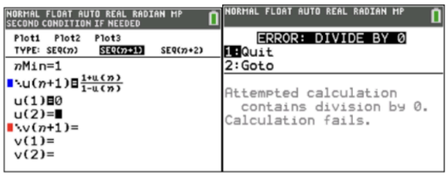
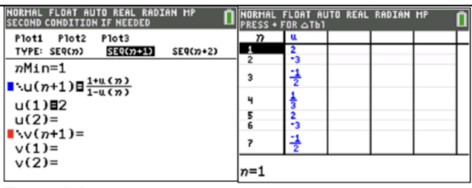
HCI 2024 Prelims Paper 2 Solutions

Section A: Pure Mathematics

Qn	Suggested Solutions
1 (a)	$\tan(A-B)=\dfrac{\tan A-\tan B}{1+\tan A\tan B}$ $\therefore\ A-B=\tan^{-1}\left(\dfrac{\tan A-\tan B}{1+\tan A\tan B}\right)$ ...(*) Let $A=\tan^{-1}\left(\dfrac{1}{x}\right)$. $\therefore\ \tan A=\dfrac{1}{x}$ Let $B=\tan^{-1}\left(\dfrac{1}{1+x}\right)$. $\therefore\ \tan B=\dfrac{1}{1+x}$ From (*): $\therefore\ A-B=\tan^{-1}\left(\dfrac{1}{x}\right)-\tan^{-1}\left(\dfrac{1}{1+x}\right)$ $=\tan^{-1}\left(\dfrac{\frac{1}{x}-\frac{1}{1+x}}{1+\frac{1}{x(1+x)}}\right)$ $=\tan^{-1}\left(\dfrac{1}{x(1+x)+1}\right)$ $=\tan^{-1}\left(\dfrac{1}{x^2+x+1}\right)$ (shown)
1(b)	$\sum_{r=1}^{n}\tan^{-1}\left(\dfrac{1}{r^2+r+1}\right)$ $=\sum_{r=1}^{n}\left[\tan^{-1}\left(\dfrac{1}{r}\right)-\tan^{-1}\left(\dfrac{1}{1+r}\right)\right]$ $=\left\{\begin{aligned}&\tan^{-1}(1) &&-\tan^{-1}\left(\tfrac{1}{2}\right)\\ &+\tan^{-1}\left(\tfrac{1}{2}\right) &&-\tan^{-1}\left(\tfrac{1}{3}\right)\\ &\vdots &&\vdots\\ &+\tan^{-1}\left(\tfrac{1}{n-1}\right) &&-\tan^{-1}\left(\tfrac{1}{n}\right)\\ &+\tan^{-1}\left(\tfrac{1}{n}\right) &&-\tan^{-1}\left(\tfrac{1}{n+1}\right)\end{aligned}\right.$ $=\tan^{-1}(1)-\tan^{-1}\left(\dfrac{1}{n+1}\right)$ $=\dfrac{\pi}{4}-\tan^{-1}\left(\dfrac{1}{n+1}\right)$ where $k=\dfrac{\pi}{4}$, $\mathrm{f}(n)=\tan^{-1}\left(\dfrac{1}{n+1}\right)$. (shown)

2 **[5]** **(a)**	$u_2 = \dfrac{1+u_1}{1-u_1} = \dfrac{1+0}{1-0} = 1$ $u_3 = \dfrac{1+u_2}{1-u_2} = \dfrac{1+1}{1-1}$, which is undefined • The sequence ends at $u_2 = 1$ as the subsequent terms are undefined. OR • There are only two terms and terminate(ends) at the 2nd term. 
(b)	 From GC, $u_2 = -3$, $u_3 = -\dfrac{1}{2}$, $u_4 = \dfrac{1}{3}$, $u_5 = 2$, $u_6 = -3$
(c)	From observation, the sequence repeats with a period of 4.
	$\sum_{r=1}^{4n} u_r = (u_1+u_2+u_3+u_4)+(u_5+...+u_8)+...+(u_{4n-1}+u_{4n})$ $= \left[2-3+\left(-\dfrac{1}{2}\right)+\dfrac{1}{3}\right]\times\dfrac{4n}{4}$ $= -\dfrac{7}{6}n$

