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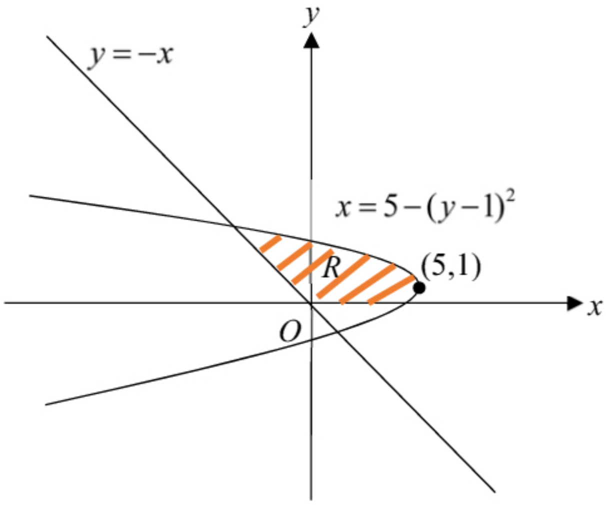

The shaded region R bounded by the curve $y = -x$, the line $y = -x$ and the x-axis is rotated about the x-axis through 360° . Find the volume of the solid formed, leaving your answer to 2 decimal places. [4]

2 **(i)** Solve the inequality $\dfrac{x^2 - ax - a}{x - a} \geq a$, where a is a positive real constant, leaving your answer in terms of a. [4]

(ii) Hence, by using a suitable value for a, solve the inequality

$$\frac{4e^{2x} - e^x - 1}{4e^x - 1} \geq \frac{1}{4}$$

leaving your answer in exact form. [3]

3 The parametric equations of a curve C are $x = at$, $y = at^3$, where a is a positive constant.

(i) The point P on the curve has parameter p and the tangent to the curve at point P cuts the y-axis at S and the x-axis at T. The point M is the midpoint of ST. Find a Cartesian equation of the curve traced by M as p varies. [5]

(ii) Find the exact area bounded by the curve C, the line $x = 0$, $x = 3$ and the x-axis, giving your answer in terms of a. [3]

4 It is given that $y=\sin^{-1}x\cos^{-1}x$, where $-1\le x\le 1$.

(i) Show that $\sqrt{1-x^2}\,\dfrac{\mathrm{d}y}{\mathrm{d}x}=\cos^{-1}x-\sin^{-1}x$. [1]

(ii) Show that $\left(1-x^2\right)\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}-x\dfrac{\mathrm{d}y}{\mathrm{d}x}=-2$ [2]

(iii) Hence find the exact value of *A*, *B* and *C* if *y* can be expressed as $Ax+Bx^2+Cx^3$, up to (and including) the term in x^3. [4]

(iv) A student used **(iii)** to estimate that $\sin^{-1}(0.8)\cos^{-1}(0.8)\approx 0.8A+0.8^2B+0.8^3C$. Explain, with working, if his estimate is a good one. [1]

5 **(a)** Referred to the origin *O*, the points *A* and *B* have position vectors $\mathbf{a}$ and $\mathbf{b}$. Point *C* is on the line which contains *A* and is parallel to $\mathbf{b}$. It is given that the vectors $\mathbf{a}$ and $\mathbf{b}$ are both of magnitude 2 units and are at an angle of $\sin^{-1}(1/6)$ to each other. If the area of triangle *OAC* is 3 units^2, use vector product to find the possible position vectors of *C* in terms of $\mathbf{a}$ and $\mathbf{b}$. [5]

(b) Referred to the origin *O*, the points *P* and *Q* have position vectors $\mathbf{p}$ and $\mathbf{q}$ where $\mathbf{p}$ and $\mathbf{q}$ are non-parallel, non-zero vectors. Point *R* is on *PQ* produced such that $PQ:QR=1:\lambda$. Point *M* is the mid-point of *OR*.

(i) Find the position vector of *R* in terms of λ, $\mathbf{p}$ and $\mathbf{q}$. [1]

F is a point on *OQ* such that *F*, *P* and *M* are collinear.

(ii) Find the ratio *OF*:*FQ*, in terms of λ. [4]

6 **Do not use a calculator in answering this question.**

(a) It is given that two complex numbers *z* and *w* satisfy the following equations

$$13z=(4-7\mathrm{i})\,w,$$
$$z-2w=5-4\mathrm{i}.$$

Find *z* and *w*. [4]

(b) It is given that $q=-\sqrt{3}-\mathrm{i}$.

(i) Find an exact expression for q^6, giving your answer in the form $re^{\mathrm{i}\theta}$, where $r>0$ and $0\le\theta<2\pi$. [3]

(ii) Find the three smallest positive whole number values of *n* for which $\dfrac{q^n}{q^*}$ is purely imaginary. [4]

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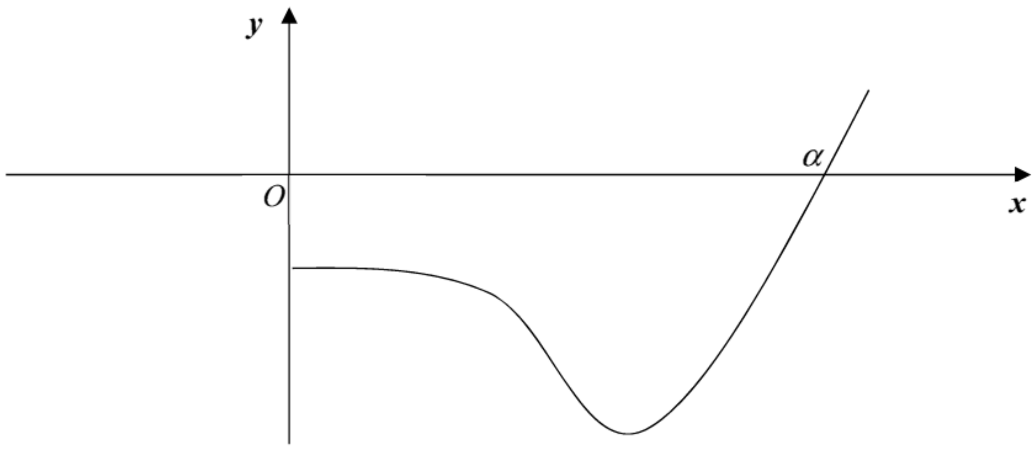

