Name : _____

Class	Index Number

METHODIST GIRLS' SCHOOL

Founded in 1887

PRELIMINARY EXAMINATION 2022
Secondary 4

Wednesday	**MATHEMATICS**	**4048/02**
3 August 2022	**Paper 2**	2 h 30 min

Candidates answer on the Question Paper.

INSTRUCTIONS TO CANDIDATES

Write your name, class and index number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 100**.**

100

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1+\frac{\text{r}}{100}\right)^n$$

Mensuration

Curved surface area of a cone = πrl

Surface area of a sphere = $4\pi r^2$

Volume of a cone = $\frac{1}{3}\pi r^2 h$

Volume of a sphere = $\frac{4}{3}\pi r^3$

Area of a triangle ABC = $\frac{1}{2}ab\sin C$

Arc length = $r\theta$, where θ is in radians

Sector area = $\frac{1}{2}r^2\theta$, where θ is in radians

Trigonometry

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f}-\left(\frac{\sum fx}{\sum f}\right)^2}$$

1 **(a)** Simplify $\frac{4}{x-3}+\frac{5}{(3-x)^2}$ as a single fraction in is lowest term.

Answer ……. [2]

(b) Simplify $\left(\frac{64x^6}{y^{-3}}\right)^{\frac{2}{3}}$, leaving your answer in positive index.

Answer ……. [2]

(c) Solve these simultaneous equations.

$$\frac{2}{3}x-\frac{1}{6}y=\frac{4}{3}$$

$$3x+\frac{3}{4}y=16$$

Answer x = ...

y = ... [3]

(d) **(i)** Express $5x^2 - 4x - 7$ in the form $a(x+b)^2 + c$.

Answer ……. [2]

(ii) Hence, solve $5x^2 = 4x + 9$.

Answer ……. [3]

2 **(a)** A fruit basket contains 2 apples and 2 oranges. A second bag contains 1 apple, 1 orange and 2 kiwis. Ben picks one fruit from each bag.

(i) Draw a possibility diagram to show all possible outcome.

Answer

[2]

(ii) Find the probability that Ben picks

(a) two apples,

Answer ……. [1]

(b) different fruits,

Answer ……. [1]

(c) at least one orange.

Answer ……. [1]

(b) Dream Renovation Company has three different packages. The table below shows the quantity of each type of work included in the different packages.

Package	Plumbing Work	Window	Carpentry Work
A	6	5	6
B	4	2	7
C	5	3	9

This information is represented by the matrix $\mathbf{Q} = \begin{pmatrix} 6 & 5 & 6 \\ 4 & 2 & 7 \\ 5 & 3 & 9 \end{pmatrix}$.

The unit cost price of each work is as follows.

Plumbing Work	\$180
Window Work	\$420
Carpentry Work	\$390

(i) **(a)** Write down the 3×1 matrix **P** to represent the unit cost of each work.

Answer **P** = ……. [1]

(b) Evaluate **QP**.

Answer **QP** = …….. [1]

(c) State what the elements of **QP** represent.

Answer……....

…….....…. [1]

(ii) A condominium's manager, Amy, decided to engage Dream Renovation Company to do renovation works. She ordered 85 package A, 90 Package B and 85 Package C. A matrix **Y** when multiplied by **QP**, will give the total cost of this renovation.