3
[7]
(a)

$2y^3 - y^2 = xe^x$

Differentiate implicitly throughout with respect to x:

$6y^2 \frac{dy}{dx} - 2y\frac{dy}{dx} = xe^x + e^x$

$\frac{dy}{dx} = \frac{e^x(x+1)}{2y(3y-1)}$

When tangent // to y-axis,

$2y(3y-1) = 0$

$y = 0, \ \frac{1}{3}$

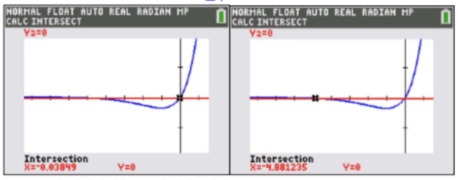
When $y = 0, x = 0$ is equation of the tangent // to y-axis

When $y = \frac{1}{3}$

$2\left(\frac{1}{3}\right)^3 - \left(\frac{1}{3}\right)^2 = xe^x$

$xe^x + \frac{1}{27} = 0$

From GC, $Y_1 = xe^x + \frac{1}{27}$, $Y_2 = 0$



$x = -0.03849, -4.881235$

Note: The more accurate answer is $x = -0.038490398$

Therefore, the equations of 3 tangents that are // to y-axis are:

$x = 0, \quad x = -0.0385 \text{ (3 s.f.)}, \quad x = -4.88 \text{ (3 s.f.)}$

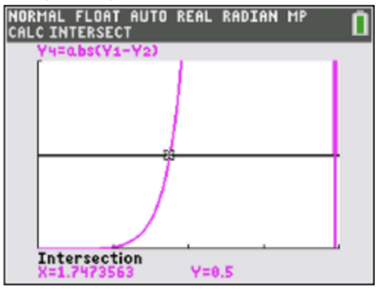
(b)

Gradient $= \pm\tan\left(\frac{\pi}{2} - \frac{\pi}{6}\right) = \pm\sqrt{3}$ or ± 1.73 (3 s.f.)

Alternative Solution

$\frac{1}{m} = \pm\tan\frac{\pi}{6} \Longrightarrow m = \pm\sqrt{3}$

4 **[5]** **(a)**	Let the acute angle between the vectors $\underset{\sim}{m}$ and $\underset{\sim}{n}$ be θ and the unit vector perpendicular to $\underset{\sim}{m}$ and $\underset{\sim}{n}$ be $\hat{\underset{\sim}{p}}$. Applying definition of dot (scalar) product, $\underset{\sim}{m}\bullet\underset{\sim}{n}=\lvert\underset{\sim}{m}\rvert\lvert\underset{\sim}{n}\rvert\cos\theta$ $\left(\underset{\sim}{m}\bullet\underset{\sim}{n}\right)^2=\lvert\underset{\sim}{m}\rvert^2\lvert\underset{\sim}{n}\rvert^2\cos^2\theta$ Applying definition of cross (vector) product, $\underset{\sim}{m}\times\underset{\sim}{n}=\lvert\underset{\sim}{m}\rvert\lvert\underset{\sim}{n}\rvert\sin\theta\,\hat{\underset{\sim}{p}}$ $\lvert\underset{\sim}{m}\times\underset{\sim}{n}\rvert=\lvert\underset{\sim}{m}\rvert\lvert\underset{\sim}{n}\rvert\sin\theta\left\lvert\hat{\underset{\sim}{p}}\right\rvert$ $\lvert\underset{\sim}{m}\times\underset{\sim}{n}\rvert=\lvert\underset{\sim}{m}\rvert\lvert\underset{\sim}{n}\rvert\sin\theta$ (since $\left\lvert\hat{\underset{\sim}{p}}\right\rvert=1$) $\lvert\underset{\sim}{m}\times\underset{\sim}{n}\rvert^2=\lvert\underset{\sim}{m}\rvert^2\lvert\underset{\sim}{n}\rvert^2\sin^2\theta$ $\therefore\left(\underset{\sim}{m}\bullet\underset{\sim}{n}\right)^2+\lvert\underset{\sim}{m}\times\underset{\sim}{n}\rvert^2$ $=\lvert\underset{\sim}{m}\rvert^2\lvert\underset{\sim}{n}\rvert^2\cos^2\theta+\lvert\underset{\sim}{m}\rvert^2\lvert\underset{\sim}{n}\rvert^2\sin^2\theta$ $=\lvert\underset{\sim}{m}\rvert^2\lvert\underset{\sim}{n}\rvert^2\left(\cos^2\theta+\sin^2\theta\right)$ since $\sin^2\theta+\cos^2\theta=1$ $=\lvert\underset{\sim}{m}\rvert^2\lvert\underset{\sim}{n}\rvert^2$ (Shown)
(b)	$\underset{\sim}{v}\times\left(\underset{\sim}{p}-\underset{\sim}{q}\right)=\underset{\sim}{0}$ $\Rightarrow\underset{\sim}{v}=\underset{\sim}{0}$ (rej.) or $\underset{\sim}{p}-\underset{\sim}{q}=\underset{\sim}{0}$ (rej. $\because$ *P* and *Q* are distinct points)
	$\therefore\underset{\sim}{v}\,/\!/\left(\underset{\sim}{p}-\underset{\sim}{q}\right)$ $\Rightarrow\underset{\sim}{p}-\underset{\sim}{q}=m\underset{\sim}{v}$ $\Rightarrow\underset{\sim}{p}=\underset{\sim}{q}+m\underset{\sim}{v}$, where $m\in\mathbb{R}\setminus\{0\}$ Since $\underset{\sim}{p}$ satisfies the equation of l_2 Therefore the lines l_1 and l_2 intersect at the point *P*.

