RVHS H2 Mathematics Remedial Programme

Topic: Vectors III

<u>Basic Mastery Questions</u>

1. **ACJC Promo 9758/2020/Q8(i), (ii)**
The lines l and m are defined by the equations

$$l: \mathbf{r} = \mathbf{i} - \mathbf{k} + \lambda(2\mathbf{i} - 6\mathbf{j} + 3\mathbf{k}),$$
$$m: \frac{x-1}{4} = \frac{a-y}{a} = \frac{z+3}{4}.$$

(i) Given that the lines intersect, show that $a = 6$. [2]

(ii) Find the position vector of N, the foot of perpendicular from the point $A(5, 0, 1)$ to the line l. [3]

Answers: (ii) $\overrightarrow{ON} = \frac{1}{7}\begin{pmatrix} 11 \\ -12 \\ -1 \end{pmatrix}$

2. **SAJC Promo 9758/2020/Q10**

Relative to an origin O, the points A and C have position vectors $3\mathbf{i} + 2\mathbf{k}$ and $4\mathbf{i} + \mathbf{j}$ respectively. The point C is such that $OABC$ is a parallelogram and the point E divides AB in the ratio 1:3. The point M is the midpoint of CA.

(i) Find the cartesian equation of line CA and the vector equation of line OE. [5]
(ii) Find the position vector of point F, the point of intersection of CA and OE. Hence deduce the ratio $OF : OE$. [3]

(iii) Find angle OMC, giving your answer correct to the nearest 0.1°. [3]

(iv) Find the exact length of projection of OM onto CA. [2]

Answers: (i) $\mathbf{r} = \mu\begin{pmatrix} 16 \\ 1 \\ 8 \end{pmatrix}, \quad \mu \in \mathbb{R}$ (ii) $\overrightarrow{OF} = \frac{1}{5}\begin{pmatrix} 16 \\ 1 \\ 8 \end{pmatrix}$, $OF : OE = 4 : 5$

(iii) $\angle OMC = 102.8^\circ$ (iv) $\frac{1}{3}\sqrt{6}$ units

Standard Questions

1. **ACJC Promo 9758/2021/Q6**

The Cartesian equation of line L_1 is $\dfrac{x-2}{a}=\dfrac{y+2}{b}=\dfrac{z-3}{c}$, where a,b,c are constants.

The line L_2 is parallel to the vector $4\mathbf{i}+3\mathbf{j}$. The line L_3 passes through the origin and the point with position vector $\mathbf{j}+\mathbf{k}$.

(i) Given that L_1 is perpendicular to L_2, form an equation relating a and b. [1]

(ii) Given that L_1 intersects L_3, show that $5a+2b-2c=0$. [3]

(iii) Hence express a and b in terms of c. [1]

Find the acute angle between L_1 and L_3. [2]

Answers: (i) $4a+3b=0$ (iii) $a=\dfrac{6}{7}c;\ b=-\dfrac{8}{7}c$ (iv) 86.7°

2. **CJC Promo 9758/2021/Q9**

With reference to the origin O, the points A and B have position vectors $\mathbf{a}=-\mathbf{i}+\mathbf{j}+2\mathbf{k}$ and $\mathbf{b}=2\mathbf{j}+5\mathbf{k}$ respectively.

(i) Find a vector equation of the line l_1 that passes through point A and is parallel to the vector $\mathbf{a}$.

(ii) Find the exact length of projection of $\mathbf{b}$ on l_1. Hence find d, the exact perpendicular distance from the point B to l_1. [4]

(iii) Using the value of d found in part **(ii)**, find the position vector of the point C, the foot of perpendicular from the point B to l_1. [3]

(iv) The line l_2 passes through point B and is parallel to vector $\mathbf{b}$. Find a cartesian equation of l_3 which is the reflection of l_2 in l_1. [3]

Answers: (i) $\mathbf{r}=\begin{pmatrix}-1\\1\\2\end{pmatrix}+k\begin{pmatrix}-1\\1\\2\end{pmatrix}=\lambda\begin{pmatrix}-1\\1\\2\end{pmatrix},\ \lambda\in\mathbb{R}$ (ii) $2\sqrt{6}$; $\sqrt{5}$

(i) $l_1 : \mathbf{r}=\begin{pmatrix}-1\\1\\2\end{pmatrix}+k\begin{pmatrix}-1\\1\\2\end{pmatrix}=\lambda\begin{pmatrix}-1\\1\\2\end{pmatrix},\ \lambda\in\mathbb{R}$ $l_1 : \mathbf{r}=\begin{pmatrix}-1\\1\\2\end{pmatrix}+k\begin{pmatrix}-1\\1\\2\end{pmatrix},\ k\in\mathbb{R}$ or AEF