Name:	Index No.:	Class:

PRESBYTERIAN HIGH SCHOOL

ADDITIONAL MATHEMATICS **4049/02**
Paper 2

26 August 2021 Thursday 2 hrs 15 min

2021 SECONDARY FOUR EXPRESS PRELIMINARY EXAMINATIONS

INSTRUCTIONS TO CANDIDATES

DO NOT OPEN THIS QUESTION PAPER UNTIL YOU ARE TOLD TO DO SO.

Write your name, index number and class on the spaces provided above.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
Write your answers on the spaces provided below each question.
Give non exact numerical answers correct to 3 significant figures or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 90.

Qn	1	2	3	4	5	6	7	8	9	10	11	*Marks Deducted*
Marks												

Category	Accuracy	Units	Symbols	Others
Question No.				

Total Marks

90

Setter: Mr Tan Chee Wee
Vetter: Mrs Yim Meng Choo

This question paper consists of **18** printed pages (including this cover page) and **0** blank page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \square + \binom{n}{r}a^{n-r}b^r + \square + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!\,r!} = \frac{n(n-1).....(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1.$$

$$\sec^2 A = 1 + \tan^2 A.$$

$$\cos ec^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \square \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \square \tan A \tan B}$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

$$a^2 = b^2 + c^2 - 2bc\cos A.$$

$$\Delta = \frac{1}{2}ab\sin C.$$

Answer all questions in the space provided.

1 The equation of a curve is $y = x\left(4 + x^2\right)^3$.

(i) Find an expression for $\frac{dy}{dx}$. [3]

(ii) Explain whether y is an increasing or decreasing function. [2]

4 The equation of the curve is $y = (a+3)x^2 + ax + 1$, where a is a constant.

(a) When $a = 4$, find the set of values of x for which $y - 4 > 0$. [3]

(b) **(i)** Find the range of values of a for which the curve has no real roots. [3]

(ii) Hence explain why the curve cannot lies completely below the x-axis. [1]

3 The expression $f(x) = x^3 + ax + b$, where *a* and *b* are constants, is exactly divisible by $x - 2$ and leaves a remainder of 30 when divided by $x - 3$.

(i) Find the value of *a* and of *b*. [4]

(ii) Determine by showing all necessary working, the number of real root(s) of the equation $f(x) = 0$. [4]

4 **(i)** Prove the identity $\frac{1}{\sec A+1}+\frac{1}{\sec A-1}=2\operatorname{cosec} A\cot A$. [4]

(ii) Hence find all the angles between 0 and 2π for which $\frac{1}{\sec A+1}+\frac{1}{\sec A-1}=\frac{4}{3}$. [5]

5 It is given that $f(x)$ is such that $f'(x) = \cos 2x + \sin 3x$.

Given that $f(\pi) = 0$, show that $f''(x) + 9f(x) = a + b\sin 2x$ where *a* and *b* are constants. [7]

6 The table below shows the experimental values of two variables x and y.

x	1	2	3	4	5	6
y	63	127	258	510	1000	2100

It is known that x and y are related by an equation of the form $y = \frac{b^x}{10^a}$, where a and b are constants.

(i) By plotting $\lg y$ against x, obtain a straight line graph to represent the above data.