5 **(a)**	$g(x)$ $=\dfrac{1}{(1+\cos x)^2}$ $=(1+\cos x)^{-2}$ $\approx\left(1+1-\dfrac{x^2}{2}+\dfrac{x^4}{24}\right)^{-2}$, since $\cos x\approx 1-\dfrac{x^2}{2}+\dfrac{x^4}{24}$ $=\left(2-\dfrac{x^2}{2}+\dfrac{x^4}{24}\right)^{-2}$ $=2^{-2}\left(1-\dfrac{x^2}{4}+\dfrac{x^4}{48}\right)^{-2}$ $=\dfrac{1}{4}\left[1+\dfrac{(-2)}{1}\left(-\dfrac{x^2}{4}+\dfrac{x^4}{48}\right)+\dfrac{(-2)(-3)}{2!}\left(-\dfrac{x^2}{4}+\dfrac{x^4}{48}\right)^2+\ldots\right]$ $=\dfrac{1}{4}\left[1+\dfrac{1}{2}x^2-\dfrac{1}{24}x^4+\dfrac{3}{16}x^4+\ldots\right]$ $\approx\dfrac{1}{4}+\dfrac{1}{8}x^2+\dfrac{7}{192}x^4$
(b)	$Y_1=\dfrac{1}{(1+\cos x)^2}$ $Y_2=\dfrac{1}{4}+\dfrac{1}{8}x^2+\dfrac{7}{192}x^4$ $y=\lvert Y_1-Y_2\rvert$  $\lvert Y_1-Y_2\rvert\le 0.5$ From GC, $\{x\in\mathbb{R}:\quad 0\le x\le 1.75\}$ (3 s.f.)

6 **[10]** **(a)**	$\lvert u \rvert^2$ $= \lvert x+\mathrm{i}y \rvert^2$ $= \left[\sqrt{x^2+y^2}\right]^2$ $= x^2+y^2$ $= (x+\mathrm{i}y)(x-\mathrm{i}y)$
(b)	**<u>Method 1</u>**: $\lvert z+w \rvert^2 = \lvert z-w \rvert^2$ $(z+w)(z+w)^* = (z-w)(z-w)^*$ $(z+w)(z^*+w^*) = (z-w)(z^*-w^*)$ $zz^*+zw^*+wz^*+ww^* = zz^*-zw^*-wz^*+ww^*$ $2(zw^*+wz^*) = 0$ $zw^*+wz^* = 0$

