CHIJ St Joseph Convent Elementary Mathematics Preliminary Examinations Paper 2

Solutions

Qn	Solution
1(a)	$\frac{2x-3y}{6x^2-5xy-6y^2}=\frac{2x-3y}{(3x+2y)(2x-3y)}$ $=\frac{1}{3x+2y}$
1(b)	$\frac{1}{y-2x}-\frac{x+3}{4x^2-y^2}$ $=\frac{1}{y-2x}-\frac{x+3}{(2x-y)(2x+y)}$ $=\frac{-1}{2x-y}-\frac{x+3}{(2x-y)(2x+y)}$ $=\frac{-1(2x+y)-(x+3)}{(2x-y)(2x+y)}$ $=\frac{-2x-y-x-3}{(2x-y)(2x+y)}$ $=\frac{-3x-y-3}{(2x-y)(2x+y)}$
1(c)	$\frac{1}{3}xy = 2\sqrt{(1-x)y}$ $\frac{1}{9}x^2y^2 = 4(1-x)y$ $\frac{1}{9}x^2y=4-4x$ $x^2y=36-36x$ $y=\frac{36-36x}{x^2}$
2a)	Amount of water used in January = $\frac{100}{x}$
b)	Amount of water used in December = $\frac{100}{x+0.05}$

c)	$\frac{100}{x+0.05}+2=\frac{100}{x}$ $\frac{100+2(x+0.05)}{x+0.05}=\frac{100}{x}$ $100x+2x^2+0.1x=100x+5$ $2x^2+0.1x-5=0$ $20x^2+x-50=0$ (shown)
d)	$x=\frac{-1\pm\sqrt{1^2-4(20)(-50)}}{2(20)}$ $=\frac{-1\pm\sqrt{4001}}{40}$ $=1.5563$ or -1.6063 ≈ 1.56 or -1.61 Therefore price of water in January = $1.56/m^3
e)	Amount of water used in December = $=\frac{100}{1.5563+0.05}$ $=62.254\approx 62.3\text{ m}^3$
3a)	a = 25, b = 36, c = 60
b)	No of dots = $=(25+1)^2=676$
c)	$y=(n+1)^2$
d)	$z=n+n(2n+1)/2n+2n^2/2n(n+1)$

e)	$2n+2n^2=5100$ $n^2+n-2550=0$ $n=50$ or -51 (reject) therefore D = $(50+1)^2$ = 2601
4(ai)	$a=1.85,\ b=1.85,\ c=1.6$
4(aii)	$S=\begin{pmatrix}1.85 & 0 & 0\\ 0 & 1.85 & 0\\ 0 & 0 & 1.60\end{pmatrix}\begin{pmatrix}0.70\\ 0.80\\ 1.40\end{pmatrix}$ $S=\begin{pmatrix}1.295\\ 1.48\\ 2.24\end{pmatrix}$ $S=\begin{pmatrix}1.30\\ 1.48\\ 2.24\end{pmatrix}$
4(b)	$T=\begin{pmatrix}1 & 1 & 1\end{pmatrix}\begin{pmatrix}25 & 14 & 0\\ 18 & 20 & 19\\ 13 & 20 & 7\end{pmatrix}\begin{pmatrix}1.295\\ 1.48\\ 2.24\end{pmatrix}$ $T=\begin{pmatrix}56 & 54 & 26\end{pmatrix}\begin{pmatrix}1.295\\ 1.48\\ 2.24\end{pmatrix}$ $T=\begin{pmatrix}210.68\end{pmatrix}$
4(c)	Total number of workers in the company = 56 + 54 +26 + *x* = 136 + *x*

	$\left(\frac{40}{136+x}\right)\left(\frac{18}{135+x}\right)=\frac{12}{329}$ $40\times18\times329 \quad =12\times(136+x)\times(135+x)$ $236880 \quad = 12(136+x)(135+x)$ $x^2+271x-1380 = 0$ $(x-5)(x+276) = 0$ $x \quad =5, \ -276$ (rej) Total number of workers in the company = 136 + 5 =141
	$\left(\frac{40}{136+x}\right)\left(\frac{18}{135+x}\right)=\frac{12}{239}$ $172080 \quad = 12\times(136+x)\times(135+x)$ $14340 \quad = 18360+271x+x^2$ $x^2+271x+4020 = 0$ $x = \frac{-271\pm\sqrt{271^2-4(1)(4020)}}{2(1)}$ $x \quad =-15.749, \ -255.25$ $x \quad =-15.7, \ -255$
5(ai)	$\overrightarrow{AB}=\overrightarrow{AO}+\overrightarrow{OB}$ $=-4\underset{\sim}{a}+3\underset{\sim}{b}$
5(aii)	$\overrightarrow{AM}=\frac{1}{2}\overrightarrow{AB}$ $=\frac{1}{2}(-4\underset{\sim}{a}+3\underset{\sim}{b})$ $=-2\underset{\sim}{a}+\frac{3}{2}\underset{\sim}{b}$
5(aiii)	$\overrightarrow{OM}=\overrightarrow{OA}+\overrightarrow{AM}$ $=4\underset{\sim}{a}-2\underset{\sim}{a}+\frac{3}{2}\underset{\sim}{b}$ $=2\underset{\sim}{a}+\frac{3}{2}\underset{\sim}{b}$ $or \ \frac{1}{2}(4\underset{\sim}{a}+3\underset{\sim}{b})$