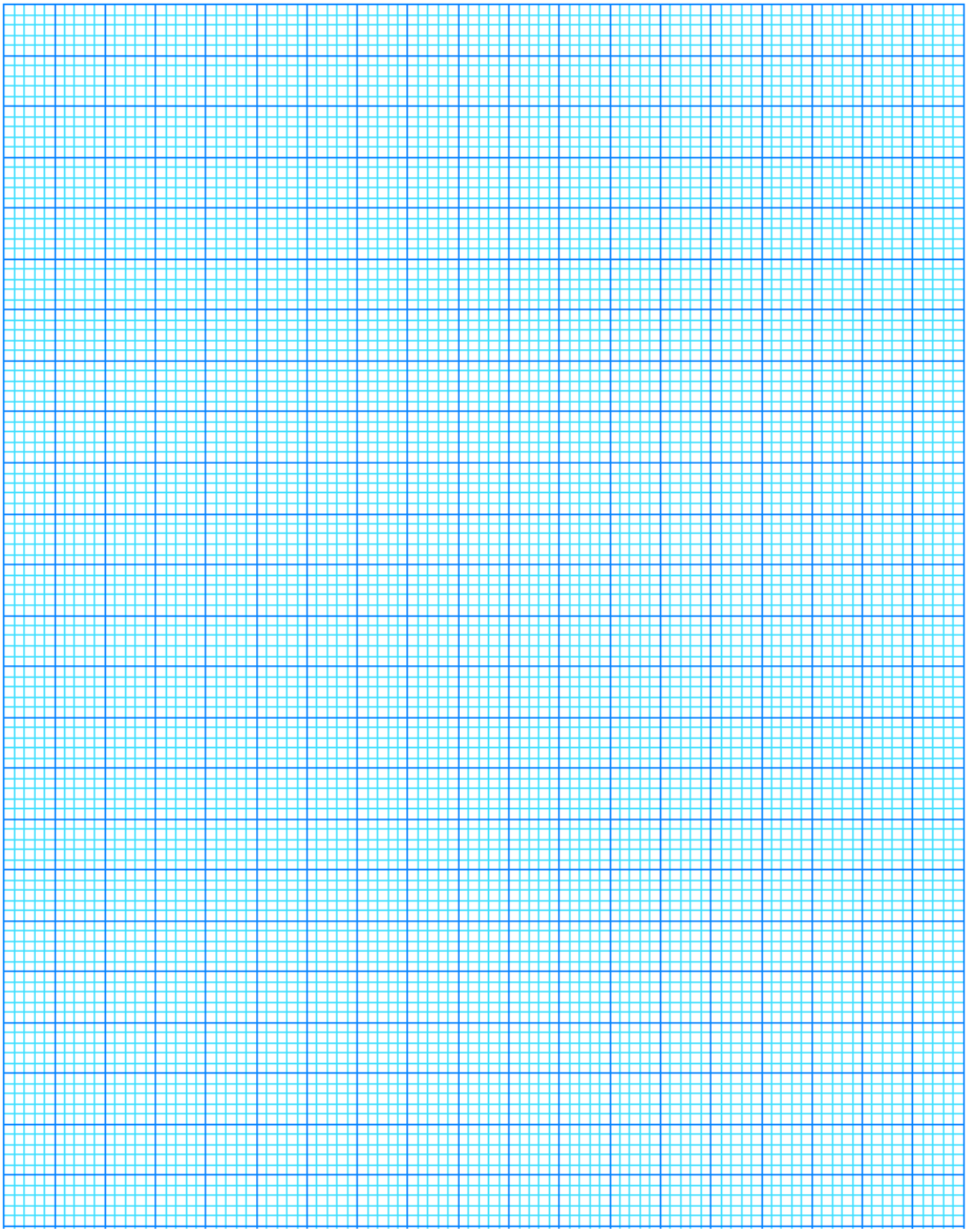
[3]

(ii) Use your graph to estimate the value of a and of b. [3]

(iii) Use your graph to find the value of x when $y = 160$. [1]

(iv) Explain how would you use the graph to find the value of x for which $(10b)^x = 10^{a+1}$.