(method 2 omitted - avoid)

(c)	**<u>Method 1</u>**: $zw^*+wz^*=0$ $zw^*+(w^*z)^*=0$ $(w^*z)+(w^*z)^*=0$ $2\,\mathrm{Re}(w^*z)=0$ w^*z is purely imaginary.
	<u>Method 2</u>: zw^* $=(x+\mathrm{i}y)(a-\mathrm{i}b)$ $=(ax+by)+\mathrm{i}(ay-bx)$ From part (b), Since $ax+yb=0$ zw^* $=0+\mathrm{i}(ay-bx)$ $\therefore \mathrm{Re}(zw^*)=0$
(d)	$w=-1+\mathrm{i}\sqrt{3} \quad \Rightarrow \arg(w)=\dfrac{2\pi}{3}$ $\arg(zw^*)$ $=\arg(z)+\arg(w^*)$ $=\arg(z)-\arg(w)$ $=\theta-\dfrac{2\pi}{3}$ Since zw^* is purely imaginary,
	$\theta-\dfrac{2\pi}{3}=\dfrac{\pi}{2}+k\pi, \quad k\in\mathbb{Z}$ $\theta=\dfrac{7\pi}{6}+k\pi$ $\theta=\dfrac{\pi}{6}$ or $\theta=-\dfrac{5\pi}{6}$ since $-\pi<\theta\le\pi$.

Section B: Statistics

7 (a)

$M \sim \mathrm{B}(n, p)$

$\mathrm{P}(2 \le M \le 3) = \mathrm{P}(M = 2) + \mathrm{P}(M = 3)$

$= \binom{n}{2} p^2 (1-p)^{n-2} + \binom{n}{3} p^3 (1-p)^{n-3}$

$= \frac{n(n-1)}{2} p^2 (1-p)^{n-2} + \frac{n(n-1)(n-2)}{6} p^3 (1-p)^{n-3}$

7 (b)

$M \sim \mathrm{B}(10, p)$

$\mathrm{P}(2 \le M \le 3)$

$\mathrm{P}(2 \le M \le 3)$

$= \mathrm{P}(M = 2) + \mathrm{P}(M = 3)$

$= \binom{10}{2} p^2 (1-p)^8 + \binom{10}{3} p^3 (1-p)^7$

$= 45 p^2 (1-p)^8 + 120 p^3 (1-p)^7$

$= k$

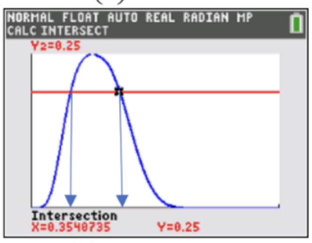
Let X be the number of days in which $2 \le M \le 3$ out of 5 working days

$X \sim \mathrm{B}(5, k)$

$\mathrm{P}(X = 3) = 0.25$

$\binom{5}{3} k^3 (1-k)^2 = 0.25$

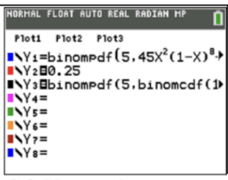
Let $Y_1 = \binom{5}{3} k^3 (1-k)^2$ and $Y_2 = 0.25$

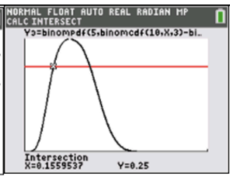


From GC,

$p = 0.1559537$ or 0.3540735

$p = 0.156$ or 0.354 (3 s.f.)





