ORCHID PARK SECONDARY SCHOOL
Preliminary Examination 2024

Marker 1	Chan Ho Lun
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MATHEMATICS **4049/01**

Paper 1 **20 October 2024**

Secondary 4 Express / 5 Normal (Academic) **2 hours 15 minutes**

Setter: Mr Chan Ho Lun **90 Marks**

Additional Materials: NIL

Answer Cover Page

Note:
-2m maximum for not giving 1 dp for angles for whole paper.
-2m maximum for not giving 3 sf for inaccurate answers for whole paper.

Qn			Solution	Marks	Remarks
1			$y = px - 5$______________(1) $y = 3x^2 + 4x - 2$_______(2) Sub (1) into (2): $3x^2 + 4x - 2 = px - 5$ $3x^2 + (4 - p)x + 3 = 0$ Line meets curve => Discriminant ≥ 0: $(4 - p)^2 - 4(3)(3) \geq 0$ $16 - 8p + p^2 - 36 \geq 0$ $p^2 - 8p - 20 \geq 0$ $(p - 10)(p + 2) \geq 0$ $p \leq -2 \ or \ p \geq 10$	M1 M1 M1 A1	
2			Let $\frac{5x^2-6x+13}{(x-1)(x^2+3)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+3}$ $5x^2 - 6x + 13 = A(x^2 + 3) + (Bx + C)(x - 1)$ Method 1: When $x = 1$, $12 = 4A$ $A = 3$ When $x = 0$, 13 = 3(3) + C (-1) $C = -4$ When $x = -1$, $24 = 3(4) + (-B - 4)(-2)$ $B = 2$ Hence, $\frac{5x^2-6x+13}{(x-1)(x^2+3)} = \frac{3}{x-1} + \frac{2x-4}{x^2+3}$	M1 M1 M1 M1 A1	
3			RHS $= cosec\ \theta \sec\theta - 2\tan\theta$ $= \frac{1}{\sin\theta}\left(\frac{1}{\cos\theta}\right) - \frac{2\sin\theta}{\cos\theta}$ $= \frac{1}{\sin\theta\cos\theta} - \frac{2\sin^2\theta}{\sin\theta\cos\theta}$ $= \frac{1 - 2\sin^2\theta}{\sin\theta\cos\theta}$ $= \frac{\cos 2\theta}{\frac{1}{2}\sin 2\theta}$ $= \frac{2\cos 2\theta}{\sin 2\theta} = 2\cot 2\theta = LHS$	M1 M1 M2 A1	for changing either cosec, sec or tan correctly for common denominator 1 mark for $\cos 2\theta$, 1 mark for $\sin 2\theta$

4	(a)		$5e^{2x+1} + 10xe^{2x+1}$	B2	1 mark for each term
	(b)		$5xe^{2x+1} = \frac{5}{2}\ e^{2x+1} + 10\int x\ e^{2x+1}\ dx + c_1$ M1 M1 where c_1 is an arbitrary constant $5xe^{2x+1} - \frac{5}{2}e^{2x+1} - c_1 = 10\int x\ e^{2x+1}\ dx$ $\int x\ e^{2x+1}\ dx = \frac{1}{2}xe^{2x+1} - \frac{1}{4}e^{2x+1} + c$ where $c = -\frac{1}{10}c_1$	M1 A1	
Qn			**Solution**	**Marks**	**Remarks**
5			$1 + 3(1 - \cos^2\theta) = 4\cos\theta$ $-3\cos^2\theta - 4\cos\theta + 4 = 0$ Let $y = \cos\theta$ $-3y^2 - 4y + 4 = 0$ $3y^2 + 4y - 4 = 0$ $(3y - 2)(y + 2) = 0$ $y = \frac{2}{3}$ or $y = -2$ $\cos\theta = \frac{2}{3}\ or\ \cos\theta = -2\ (rej)$ $\theta = 0.841\ or - 0.841$ (3 s.f.)	M1 M1 M2 A2	
6	(a)		$f'(x)$ $= \frac{(x-2k)(2x) - x^2(1)}{(x-2k)^2}$ $= \frac{x^2 - 4kx}{(x-2k)^2}$	M1 A1	

6	(b)		$g'(x)$ $= x^2 - 4kx$ $= x(x - 4k)$ g decreases => $g'(x) < 0$ $x(x - 4k) < 0$ $0 < x < 4k$ Since $x > 2k$, then $2k < x < 4k$______(1) But it is also given that $k < x < 6$, i.e. $2k < x < 12$___(2) By (1) and (2), $4k = 12$ $k = 3$	M1 M1 M1 A1	
7			For the function to be always positive, $k > 0$ (so that the graph is U-shaped)___(1) We also require discriminant < 0 (so that graph never cuts *x*-axis) $2^2 - 4(k)(-2k - 3) < 0$ $2k^2 + 3k + 1 < 0$ $(k + 1)(2k + 1) < 0$ $-1 < k < -\frac{1}{2}$ _____________________(2) But (1) and (2) cannot happen at the same time (*k* cannot be positive but yet also be between -1 and $-\frac{1}{2}$) ∴ There is no value of *k* for which the function is positive (proven)	M1 M1 M1 M1 M1 A1	