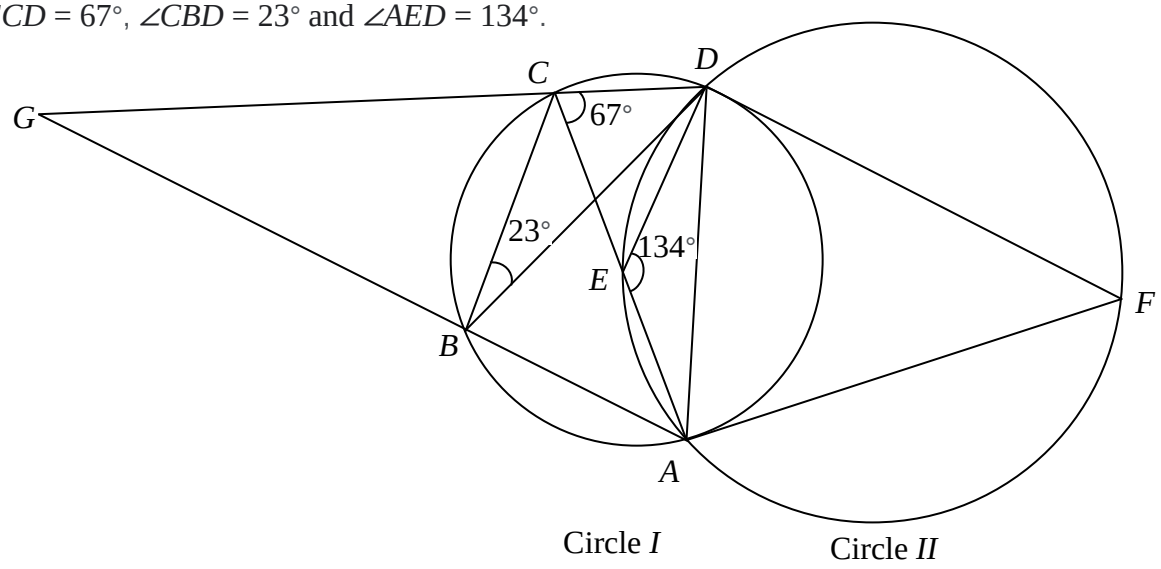
(a) State matrix **Y**.

Answer **Y**= ……. [1]

(b) Using matrix multiplication, find the total cost of this renovation.

Answer $ ……. [2]

3 In the diagram, the points *A, B, C, D* lie on circle *I*. The points *A, E, D, F* lie on circle *II*. *AEC* is a straight line. *DF* and *AF* are tangents to circle *I*. *CD* and *AB* are extended to meet at point *G*.
$\angle ECD = 67°$, $\angle CBD = 23°$ and $\angle AED = 134°$.



(a) Stating your reason clearly, find

(i) $\angle AFD$,

Answer $\angle AFD$ = …….. [1]

(ii) $\angle CAD$,

Answer $\angle CAD$ = …….. [1]

(iii) a line that has the same length as DF.

Answer line ……..... [2]

(b) Explain why the centre of circle I lie on the circumference of circle II.

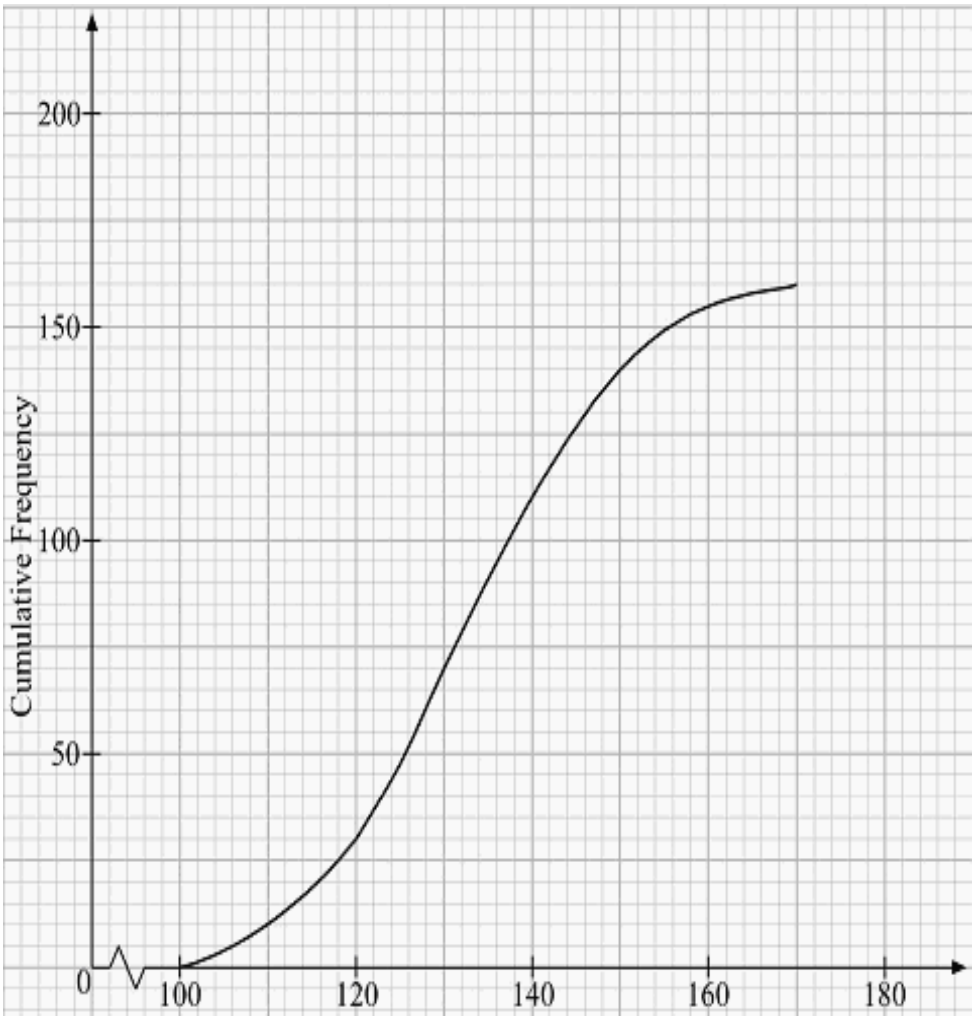
[3]