GC Keystrokes:

- $\mathrm{P}(2 \le M \le 3)$
  $= \mathrm{P}(M \le 3) - \mathrm{P}(M \le 1)$
  $= \mathrm{binomcdf}(10, X, 3) - \mathrm{binomcdf}(10, X, 1)$

- $\mathrm{P}(2 \le M \le 3)$
  $= \mathrm{P}(M = 2) + \mathrm{P}(M = 3)$
  $= \mathrm{binompdf}(10, X, 2) + \mathrm{binompdf}(10, X, 3)$

- $\mathrm{P}(X = 3)$
  $= (5, \mathrm{binomcdf}(10, x, 3) - \mathrm{binomcdf}(10, x, 1), 3)$

8(a)

A B C D E

$^4C_1$ $^4C_1$ $^4C_1$ $^4C_1$ $^4C_1$ ← Each door can be painted in any of the 4 colours

No. of ways $= (^4C_1)^5 = 1024$

8(b)

A B C D E

Consider each door A to E in sequence → $^4C_1$ $^3C_1$ $^3C_1$ $^3C_1$ $^3C_1$

$^4C_1$ ways to paint A

$^3C_1$ ways to paint B different from A

$^3C_1$ ways to paint C different from B

$^3C_1$ ways to paint D different from C

$^3C_1$ ways to paint E different from D

No. of ways $= {}^4C_1 \times (^3C_1)^4 = 324$

8(c)

If all 4 colours are used, there must be 2 doors out of 5 painted with the same colour.

Method 1: (consider colours to be used)

No. of ways $= {}^4C_1 \times \dfrac{5!}{2!} = 240$

$^4C_1$ ways to choose a colour to paint 2 doors

$\frac{5!}{2!}$ ways to arrange 5 colours in a row, with 1 colour appearing twice

Method 2: (consider doors to be painted)

No. of ways $= {}^5C_2 \times {}^4C_1 \times 3! = 240$

$^5C_2$ ways to choose 2 doors out of 5 to paint same colour

$^4C_1$ ways to choose a colour to paint the 2 chosen doors

3! ways to paint remaining 3 doors with 3 colours

9 (a)

$S \sim \mathrm{N}\left(\mu, \sigma^2\right)$

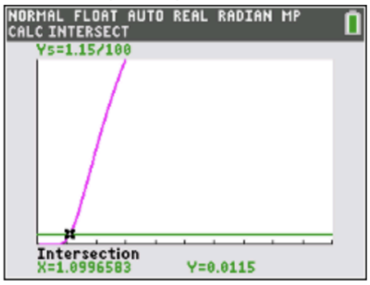
Since $\mathrm{P}(S < 80.5) = \mathrm{P}(S > 84.5)$,

By symmetry, $\mu = \dfrac{80.5 + 84.5}{2} = 82.5$.

<u>**Method 1**</u>: Using GC

$S \sim \mathrm{N}\left(82.5, \sigma^2\right)$

Let $Y_1 = \mathrm{P}(S > 85)$ and $Y_2 = 0.0115$



From GC,

$\sigma = 1.0996583 = 1.10$ (3 s.f.) (Shown)

<u>**Method 2**</u>: Using Standard Normal Distribution

$\mathrm{P}(S > 85) = 0.0115$

$\mathrm{P}\left(Z > \dfrac{85-\mu}{\sigma}\right) = 0.1155$

$\dfrac{85 - 82.5}{\sigma} = 2.27343$

$\sigma = 1.099657 = 1.10$ (3 s.f.) (Shown)

9 (b)

$C \sim \mathrm{N}\left(83, 1.5^2\right)$

Let $W = C - S$.

$W \sim \mathrm{N}\left(83 - 82.5, 1.1099657^2 + 1.5^2\right)$

i.e. $W \sim \mathrm{N}(0.5, 3.45925)$

or $W \sim \mathrm{N}(0.5, 3.46)$ [if use σ=1.10]

$\mathrm{P}(0 < C - S \le 2)$

$= \mathrm{P}(0 \le W \le 2)$

$= 0.39599$ or 0.39595 [if use σ=1.10]

$= 0.396$ (3 s.f.)

9 (c)

$\mathrm{P}(C - S > 1.5 \mid 0 < C - S \le 2)$

$= \dfrac{\mathrm{P}(1.5 < C - S \le 2)}{\mathrm{P}(0 < C - S \le 2)}$

$= \dfrac{0.0854258}{0.39599}$ or $= \dfrac{0.0854208}{0.39595}$ [if use σ=1.10]

$= 0.215727$ or $= 0.215734$ [if use σ=1.10]

$= 0.216$ (3 s.f.)