5(aiv)	$\overrightarrow{PQ} = \overrightarrow{PO} + \overrightarrow{OQ}$ $= -7\underset{\sim}{a} + 6\underset{\sim}{b}$
5(av)	$\overrightarrow{MQ} = \overrightarrow{MO} + \overrightarrow{OQ}$ $= -2\underset{\sim}{a} - \frac{3}{2}\underset{\sim}{b} + 6\underset{\sim}{b}$ $= -2\underset{\sim}{a} + \frac{9}{2}\underset{\sim}{b}$ $Or \quad \frac{1}{2}\left(-4\underset{\sim}{a} + 9\underset{\sim}{b}\right)$
5(avi)	$\overrightarrow{MP} = \overrightarrow{MO} + \overrightarrow{OP}$ $= -2\underset{\sim}{a} - \frac{3}{2}\underset{\sim}{b} + 7\underset{\sim}{a}$ $= 5\underset{\sim}{a} - \frac{3}{2}\underset{\sim}{b}$ $Or \quad \frac{1}{2}\left(10\underset{\sim}{a} - 3\underset{\sim}{b}\right)$
5(bi)	$\frac{\text{area of } \Delta OAM}{\text{area of } \Delta AMP} = \frac{\frac{1}{2}OA(h)}{\frac{1}{2}AP(h)}$ $= \frac{4}{3}$
5(bii)	$\frac{\text{area of } \Delta OMP}{\text{area of } \Delta OMB} = \frac{OMP}{OAM} \times \frac{OAM}{OMB}$ $= \frac{\frac{1}{2}OP(h_1)}{\frac{1}{2}OA(h_1)} \times \frac{1}{1}$ $= \frac{7}{4}$

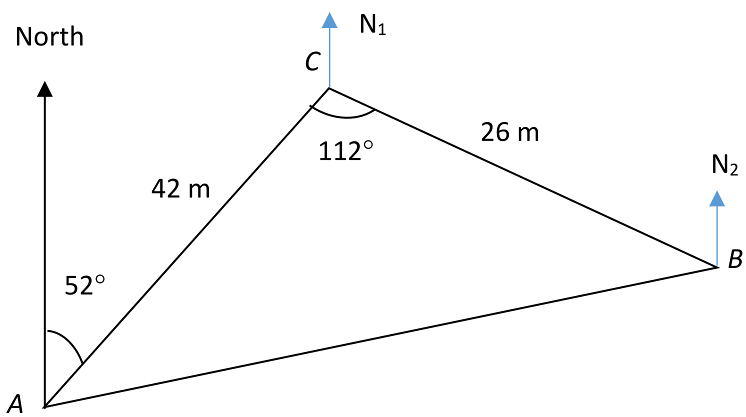
5(c)	$\overrightarrow{PR} = \frac{7}{15}\overrightarrow{PQ}$ $= \frac{7}{15}(-7\underset{\sim}{a} + 6\underset{\sim}{b})$ $= -\frac{49}{15}\underset{\sim}{a} + \frac{14}{5}\underset{\sim}{b}$ $\overrightarrow{OR} = \overrightarrow{OP} + \overrightarrow{PR}$ $= 7\underset{\sim}{a} - \frac{49}{15}\underset{\sim}{a} + \frac{14}{5}\underset{\sim}{b}$ $= \frac{56}{15}\underset{\sim}{a} + \frac{14}{5}\underset{\sim}{b}$ $= \frac{14}{15}(4\underset{\sim}{a} + 3\underset{\sim}{b})$ $\overrightarrow{OM} = h\overrightarrow{OR}$ $OM = hOR$ $h = \frac{OM}{OR}$ $h = \frac{\frac{1}{2}}{\frac{14}{15}}$ $h = \frac{15}{28}$
6ai)	median = 3.6 to 3.7
ii)	Lower quartile = 2.6 Upper quartile = 4.5 to 4.7 Interquartile range = 1.9 to 2.0
iii)	10% at least *x* hours 90% of 600 = 540 ➔ 7 to 7.2 Therefore *x* = 7 to 7.2

bi)	median = 4.2
ii)	Lower quartile = 2.8 Upper quartile = 7.4 Interquartile range = 7.4 – 2.8 = 4.6
ci)	Disagree, because the median of 4.2 hours for Group B is **higher** than the **median** of 3.6 (3.6 -3.7) hours for Group A, implying that people in Group B spent more time on their smart phones than Group A.
ii)	Group A is more consistent on the use of the smart phones than Group B because Group A had a **smaller interquartile range** of 2 compared with 3.6 for Group B
7ai)	Area of segment = $\frac{240}{12} = 20$ Area of sector – area of triangle AOB = 20 $\frac{1}{2}r^2(1.5) - \frac{1}{2}r^2(\sin 1.5) = 20$ $\frac{1}{2}r^2(1.5 - \sin 1.5) = 20$ $r^2 = \frac{40}{1.5 - \sin 1.5}$ = 79.601 r = 8.9219 $\approx$ 8.92
ii)	$AB^2 = 8.9219^2 + 8.9219^2 - 2(8.9219)(8.9219)\cos 1.5$ = 147.93 AB = 12.162 surface area of water = 12.162 $\times$12 = 145.94 $\approx$ 146 cm^2