(c) Show that triangle *BCG* is similar to triangle *DAG*. State your working and reasons clearly.

Answer

[2]

4 The cumulative frequency curve shows the distribution of the masses of 160 chicks from Dairy Farm.



(a) From the curve, find

(i) the median mass,

Answer ……. [1]

(ii) the interquartile range,

Answer ……. [1]

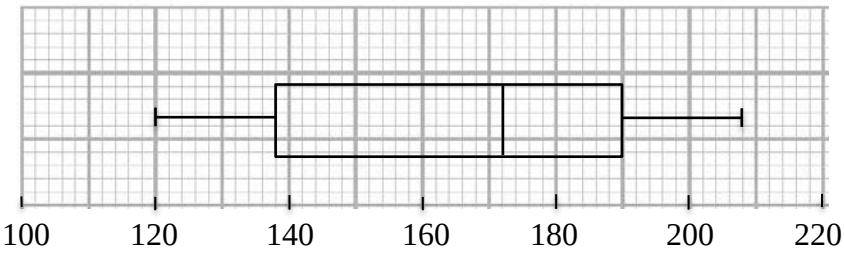
(b) Given that 62.5% of the chicks have a mass of more than x g, find the value of x.

Answer ……. [2]

(c) Chicks that have mass less than 120g are classified as malnourished. Two chicks are selected at random. Find the probability that at least 1 chick selected is malnourished.

Answer ……. [2]

(d) The box-and-whisker plot below shows the distribution of the weight of the chicks one month later.



Make two comparisons between the distributions of the weights of the chicks in the one month.

Answer …….....

…….....

……..... [3]

5 At a bowling alley, Patrick rolls a bowling ball towards the pins. The ball travels 18 metres at an average speed of x metres per second.

(a) Write an expression, in terms of x, for the time taken, in seconds, for the ball to reach the pins.

Answer …….. s [1]

(b) Patrick then picks up a second bowling ball and rolls it. The average speed of the second ball is 0.5 metres per second greater than the average speed of the first ball.

(i) Write an expression, in terms of x, for the time taken, in seconds, for the second ball to reach the pins.

Answer ……. s [1]

(ii) There is a 54-second time gap between Patrick rolling the second ball and when the first ball hit the pins. The total time taken when Patrick rolls off his first ball to the time when the second ball hits the pins is 1 minute.
Write down an equation in x, and show that it simplifies to $2x^2 - 11x - 3 = 0$.

Answer

[2]

(c) **(i)** Solve the equation $2x^2 - 11x - 3 = 0$, giving both answers correct to two decimal places.

Answer x = …… or …… [3]

(ii) Explain why one of the solutions in **(c)(i)** is rejected.