10 (a)	The population consists of all fresh graduates with a B.Sc degree. While universities may have data on students before graduation, these graduates can work in various industries across Singapore after graduation. To obtain a truly random sample, every graduate must have an equal chance of being selected, and the selection process must be independent. However, several challenges make this difficult: • Not all graduates will be employed immediately after graduation, making it harder to gather salary data and select a truly random sample. • It may be difficult to track where fresh graduates are employed, as their contact details may have changed since leaving university. • Some graduates may be unwilling to respond to the survey, particularly if they are uncomfortable sharing salary information. • Graduates in different job sectors or industries (e.g., private vs. public) may have different levels of transparency regarding salary data. For example, starting salaries in the private sector may be confidential, further complicating data collection.

10 (b)	$\sum(x-3600)=1000$ $\sum x-\sum 3600=1000$ $\sum x=1000+\sum 3600$ $\sum x=1000+80(3600)$ An unbiased estimate of population mean is = $\bar{x}=\frac{\sum x}{80}$ $\bar{x}=\frac{1000}{80}+3600$ $=3612.5$ $=\frac{7225}{2}$ An unbiased estimate of population variance is $=s^2$ $s^2=\frac{1}{79}\left[205000-\frac{1000^2}{80}\right]$ $=\frac{192500}{79}$ $=2436\frac{56}{79}$ $=2436.708861$

10 (c)

Let μ and σ^2 be the **population mean and variance** of starting monthly salaries of fresh graduates with B.Sc.

Test $H_0: \mu = 3600$
Against $H_1: \mu > 3600$

Perform a 1-tailed test at 5% level of significance.
Under H_0, since $n = 80$ is large, by **Central Limit Theorem**, $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$ approximately

Test Statistic: $Z = \frac{\bar{X} - \mu}{\frac{S}{\sqrt{n}}} \sim N(0,1)$ approximately

At 5% level of significance, p-value
$= 0.01175874 \approx 0.0118$ (3 s.f)

Since p-value $= 0.0118 < 0.05$, we reject H_0 and conclude that there is **sufficient evidence** at **5%** **level of significance** that the **population mean** **monthly salary** is **higher than** **$3600**. Therefore **First-Pay's claim is justifiable**.

10 (d)

Since the sample size of 80 is large, by **Central Limit Theorem**, **sample mean** **monthly salary** of fresh graduates with B.Sc, $\bar{X}$ follows a **normal distribution** **approximately**. Thus, no assumption on the population, X is needed.

10 (e)

"5% level of significance" means that there is a 5% probability that we **wrongly conclude** that **population mean** **monthly salaries** of fresh graduates with B.Sc. is **higher than** $3600 **when it is in fact** $3600.

10 (f)

Test $H_0: \mu = 3600$
Against $H_1: \mu > 3600$

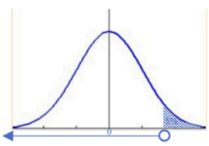
Perform a 1-tailed test at 5% level of significance.
$s^2 = \frac{60}{59} \times 355^2 = 128161.0169$

Under H_0, since $n = 60$ is large, by **Central Limit Theorem**, $\bar{Y} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$ approximately.

Test Statistic: $Z = \frac{\bar{Y} - \mu}{\frac{S}{\sqrt{60}}} \sim N(0,1)$ approximately

At 5% level of significance,
reject H_0 when critical region is $z \geq 1.644853626$

Since H_0 is **not** rejected,



$\frac{\bar{y} - 3600}{\sqrt{\frac{128161.0169}{60}}} < 1.644853626$

$\bar{y} < 1.644853626 \times \sqrt{\frac{128161.0169}{60}} + 3600$

$0 < \bar{y} < 3676.020304$

Largest $\bar{y} = 3676$ (nearest dollar)

11 (a)