[2]

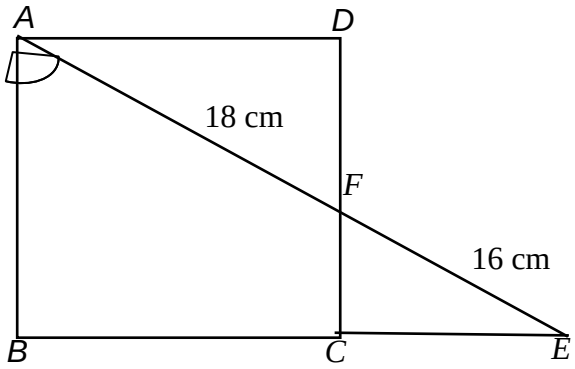


7 **(i)** Express $\dfrac{5x^2-12x-3}{(x-2)\left(x^2+3\right)}$ in partial fractions. [5]

(ii) Differentiate $\ln\left(x^2+3\right)$ with respect to x. [1]

(iii) Using the results from **(i)** and **(ii)**, determine $\int \frac{5x^2-12x-3}{(x-2)\left(x^2+3\right)}dx$. [4]

8 The diagram shows a rectangle *ABCD*. A line through *A* intersects *CD* at *F* and *BC* produced at *E*. It is given that $\angle BAE = \theta°$, *AF* =18 cm and *FE* =16 cm.



(i) Show that the perimeter, *P* cm, of the rectangle is given by $P = 68\cos\theta + 36\sin\theta$. [3]

(ii) Express P in the form $R\cos(\theta-\alpha)$. [3]

(iii) Find the value of θ when the perimeter is 62 cm. [2]

9 A circle C_1 has equation $x^2 + y^2 - 6x + 4y = 12$.

(i) Find the radius and the coordinates of the centre of C_1. [3]

(ii) Show that the equation of the tangent to the circle at the point $(7, -5)$ is $3y = 4x + k$ where k is an integer. [3]

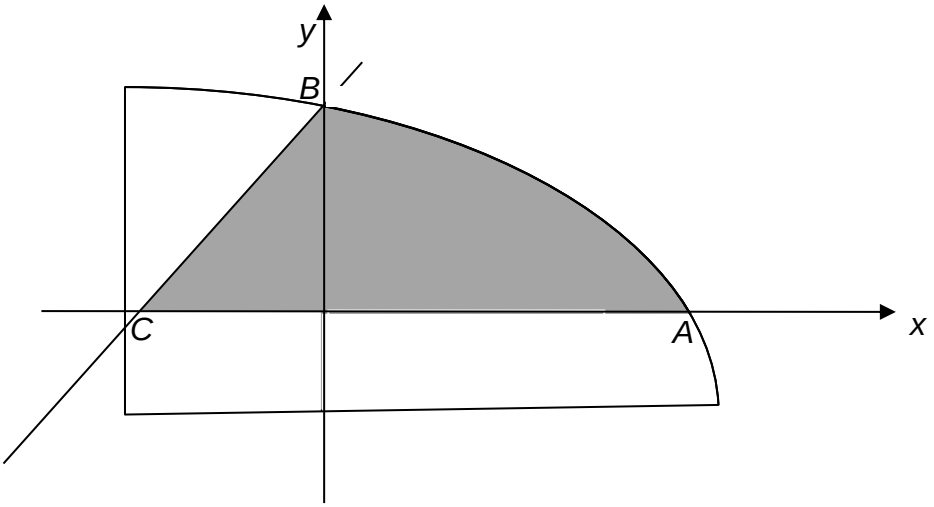
(iii) Another circle C_2 has centre $(-8, 4)$ and radius 7 cm. Find the shortest distance between the 2 circles. [2]

10 **(a)** **Without using a calculator,** find the value of 6^x given that $12^{x-2} = 3^{4-x}$. [4]

(b) Solve the equation $\log_2 x = 6 + \log_{16} x$. [3]

(c) Solve the equation $\lg(x-4)+2\lg 3=1+\lg\left(\frac{x}{2}\right)$. [3]

11



The diagram shows part of the curve $y = 27 + \frac{8}{(x-2)^3}$ which crosses the axes at points A and at B

(i) Find the coordinates of A and of B. [2]

The normal to the curve at B cuts the x-axis at C.

Find

(ii) the equation of the normal, [3]

(iii) the area of the shaded region. [4]

END OF PAPER