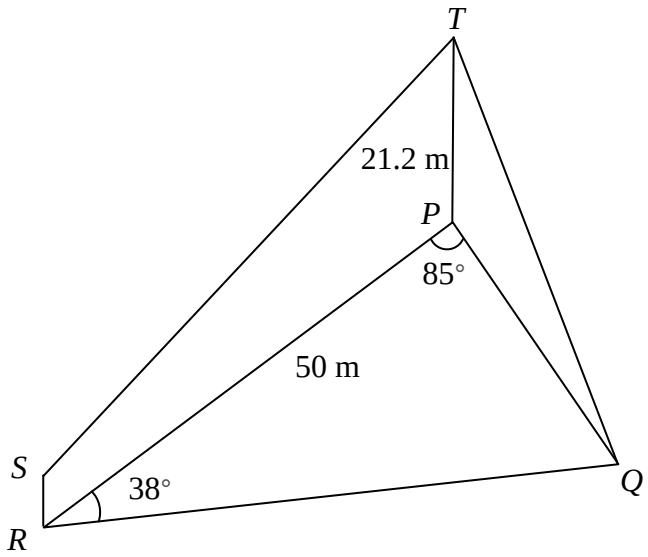
Answer.....

.....[1]

(d) How much longer, in seconds, does the first ball take to hit the pins than the second ball?

Answer …… s [2]

6 P, Q and R are three points on a horizontal ground. It is given that $PR = 50$ m, $\angle RPQ = 85°$, $\angle PRQ = 38°$. The height of the vertical tower TP is 21.2 m. S is the top of a vertical statue standing at R.



(a) Find

(i) the distance PQ,

Answer PQ = ... m [2]

(ii) the area of triangle PQR,

Answer ..... m^2 [2]

(iii) the greatest angle of elevation of the tower when viewed from a man walking along RQ.

Answer ……° [4]

(b) If the angle of depression of S from T is 20°, find the height of the statue.

Answer …….. m [3]

7 **(a)** Given that $\overrightarrow{RS} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$ and the position vector of R is $\begin{pmatrix} 3 \\ -5 \end{pmatrix}$, find

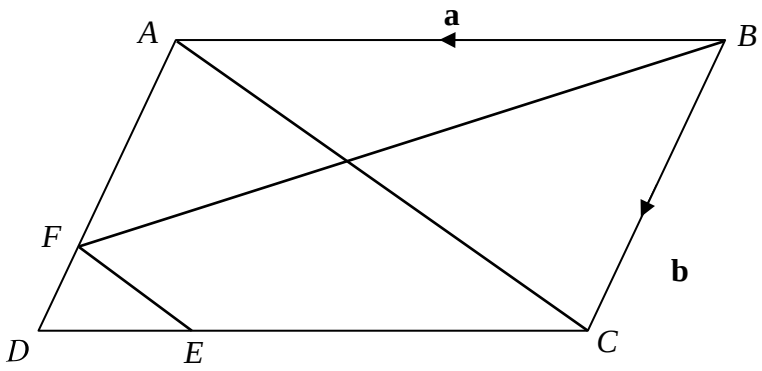
(i) $\left|\overrightarrow{RS}\right|$,

Answer …… units [1]

(ii) the coordinates of S.

Answer (…… , ……) [2]

(b)



$ABCD$ is a parallelogram. $\overrightarrow{AF} = 2\overrightarrow{FD}$, $\overrightarrow{ED} = \frac{1}{3}\overrightarrow{CD}$, $\overrightarrow{BA} =$ **a** and $\overrightarrow{BC} =$ **b.**

(i) Express, as simply as possible, in terms of **a** and/or **b**,

(a) $\overrightarrow{EC}$,

Answer …… [1]

(b) $\overrightarrow{DF}$,

Answer …… [1]

(c) $\overrightarrow{CA}$,

Answer ..... [1]

(d) Are $\overrightarrow{FE}$ and $\overrightarrow{CA}$ parallel vectors? Explain your answer with working.