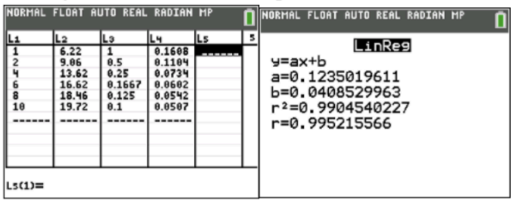
$$h=\frac{an}{b+n}$$

$$\frac{1}{h}=\frac{b+n}{an}$$

$$\frac{1}{h}=\left(\frac{b}{a}\right)\left(\frac{1}{n}\right)+\frac{1}{a}$$

From GC,

$L_3=\frac{1}{L_1}$, where $L_1=n$, $L_4=\frac{1}{L_2}$, where $L_2=h$



$$\frac{1}{h}=0.1235019611\frac{1}{n}+0.0408529963$$

$$\frac{1}{h}=0.124\frac{1}{n}+0.0409 \text{ (3 s.f.)}$$

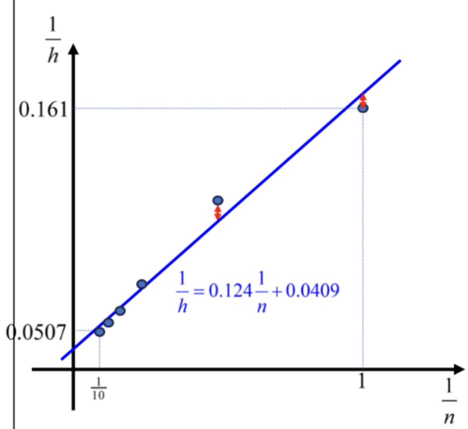
$$\frac{b}{a}=0.1235019611$$

$$\frac{1}{a}=0.0408529963 \Rightarrow a=24.47800893$$

$$b=0.1235019611\times 24.47800893=3.023082082$$

$\therefore a=24.478$ and $b=3.023$ (3 d.p.)

11(b) Red vertical line on the graph represents the residuals



11 (c)

$$h=\frac{an}{b+n} \quad \text{or} \quad \frac{1}{h}=\left(\frac{b}{a}\right)\left(\frac{1}{n}\right)+\frac{1}{a}$$

$$h=a-\frac{ab}{b+n}$$

As $n\to\infty$, $h\to a$ or $\frac{1}{h}\to\frac{1}{a}$

- The **theoretical maximum height** of the plant specimen.
- The **maximum height** of the plant specimen in the **long run.**

11 (d)

Since $h \geq 18$,

$\frac{1}{h} \leq \frac{1}{18}$

$\frac{1}{h} = 0.1235019611\left(\frac{1}{n}\right) + 0.0408529963$

$0.1235019611\left(\frac{1}{n}\right) + 0.0408529963 \leq \frac{1}{18}$

$n \geq 8.400031487$

Minimum number of months is 9.

The estimate is **reliable** since

- $h = 18$ cm is within the data range $[6.22, 19.72]$ and
- scatter diagram in part **(b)** shows that there is a strong positive linear relationship between $\frac{1}{h}$ and $\frac{1}{n}$.
- $r = 0.995215566 = 0.995$ (3 s.f.) is close to 1 indicating a strong positive linear relationship between $\frac{1}{h}$ and $\frac{1}{n}$.

11(f)

$L_4 = \frac{1}{h}$

From least squares regression line:

$L_5 = \frac{1}{h} = 0.1235019611\left(\frac{1}{n}\right) + 0.0408529963$, or

$L_5 = 0.12350L_3 + 0.040852$ (5 s.f.)

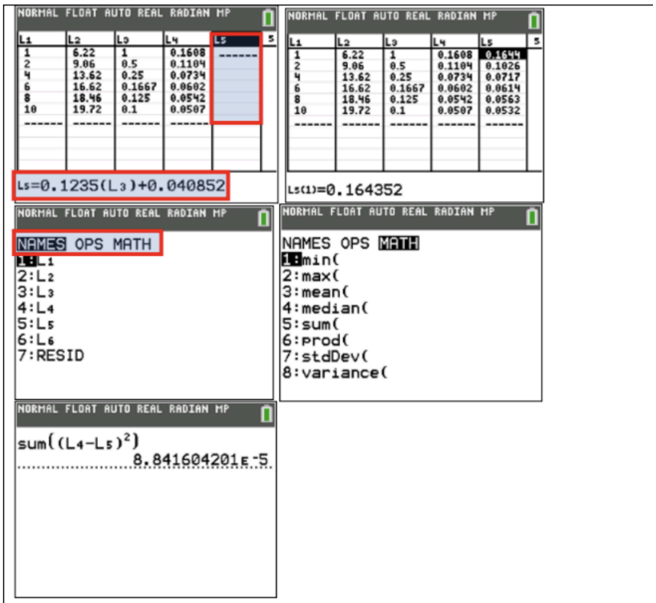
From GC,

sum of squares of residual

$= \text{Sum } (L_4 - L_5)^2$

$= 8.8416 \times 10^{-5}$ (5 s.f.)