Qn			Solution	Marks	Remarks
8			$y - x = 2$ $y = x + 2$ ________________________(1) $y^2 = 4(2x + 1)$ ___________________(2) Sub (1) into (2): $(x + 2)^2 = 4(2x + 1)$ $x^2 + 4x + 4 = 8x + 4$ $x^2 - 4x = 0$ $x(x - 4) = 0$ $x = 0 \; or \; x = 4$ Sub into (1): $y = 2 \; or \; y = 6$ The two points are (0, 2) and (4, 6)	M1 M1 M1 M1 A1	
9	(a)		$\frac{dy}{dx} = 3x^2 + m$ When $x = 2, \frac{dy}{dx} = 0 \; (given)$: $0 = 3(2)^2 + m$ $m = -12$	M1 A1	
	(b)		$\frac{dy}{dx} = 3x^2 - 12$ For stationary points, $\frac{dy}{dx} = 0$ $3x^2 - 12 = 0$ $x^2 = 4$ $x = 2 \; or -2$ When $x = -2$, $y = (-2)^3 - 12(-2) - 15$ $y = 1$ B is (-2, 1)	M1 A1	

	(c)		For gradient to be a min, $\frac{d^2y}{dx^2} = 0$ $6x = 0$ $x = 0$ When $x = 0$, $y = -15$ P is (0, -15)	M1 M1 A1	
	(d)		$\frac{d^3y}{dx^3} = 6$ Since $\frac{d^3y}{dx^3}$ is positive, the gradient is a minimum.	M1 A1	
Qn			**Solution**	**Marks**	**Remarks**
10	(a)		Method 1: $(x-2)^2 + (y-3)^2 - 2^2 - 3^2 - 12 = 0$ $(x-2)^2 + (y-3)^2 = 25$ Method 2: $2g = -4$ $2f = -6$ $g = -2$ $f = -3$ $g^2 + f^2 - c = 4 + 9 + 12 = 25$ Centre = (2, 3) Radius = 5 units	M1 M1 M1 M1 A1 A1	
	(b)		When $y = 0$, $x^2 - 4x - 12 = 0$ $(x-6)(x+2) = 0$ $x = 6$ or $x = -2$ The points are (6, 0) and (-2, 0)	M1 A2	
	(c)		Centre = (-2, 3), radius = 5 units Eqn: $(x+2)^2 + (y-3)^2 = 25$	M1 A1 /B2	
	(d)		Since the centre is only 2 units away from the *y*-axis and the radius is 5 units, the circle will cut the *y*-axis and does not lie entirely in the 2nd quadrant.	M1 A1	Accept any similar answer (e.g. centre is 3 units away from *x*-axis)

11	(a)		$A = \pi r^2 + 2\pi kr + c$ where c is an arbitrary constant $When\ r = 0, A = 0$: (when there is no radius, there is no area) $c = 0$ Hence, $A = \pi r^2 + 2\pi kr$	M1 A1	
	(b)		Curved surface area: $2\pi rh = 2\pi kr$ $h = k$	B1	
	(c)		$\frac{dr}{dt} = \frac{5}{2}\ln(2t+1) + c_1$ where c_1 is an arbitrary constant $When\ t = 0, \frac{dr}{dt} = 3$: $3 = c_1$ $\frac{dr}{dt} = \frac{5}{2}\ln(2t+1) + 3$	M1 A1	
	(d)		$\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$ $\frac{dA}{dt} = 2\pi(r+k) \times [\frac{5}{2}\ln(2t+1) + 3]$ When $r = 15, t = 4, k = 10$: $\frac{dA}{dt} = 2\pi(15+10) \times \left(\frac{5}{2}\ln 9 + 3\right)$ $\frac{dA}{dt} =$	M1 M1 A1	

Qn			Solution	Marks	Remarks
12	(a)		$4x + 4y = 100$ $y = 25 - x$	M1 A1	
	(b)		A $= x^2 + y^2$ $= x^2 + (25-x)^2$ $= 2x^2 - 50x + 625$ $= 2\left[\left(x - \frac{25}{2}\right)^2 - \left(\frac{25}{2}\right)^2 + \frac{625}{2}\right]$ $= 2\left(x - \frac{25}{2}\right)^2 + \frac{625}{2}$	 M1 M1 M1 A1	

	(c)		Min area = $\frac{625}{2}$ when $x = \frac{25}{2}$	B1 B1	
	(d)		$25 - x_1$ (E.g. if $x_1 = 10$, then $25 - x_1 = 15$ will give the same area)	B1	
13	(a)		$\left(\frac{3-7}{2}, \frac{2h-10}{2}\right)$ $= (-2, h-5)$	M1 A1	
	(b)		$\frac{2h+10}{3+7}$ $= \frac{h+5}{5}$	M1 A1	
	(c)		Let the equation of the perpendicular bisector be $\frac{y-(h-5)}{x-(-2)} = -\frac{5}{h+5}$ Sub (h, 3): $\frac{3-(h-5)}{h-(-2)} = -\frac{5}{h+5}$ $h^2 - 8h - 50 = 0$ $h = \frac{8 \pm \sqrt{8^2 - 4(1)(-50)}}{2(1)}$ $h = 12.1 \text{ or } -4.12 \ (3\ s.f.)$	 M2 M1 M1 M1 A2	1 mark for substituting M 1 mark for gradient