Answer

[2]

(ii) Calculate the value of

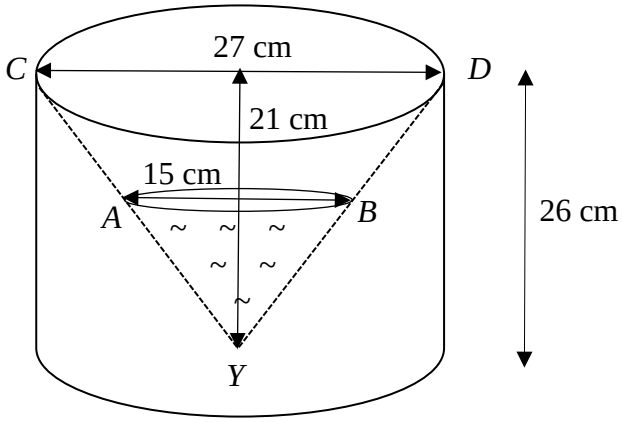
(a) $\dfrac{\text{area of } \Delta DEF}{\text{area of } \Delta DCA}$,

Answer ..... [1]

(b) $\dfrac{\text{area of } \Delta DEF}{\text{area of parallelogram } ABCD}$.

Answer ..... [1]

8



The figure shows a solid cylinder of height 26 cm with the cone *CYD* removed from it. The diameter and height of the big cone *CYD* is 27 cm and 21 cm respectively. Water is poured into the container forming a small cone *AYB* with diameter of 15 cm as shown in the figure.

(a) Show that the height of the cone *ABY* is $\frac{35}{3}$ cm.

Answer

[1]

(b) Calculate the length of *YB*.

Answer *YB* = ……. cm [2]

(c) The water is then emptied from the solid container. Find

(i) the total surface area of the solid container,

Answer …..... cm^2 [3]

(ii) the volume of the solid container.

Answer …..... cm^3 [2]

9 Energy is released during the breaking of molecular bonds. The amount of energy (y joules) released varies with the temperature (x °C) such that $y = 3(2^x)$. The table below shows $y = 3(2^x)$ for $-2 \le x \le 2$.

Temperature (x °C)	–2	–1	–0.5	0.5	1	1.5	2
Energy released (y joules)	0.75	1.5	2.12	p	6	8.49	12

(a) Find the value of p.

Answer p = ... [1]

(b) Using a scale of 2 cm to represent 0.5 degree Celsius, draw a horizontal x-axis for $-2 \le x \le 2$.
Using a scale of 1 cm to represent 1 joule on the y-axis, draw a vertical y-axis for $0 \le y \le 12$.
On your axes, plot the points given in the table and join them with a smooth curve. [3]

(c) Use your graph to estimate the amount of energy, in joules, that is released when the temperature is 0 degree Celsius.

Answer ..... J [1]

(d) **(i)** By drawing a tangent, find the gradient of the curve at $x = 1$.

Answer ..... [2]

(ii) State briefly what this gradient represent.

Answer .....

..... [1]

(e) By drawing a suitable straight line on the same axes, solve the equation $2^{x+1} = \frac{2x+8}{3}$ for $-2 \le x \le 2$.

Answer ..... [3]

10 (a) Jane planned a trip to Seoul, South Korea in November 2021 using the Vaccinated Travel Lane.

The table below shows the prices of a plane ticket from Singapore to Seoul in November 2021 on a particular airline. For example, the cost of a flight departing Singapore for Seoul on 4 November and a flight returning from Seoul to Singapore on 10 November was $990. Assume that all flights depart at 10 am and the flight time is 6 hours.

	Return 10 Nov (Thurs)	Return 11 Nov (Fri)	Return 12 Nov (Sat)
Depart 4 Nov (Fri)	$990	$1105	$1105
Depart 5 Nov (Sat)	$1263	$1303	$1263
Depart 6 Nov (Sun)	$1303	$1263	$990

Travellers to South Korea have to take 5 different tests. The table below shows the cost of 5 different tests.