$= 0.000088416$ (5 s.f.)



12 (a)

Probability Distribution of X

x	-2	-1	0	1	2	3
$\mathrm{P}(X=x)$	$\frac{b}{2}$	$\frac{b}{2}$	a	a	b	b

$$\sum_{\text{all x}} \mathrm{P}(X=x)=1$$

$$\frac{b}{2}+\frac{b}{2}+a+a+b+b=1$$

$$2a+3b=1$$

$$b=\frac{1-2a}{3}$$

12 (b) (i)

$$\mathrm{E}(X)=\sum_{\text{all x}} x\mathrm{P}(X=x)$$

$$\mathrm{E}(X)=-\frac{2b}{2}-\frac{b}{2}+0+a+2b+3b$$

$$\mathrm{E}(X)=a+\frac{7b}{2}$$

Since $b=\frac{1-2a}{3}$,

$$\mathrm{E}(X)$$

$$=a+\frac{7\left(\frac{1-2a}{3}\right)}{2}$$

$$=\frac{7}{6}-\frac{4a}{3}$$

12 (b) (ii)

$$\mathrm{E}\left(X^2\right)=\sum_{\text{all x}} x^2\mathrm{P}(X=x)$$

$$\mathrm{E}\left(X^2\right)$$

$$=\frac{4b}{2}+\frac{b}{2}+0+a+4b+9b$$

$$=a+\frac{31b}{2}$$

Since $b=\frac{1-2a}{3}$,

$$\mathrm{E}\left(X^2\right)$$

$$=a+\frac{31\left(\frac{1-2a}{3}\right)}{2}$$

$$=\frac{31}{6}-\frac{28a}{3}$$

$$\mathrm{Var}(X)=\mathrm{E}\left(X^2\right)-\left[\mathrm{E}(X)\right]^2$$

$$=\frac{31}{6}-\frac{28a}{3}-\left(\frac{7}{6}-\frac{4a}{3}\right)^2$$

$$=\frac{31}{6}-\frac{28a}{3}-\left(\frac{49}{36}-\frac{56a}{18}+\frac{16a^2}{9}\right)$$

$$=\frac{137}{36}-\frac{56a}{9}-\frac{16a^2}{9}$$

12 (b) (iii)	For X to be defined, $a, b > 0$ as stated in the question. From part **(a)**, $b = \frac{1-2a}{3}$ $\therefore 1 - 2a > 0$ $a \le \frac{1}{2}$ When X is defined, $\text{Var}(X)$ is defined when $0 < a < \frac{1}{2}$

12 (c)	For $a = \frac{7}{20}$, $\text{E}(X) = \frac{7}{6} - \frac{4\left(\frac{7}{20}\right)}{3} = \frac{7}{10}$ $\text{Var}(X) = \frac{137}{36} - \frac{56\left(\frac{7}{20}\right)}{9} - \frac{16\left(\frac{7}{20}\right)^2}{9} = \frac{141}{100}$ Since $n = 50$ is large, by **Central Limit Theorem**, Let $T = X_1 + X_2 + ... + X_{50} \sim \text{N}\left(50\left(\frac{7}{10}\right), 50\left(\frac{141}{100}\right)\right)$ **approximately**. $\therefore T \sim \text{N}(35, 70.5)$ **approximately**.

	$\text{P}(\lvert T - 36 \rvert < 5)$ $= \text{P}(-5 < T - 36 < 5)$ $= \text{P}(31 < T < 41)$ $= 0.445667$ $= 0.446$ (3 s.f.)