It is given that $\mathrm{f}(x) = 2x^6 - 4x^4 - 6x^2 - 7$. The diagram shows the curve with equation $y = \mathrm{f}(x)$ for $x \geq 0$. The curve crosses the positive x-axis at $x = \alpha$.

(i) Find the value of α, giving your answer correct to 3 decimal places. [1]

(ii) Show that $\mathrm{f}(x) = \mathrm{f}(-x)$ for all real values of x. What can be said about the six roots of the equation $\mathrm{f}(x) = 0$? [4]

It is given that $\mathrm{g}'(x) = \mathrm{f}(x)$, for all real values of x.

(iii) Determine the x-coordinates of all the stationary points of graph of $y = \mathrm{g}(x)$ and determine their nature. [3]

(iv) For which values of x is the graph of $y = \mathrm{g}(x)$ concave upwards? [3]

8 **(a)** **(i)** Show that, for $r \in$ ¢, $r \geq 2$,

$$\frac{1}{(r-1)!} - \frac{2}{r!} + \frac{1}{(r+1)!} = \frac{r^2-r-1}{(r+1)!}.$$ [1]

Let $S_n = \sum_{r=2}^{n} \frac{r^2-r-1}{(r+1)!}$.

(ii) Hence find S_n in terms of n. [3]

(iii) Show that S_n converges to a limit L, where L is to be determined. [2]

(iv) Find the least integer value of n such that S_n differs from L by less than 10^{-10}. [2]

(b) **(i)** Suppose that f is a continuous, strictly decreasing function defined on $[1, \infty)$, with $\mathrm{f}(x) > 0$, $x \geq 1$. According to the Maclaurin-Cauchy test, then the infinite series $\sum_{n=1}^{\infty} \mathrm{f}(n)$ is convergent if and only if the integral $\int_1^{\infty} \mathrm{f}(x)\,\mathrm{d}x$ is finite. By applying the Maclaurin-Cauchy test on the function f defined by $\mathrm{f}(x) = \frac{1}{x}$, $x \geq 1$, determine if the infinite series $\sum_{n=1}^{\infty} \frac{1}{n}$ is convergent. [2]

(ii) Let p be a positive number. By considering the Maclaurin-Cauchy test, show that if $p > 1$, the infinite series $1 + \frac{1}{2^p} + \frac{1}{3^p} + \ldots \frac{1}{n^p} + \ldots$ is convergent. [2]

9 A drilling company plans to install a straight pipeline AB through a mountain. Points (x, y, z) are defined relative to a main control site at the foot of the mountain at $(0,0,0)$, where units are metres. The x-axis points East, the y-axis points North and the z-axis points vertically upwards. Point A has coordinates $(-200,150,10)$ while point B has coordinates $(100,10,a)$, where a is an integer. Point B is at a higher altitude than Point A.

(i) Given that the pipeline AB is of length 337 metres, find the coordinates of B. [3]

A thin flat layer of rock runs through the mountain and is contained in the plane with equation $20x + y + 2z = -837$.

(ii) Find the coordinates of the point where the pipeline meets the layer of rock. [4]

To stabilise the pipeline, the drilling company decides to build 2 cables to join points A and B to the layer of rock. Point A is joined to Point P while point B is joined to Point Q.

(iii) Assuming that the minimum length of cable is to be used, find the length PQ. [2]

(iv) Show that the pipeline is at an angle of 10.8^0 to the horizontal plane.
[2]

(v) After the pipeline is completed, a ball bearing is released from point B to roll down the pipeline to check for obstacles. The ball bearing loses altitude at a rate of $0.3t$ metres per second, where t is the time (in seconds) after its release. Find the speed at which the ball bearing is moving along the pipeline 10 seconds after its release.
[3]

10 An epidemiologist is studying the spread of a disease, dengue fever, which is spread by mosquitoes, in town A. P is defined as the number of infected people (in thousands) t years after the study begins. The epidemiologist predicts that the rate of increase of P is proportional to the product of the number of infected people and the number of uninfected people. It is known that town A has 10 thousand people of which a thousand were infected initially.

(i) Write down a differential equation that is satisfied by P. [1]

(ii) Given that the epidemiologist projects that it will take 2 years for half the town's population to be infected, solve the differential equation in **(i)** and express P in terms of t. [6]

(iii) Hence, sketch a graph of P against t. [2]

A second epidemiologist proposes an alternative model for the spread of the disease with the following differential equation:

$$\frac{\mathrm{d}P}{\mathrm{d}t} = \frac{2\cos t}{(2-\sin t)^2} \qquad (*).$$

(iv) Using the same initial condition, solve the differential equation (*) to find an expression of P in terms of t. [3]

(v) Find the greatest and least values of P predicted by the alternative model. [2]

(vi) The government of town A deems the alternative model as a more realistic model for the spread of the disease as it more closely follows the observed pattern of the spread of the disease. What could be a possible factor contributing to this? [1]