4\sin x + 3) = 0$ $\cos x = 0$ or $\sin x = -\frac{3}{4}$ PV of $x = 90^\circ$ or -48.6°
10bi	$a = 4$ $c = 5$ $\pi = \frac{2\pi}{b}$ $b = 2$
10bii	$m = k + \pi$
10biii	$\frac{k+l}{2} = \frac{\pi}{4}$ or $k + l = \frac{\pi}{2}$ etc
11ai	The centre of C_1 looks like $(-r, r)$

11aii	$(-r+8)^2+(r-1)^2=r^2$ $r^2-16r+64+r^2-2r+1=r^2$ $r^2-18r+65=0$ $(r-5)(r-13)=0$ $r=5$ or 13 (rej) $(x+5)^2+(y-5)^2=25$
11bi	P (7, 10) $r=\sqrt{7^2+10^2-113}$ $=6$
11bii	$\angle QSR$ max $\Rightarrow$ *SQ* and *SR* are tangents to circle $\Rightarrow$ *SQ* = *SR* since they met at an external point
12i	$v=\int -e^{-0.3t}dt$ $=\frac{-e^{-0.3t}}{-0.3}+c$ $=\frac{10}{3}e^{-0.3t}+c$ $3=\frac{10}{3}e^{0}+c$ $c=-\frac{1}{3}$ $v=\frac{10}{3}e^{-0.3t}-\frac{1}{3}$ $0=\frac{10}{3}e^{-0.3t}-\frac{1}{3}$ $\frac{1}{10}=e^{-0.3t}$ $\ln\frac{1}{10}=-0.3t$ $t=-\frac{10}{3}\ln\frac{1}{10}$ $=\frac{10}{3}\ln 10$ }

12ii	$v = \frac{10}{3}e^{-0.3t} - \frac{1}{3}$ $s = \int \frac{10}{3}e^{-0.3t} - \frac{1}{3}dt$ $= -\frac{100}{9}e^{-0.3t} - \frac{1}{3}t + c$ $0 = -\frac{100}{9}e^{0} - \frac{1}{3}(0) + c$ $c = \frac{100}{9}$ $s = -\frac{100}{9}e^{-0.3t} - \frac{1}{3}t + \frac{100}{9}$ $t = \frac{10}{3}\ln 10$ $s = -\frac{100}{9}e^{-\ln 10} - \frac{1}{3}\left(\frac{10}{3}\ln 10\right) + \frac{100}{9}$ $= 10 - \frac{10}{9}\ln 10$ $= 7.44$
12iii	When $t = 33$, $s = 0.111$ When $t = 34$, $s = -0.223$ Since displacement changes sign from $t = 33$ to $t = 34$, the particle is again at O during the 34^{th} second.