Test		Where & When	Price
1	Pre-Departure PCR Test	Singapore, within 48 hours of departure	$91
2	On-Arrival PCR Test	Incheon Airport, upon arrival	$106 (Monday to Saturday) $212 (Sunday)
3	Self-Administered ART for those staying 8 days or more	South Korea, day 7 of arrival	$5
4	Pre-Departure ART	MOH approved hospitals within Seoul	$90
5	Supervised Self-administered ART	Quick Test Centre, within 24 hours of arrival in Singapore	$15

Jane wanted to spend 6 nights in Seoul.

(i) Which dates should Jane depart Singapore for Seoul and return to Singapore from Seoul? Justify your answer.

Answer .....

……

……

..... [2]

(ii) What would be the total cost incurred for the flight and all the PCR and ART tests throughout the trip if she travelled on the dates in **(a)(i)**?

Answer $ …….. [2]

(b) The tables below give information that Jane used to work out her total basic expenses for her **7 days and 6 nights** trip. She intended to purchase a Tourist Transportation Card to cover her public transportation costs for all 7 days.

The costs are all given in South Korean won (₩).

Estimated basic expenditure in South Korea

Hotel Accommodation	₩ 70 000 per night
Leisure (including sightseeing and theme park admission)	₩ 57 000 per day
Food & Beverages	₩ 60 000 per day

Tourist Transportation Card (for all public transportation)	
Card Duration	Price
1-day pass	₩ 15 000
3-day pass	₩ 30 500
5-day pass	₩ 44 500

*All costs are subjected to a 7.5% goods & service tax.

The exchange rate at a money exchange between South Korean won and Singapore dollars is given below.

		Singapore dollars (S$)	
Currency	Unit	Buying	Selling
South Korean won	1000	0.909	1.099

Suggest a sensible budget in Singapore dollars for Jane's basic expenditure in South Korea. Justify any decisions that you make and show your calculations clearly.

Answer

[5]

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Class	Index Number

Name : ___*Solutions*_____

METHODIST GIRLS' SCHOOL

Founded in 1887

PRELIMINARY EXAMINATION 2022

Secondary 4

Wednesday	**MATHEMATICS**	**4048/02**
3 August 2022	**Paper 2**	2 h 30 min

Candidates answer on the Question Paper.

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For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 100.

/100

This question paper consists of 25 printed pages and 3 blank pages.

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^{n}$$

Mensuration

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of a triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

1 **(a)** Simplify $\frac{4}{x-3}+\frac{5}{(3-x)^2}$ as a single fraction in is lowest term.

$\frac{4}{x-3}+\frac{5}{(3-x)^2}$

$=\frac{4}{x-3}-\frac{5}{(x-3)(3-x)}$ [M1]

$=\frac{4(3-x)-5}{(x-3)(3-x)}$

$=\frac{4x-7}{(x-3)^2}$ [A1]

or

$\frac{4}{x-3}+\frac{5}{(3-x)^2}$

$=\frac{5}{(3-x)^2}-\frac{4}{3-x}$ [M1]

$=\frac{5-4(3-x)}{(3-x)^2}$

$=\frac{4x-7}{(3-x)^2}$ [A1]

or

$\frac{4}{x-3}+\frac{5}{(x-3)^2}$ [M1]

$=\frac{4(x-3)+5}{(x-3)^2}$

$=\frac{4x-7}{(x-3)^2}$ [A1]

Answer ..... [2]

(b) Simplify $\left(\frac{64x^6}{y^{-3}}\right)^{\frac{2}{3}}$, leaving your answer in positive index.

$\left(\frac{64x^6}{y^{-3}}\right)^{\frac{2}{3}}$

$=\frac{16x^4}{y^{-2}}$ [M1 for exhibiting one law of indices]

$=16x^4y^2$ [A1]

Or $\left(4^3x^6y^3\right)^{\frac{2}{3}}=4^2x^4y^2=16x^4y^2$

Answer ..... [2]

(c) Solve these simultaneous equations.