b)	Lateral height of cone: $l^2 = 10^2 + (8.9219)^2$ $l = 13.401$ cm area of container that is in contact with water $= \frac{1}{2}(2\pi(8.9219)^2 + \frac{1}{2} \times 2\pi(8.9219)12 + \frac{1}{2} \times \pi(8.9219)(13.401)$ $= 250.07 + 336.34 + 187.80$ $= 774.21$ $\approx 774 \text{ cm}^2$
8a) **b)**	Amount received for 10 years =10(12 x 800) = $96 000 Total amount received after 15 years = 96 000 + 80 000 = $176 000 Total interest earned = 176 000 – 100 000 = $76 000

c)	Amount received after 5 years = $\frac{20}{100}\times 100000$ =\$20 000 <table><tr><th>Interpretation 1</th><th>Interpretation 2</th></tr><tr><td>$=80000\left(1+\frac{14}{100}\right)^{10}$ $=\$296577.71$ Tot Amt $=20000+296577.71$ $=\$316577.71$ Interest $=\$316577.71-100000$ $=\$216577.71$</td><td>$=80000\left(1+\frac{7}{100}\right)^{20}$ $=\$309574.76$ Tot Amt $=20000+309574.76$ $=\$329574.76$ Interest $=\$329574.76-100000$ $=\$229574.76$</td></tr></table> $180000 = P\left(1+\frac{5}{100}\right)^{15}$ $P = 180000\div\left(1+\frac{5}{100}\right)^{15}$ = \$86 583.08
9a) **b)**	a = 37 Refer to graph
ci)	Draw graph of y = x + 20 x = 1.05 or 2.8

cii)	$k = 17 – 17.5$
ciii)	Draw line y = 5x or any line with gradient = 5 $x = 2.2$ to 2.3
civ)	Draw the line of y = 2x + 20 1 < x < 3.1
d)	$2x^2 + \frac{20}{x} = 5x + 1$ $2x^3 + 20 = 5x^2 + x$ $2x^3 - 5x^2 - x + 20 = 0$ A= -5, B= -1, C=20

10(ai)	 Angle N_1CA = 180 – 52 (Interior angles, parallel lines) = 128° Angle N_1CB = 360 - 128 – 112 (angles at a point) = 120° Angle N_1BC = 180 - 120 (Interior angles) = 60° Therefore, bearing of C from B

	= 360 – 60 (angles at a point) = 300°
10(aii)	$AB^2 = 42^2 + 26^2 - 2(42)(26)cos112°$ $AB = \sqrt{42^2 + 26^2 - 2(42)(26)cos112°}$ $AB = 57.080$ $AB = 57.1$ m
10(aiii)	Area of triangle ABC = $\frac{1}{2}(42)(26)sin112°$ $= 506.24$ $= 506$ m^2
10(bi)	Let shortest distance be *d*. $\frac{1}{2}(57.080)d = 506.24$ $(57.080)d = 1012.48$ $\therefore d = 17.737$ $\therefore d = 17.7$ m
10(bii)	$tan\theta = \frac{5}{17.737}$ $\therefore \theta = tan^{-1}\left(\frac{5}{17.737}\right)$ $\therefore \theta = 15.7°$
10(ci) 10 (cii)	**Point A** because she will furthest away from the flagpole at C. $tan\theta = \frac{5}{42}$ $\therefore \theta = tan^{-1}\left(\frac{5}{42}\right)$ $\therefore \theta = 6.8°$
11(ai)	Total land residential land area = 0.17 × 719.2 = 122.264 = 122 km^2

11(aii)	Total number Singapore residents in 2030 $= 3933600\left(1+\frac{1.3}{100}\right)^{14}$ = 4713271.208 = 4710000 (3sf)
11(b)	Total number of elderly needing centre services = [4713271.208] × 0.25 × 0.1 = 117831.7802 Average number of elderly per square km = [117832] ÷ [122.264] = 963.7487 (7sf) Maximum size of one catchment area $= \pi(1)^2$ $= 3.14159$ km^2 (6sf) Average number of elderly in one catchment area **= [**964] × [3.14159] = 3028.49 (5sf) As the **number of elderly needing the centre services far exceeds the current capacity of 1000**, the AIC **needs to build more centres**.
11(c)	Possible assumptions Total land area remains constant. Even distribution of elderly in all residential areas. Population density is the same for all areas. Stable population growth continues until 2030. Do **not** accept (conditions given in question) Exactly 1000 elderly cared for in each centre All 10% of elderly need centre's services