RAFFLES INSTITUTION
2023 YEAR 6 COMMON TEST

CANDIDATE NAME

CLASS 23

MATHEMATICS 9758

3 hours

Candidates answer on the Question Paper
Additional Materials: List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your name and class on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers in the spaces provided in the question paper.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved graphing calculator is expected, where appropriate.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

FOR EXAMINER'S USE							
Section A: Pure Mathematics							TOTAL
Q1	Q2	Q3	Q4	Q5	Q6	Subtotal	
4	10	10	11	12	13	60	
Section B: Statistics							
Q7	Q8	Q9	Q10	Q11	Subtotal		
5	7	7	8	13	40		100

This document consists of ~~26~~ 7 printed pages.

RAFFLES INSTITUTION
Mathematics Department

Section A: Pure Mathematics [60 marks]

1 The points A, B and C have position vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ respectively. The points A and B are fixed while C varies.

(a) Given that $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are non-zero vectors such that $(\mathbf{c}-\mathbf{a})\times\mathbf{b}=\mathbf{b}\times(\mathbf{c}-\mathbf{a})$, find the relationship between $(\mathbf{c}-\mathbf{a})$ and $\mathbf{b}$. [2]

(b) Hence describe geometrically the set of all possible positions of the point C. [2]

2 Complete the square for $2x^2-9x+5$. [1]

The function f is defined by

$$\mathrm{f}:x\mapsto\left|2x^2-9x+5\right|,\quad \text{for } x\in\mathbb{R}.$$

(a) Sketch the graph of $y=\mathrm{f}(x)$, stating the coordinates of any turning points and points of intersection with the axes. [2]

(b) If the domain of f is further restricted to $1\le x\le k$, state with a reason the largest value of k for which the function f^{-1} exists. [2]

In the rest of the question, the domain of f is $x\in\mathbb{R}$, $1\le x\le 2$.

(c) Find the solution set of $\mathrm{f}(x)=\mathrm{f}^{-1}(x)$. [1]

The function g is defined by

$$\mathrm{g}:x\mapsto\frac{1}{x},\quad \text{for } x\in\mathbb{R},\ 0<x\le 5.$$

(d) State whether the composite function $\mathrm{f}^{-1}\mathrm{g}$ exists, justifying your answer. [2]

(e) If the domain of g is further restricted to $p\le x\le q$ and the composite function gg exists, find an equation involving p and q. [2]

3 Cadence, the pedal speed in cycling, is measured in pedal stroke revolutions per minute (RPM). For example, a cadence of 60 RPM means that one pedal makes a complete revolution 60 times in one minute. Every complete revolution of the pedal stroke of a stationary bicycle is equivalent to cycling an approximate distance of 0.006 km.

Lisa signed up for a beginner spinning class. The workout routine requires her to start cycling at 30 RPM consistently for the first minute and progressively increase the cadence by 2.5 RPM for every subsequent minute.

(a) At which minute will Lisa cycle at 120 RPM? [2]

(b) Find Lisa's average speed of cycling, in km/h, for the first 15 minutes. [3]

Celina signed up for an intermediate spinning class. The workout routine requires her to start cycling at 25 RPM consistently for the first minute and increase the cadence by 15% for every subsequent minute.

(c) At which minute does Celina's cadence first exceed 120 RPM? [2]

(d) If Lisa and Celina start their workout routine at the same time, at which minute does Celina's total distance cycled first exceed Lisa's total distance cycled? [3]

4 A curve C has parametric equations

$$x = \left(a^2 - t^2\right)^{\frac{1}{2}}, \qquad y = \ln t, \qquad \text{for } 0 < t \le a \text{ and } a > 1.$$

(a) C crosses the x-axis at the point P. Find the x-coordinate of P in terms of a. [1]

(b) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} \le 0$ for $0 < t \le a$. [3]

(c) Describe the behaviour of the tangent to C as $t \to 0$. [1]

(d) Sketch the graph of C, stating the equations of any asymptotes and the coordinates of the points where the curve meets the axes. [2]

(e) The tangent at P cuts the y-axis at the point Q and the normal at P cuts the y-axis at the point R. Find the area of triangle PQR. [4]

5 It is given that

$$\frac{\mathrm{d}w}{\mathrm{d}x}=(w-1)(2-w)\,, \quad \text{for } 1<w<2\,. \qquad \textbf{(I)}$$

(a) By solving **(I)**, show that $w=1+\dfrac{A\mathrm{e}^{x}}{1+A\mathrm{e}^{x}}$, where A is a positive constant. [5]

(b) Sketch the graph of w against x, stating the equation of any asymptotes and the coordinates of the point(s) where the curve meets the axes. [2]

It is also given that

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2-(3+2x)\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)+x^2+3x+1=0\,, \quad \text{for } 1<\frac{\mathrm{d}y}{\mathrm{d}x}-x<2\,. \qquad \textbf{(II)}$$

(c) Using the substitution $\dfrac{\mathrm{d}y}{\mathrm{d}x}=w+x$, show that **(II)** can be transformed to **(I)**.