$\frac{2}{3}x-\frac{1}{6}y=\frac{4}{3}$

$3x+\frac{3}{4}y=16$

$\frac{2}{3}x-\frac{1}{6}y=\frac{4}{3}-(1)$

$3x+\frac{3}{4}y=16-(2)$

$(2)\times 2,\ 6x+\frac{3}{2}y=32-(3)$

$(1)\times 9,\ 6x-\frac{3}{2}y=12-(4)$

$(3)-(4)$,

$3y=20$

$y=\frac{20}{3}=6\frac{2}{3}$

[M1-for using substitution or elimination method]

[A1]

sub $y=\frac{20}{3}$ into (2),

$3x=11$

$x=\frac{11}{3}=3\frac{2}{3}$

[A1]

Answer $x=$ .....

$y=$ ..... [3]

(d) (i) Express $5x^2-4x-7$ in the form $a(x+b)^2+c$.

$5x^2-4x-7$

$=5(x^2-\frac{4}{5}x)-7$

$=5\left[\left(x-\frac{4}{10}\right)^2-\left(\frac{4}{10}\right)^2\right]-7$ [M1]

$=5\left(x-\frac{2}{5}\right)^2-5\left(\frac{4}{25}\right)-7$

$=5\left(x-\frac{2}{5}\right)^2-7\frac{4}{5}$ [A1]

Answer ..... [2]

(ii) Hence, solve $5x^2=4x+9$.

$5x^2-4x-7=2$

$5\left(x-\frac{2}{5}\right)^2-7\frac{4}{5}=2$ [M1]

$\left(x-\frac{2}{5}\right)^2=\frac{49}{25}$

$x-\frac{2}{5}=\pm\frac{7}{5}$ [A2]

$x=\frac{9}{5}$ or $x=-1$

Answer ..... [3]

2 **(a)** A fruit basket contains 2 apples and 2 oranges. A second bag contains 1 apple, 1 orange and 2 kiwis. Ben picks one fruit from each bag.

(i) Draw a possibility diagram to show all possible outcome.

Answer

2nd Bag / 1st Bag	Apple	Orange	Kiwi	Kiwi
Apple	AA	AO	AK	AK
Apple	AA	AO	AK	AK
Orange	OA	OO	OK	OK
Orange	OA	OO	OK	OK

[A2]

[2]

(ii) Find the probability that Ben picks

(a) two apples,

P(two apples) = $\frac{1}{8}$ [B1]

Answer ..... [1]

(b) different fruits,

P(different kind) = $\frac{3}{4}$ [B1]

Answer ..... [1]

(c) at least one orange.

P(at least one orange) = $\frac{5}{8}$ [B1]

Answer ..... [1]

(b) Dream Renovation Company has three different packages. The table below shows the quantity of each type of work included in the different packages.

Package	Plumbing Work	Window	Carpentry Work
A	6	5	6
B	4	2	7
C	5	3	9

This information is represented by the matrix $\mathbf{Q} = \begin{pmatrix} 6 & 5 & 6 \\ 4 & 2 & 7 \\ 5 & 3 & 9 \end{pmatrix}$.

The unit cost price of each work is as follows.

Plumbing Work	\$180
Window Work	\$420
Carpentry Work	\$390

(i) **(a)** Write down the 3×1 matrix **P** to represent the unit cost of each work.

$$P = \begin{pmatrix} 180 \\ 420 \\ 390 \end{pmatrix}$$ [B1]

Answer **P** = ... [1]

(b) Evaluate **QP**.

$$QP = \begin{pmatrix} 6 & 5 & 6 \\ 4 & 2 & 7 \\ 5 & 3 & 9 \end{pmatrix}\begin{pmatrix} 180 \\ 420 \\ 390 \end{pmatrix}$$

$$= \begin{pmatrix} 5520 \\ 4290 \\ 5670 \end{pmatrix}$$

[B1]

Answer **QP** = ... [1]

(c) State what the elements of **QP** represent.

Answer The elements of **QP** represent the total cost of package A, B and C respectively.

……

……[1]

(ii) A condominium's manager, Amy, decided to engage Dream Renovation Company to do renovation works. She ordered 85 package A, 90 Package B and 85 Package C. A matrix **Y** when multiplied by **QP**, will give the total cost of this renovation.

(a) State matrix **Y**.