Hence find y in terms of x. [5]

6 The plane π_1 contains the point $(2,-2,1)$ and the line $\mathbf{r}=\begin{pmatrix}0\\1\\8\end{pmatrix}+\lambda\begin{pmatrix}0\\2\\3\end{pmatrix}$, where λ is a parameter.

(a) Find an equation of π_1 in scalar product form. [2]

The plane π_2 has vector equation $\mathbf{r}=a\begin{pmatrix}0\\-1\\4\end{pmatrix}+b\begin{pmatrix}1\\2\\-1\end{pmatrix}$, where a and b are parameters.

(b) Find the acute angle between π_1 and π_2. [3]

The line l passes through the point A with position vector $4\mathbf{i}+5\mathbf{j}+6\mathbf{k}$ and is parallel to both π_1 and π_2.

(c) Find a vector equation for l. [2]

The plane π_3 with the cartesian equation $3x+2y+mz=0$ is also parallel to l.

(d) Show that $m=-6$. [1]

(e) Find $\overrightarrow{OA}\cdot(3\mathbf{i}+2\mathbf{j}-6\mathbf{k})$. [1]

(f) Find the distance between l and π_3. [2]

(g) Hence or otherwise, find an equation of the line which is a reflection of l in π_3. [2]

Section B: Statistics [40 marks]

7 A firm claims that, on average, their workers spend no more than 8 hours a day at the construction site. The amount of time clocked, x hours, by each worker from a random sample of 150 workers, is summarised as follows.

$$\sum x = 1305 , \quad \sum x^2 = 12751 .$$

Test, at the 1% significance level, whether the firm's claim is valid. [5]

8 Colour Vision Deficiency is the decreased ability to see differences in colours. Someone with normal colour vision will be able to see 6 colours (Red, Orange, Yellow, Green, Blue, Violet) as 6 bands of colours.

Deutan, who has colour vision deficiency, is not able to differentiate the following two pairs of colours:

(I) Red and Green,

(II) Blue and Violet.

For 6 colours (Red, Orange, Yellow, Green, Blue, Violet) of a particular shade, when the above pairs of colours are placed next to each other, it will look like a single band of colour to Deutan. Some examples of his vision of these 6 colours are as follows.

Arrangement of these 6 colours in a line	**Number of bands it will appear to Deutan**
Orange, Yellow, Red, Green, Blue, Violet	4
Red, Orange, Yellow, Green, Violet, Blue	5
Violet, Red, Orange, Yellow, Green, Blue	6

Find the number of ways these 6 colours can be arranged such that Deutan will see exactly

(a) 4 bands of colour, [2]

(b) 5 bands of colour, [3]

(c) 6 bands of colour. [2]

9 Two families plan to go on a hiking trip together. There are 10 people from the Lee family, consisting of 3 married couples, 3 single men and 1 single woman. There are 14 people from the Tan family, consisting of 4 married couples, 1 single man and 5 single women. Two people are randomly chosen to organise the hiking trip.

(a) Show that the probability that they are both from the Lee family is $\frac{15}{92}$. [1]

(b) Find the probability that they are a man and woman from the same family. [3]

(c) Find the probability that they are married to each other given that they are a man and a woman from the same family. [3]

10 In this question you should state clearly the parameters of any normal distributions you use.

The masses in kilograms of chickens have the distribution $\mathrm{N}(1.5, \sigma^2)$ such that 7.65% of them have a mass greater than 1.6 kg.

(a) Find the value of σ. [2]

The masses in kilograms of ducks have the distribution $\mathrm{N}(2.0, 0.08^2)$.

(b) Find the probability that the mass of a randomly chosen chicken is less than 1.6 kg and the mass of a randomly chosen duck is more than 1.9 kg. [2]

(c) Chickens and ducks are sold at \$11 per kg and \$18 per kg respectively. Find the probability that the total selling price of 4 randomly chosen chickens differs from twice the selling price of a randomly chosen duck by at least \$5. [4]

11 By the end of 2022, the percentages of a population who has received V dose(s) of Covid-19 vaccine are shown in the table below, with no one receiving 5 or more doses of vaccine.

v	0	1	2	3	4
Percentage of population	7	a	b	69	9

(a) Given that $\mathrm{E}(V) = 2.72$, find $\mathrm{Var}(V)$. [4]

Mr Lee is doing a survey on people's perspectives on the Covid-19 vaccine.

(b) Find the probability that the total number of doses of vaccine received by 3 randomly selected surveyees is more than 10. [2]

Moderna's and Pfizer-BioNTech's bivalent vaccine are available at a particular vaccination centre.

On average 60% of walk-ins opt for Moderna's bivalent vaccine at this centre. Let M denote the number of people, out of n walk-ins surveyed by Mr Lee on a particular day, that choose to receive the Moderna's bivalent vaccine.

(c) Explain, in the context of this question, the assumptions needed to model M by a binomial distribution. [2]

Assume now that these assumptions do in fact hold.

(d) Find $\mathrm{P}(4 \le M < 8)$ when $n = 10$. [2]

(e) Find the least number of people Mr Lee needs to survey to have at least a 90% chance of at least 15 surveyees choosing to receive Moderna's bivalent vaccine. [3]