$\begin{pmatrix}85 & 90 & 85\end{pmatrix}$ --[B1]

Answer **Y**= …… [1]

(b) Using matrix multiplication, find the total cost of this renovation.

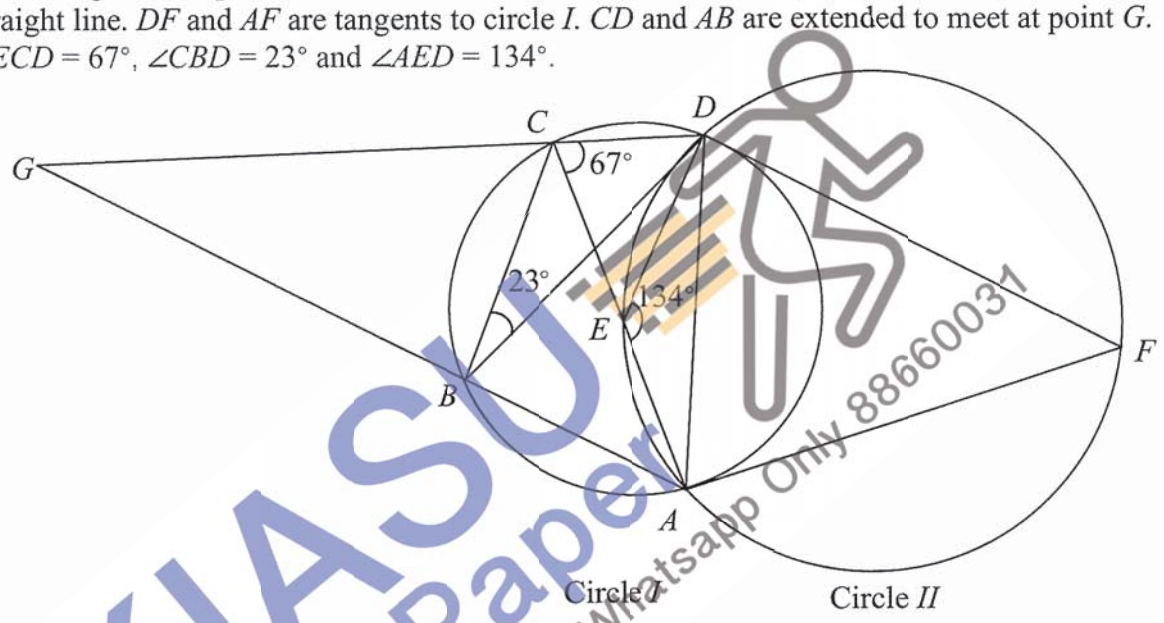
Total cost

$= \begin{pmatrix}85 & 90 & 85\end{pmatrix}\begin{pmatrix}5520\\4290\\5670\end{pmatrix}$ ---[M1] Therefore, cost is $1337250 [A1]

$= (1337250)$

Answer $ …… [2]

3 In the diagram, the points A, B, C, D lie on circle I. The points A, E, D, F lie on circle II. AEC is a straight line. DF and AF are tangents to circle I. CD and AB are extended to meet at point G. $\angle ECD = 67°$, $\angle CBD = 23°$ and $\angle AED = 134°$.



(a) Stating your reason clearly, find

(i) $\angle AFD$,

$\angle AFD = 180° - 134° = 46°$ (angles in the opp segment) [B1]

Answer $\angle AFD =$ ..... [1]

(ii) $\angle CAD$,

$\angle CAD = 23°$ (angles in the same segment) [B1]

Answer $\angle CAD =$ ..... [1]

(iii) a line that has the same length as DF.

AF (tangents from ext point) [A1, M1 for correct reason]

Answer line ..... [2]

(b) Explain why the centre of circle I lie on the circumference of circle II.

$\angle AED = 134°$
$= 2 \times 67°$
$= 2 \times \angle ACD$ [M1]

[M1]

Hence, since angle at centre is twice angle at circumference, E is the centre of circle I.
Since E lies on circle II, the centre of circle I lies on circle II. [A1]

[3]

(c) Show that triangle BCG is similar to triangle DAG. State your working and reasons clearly.

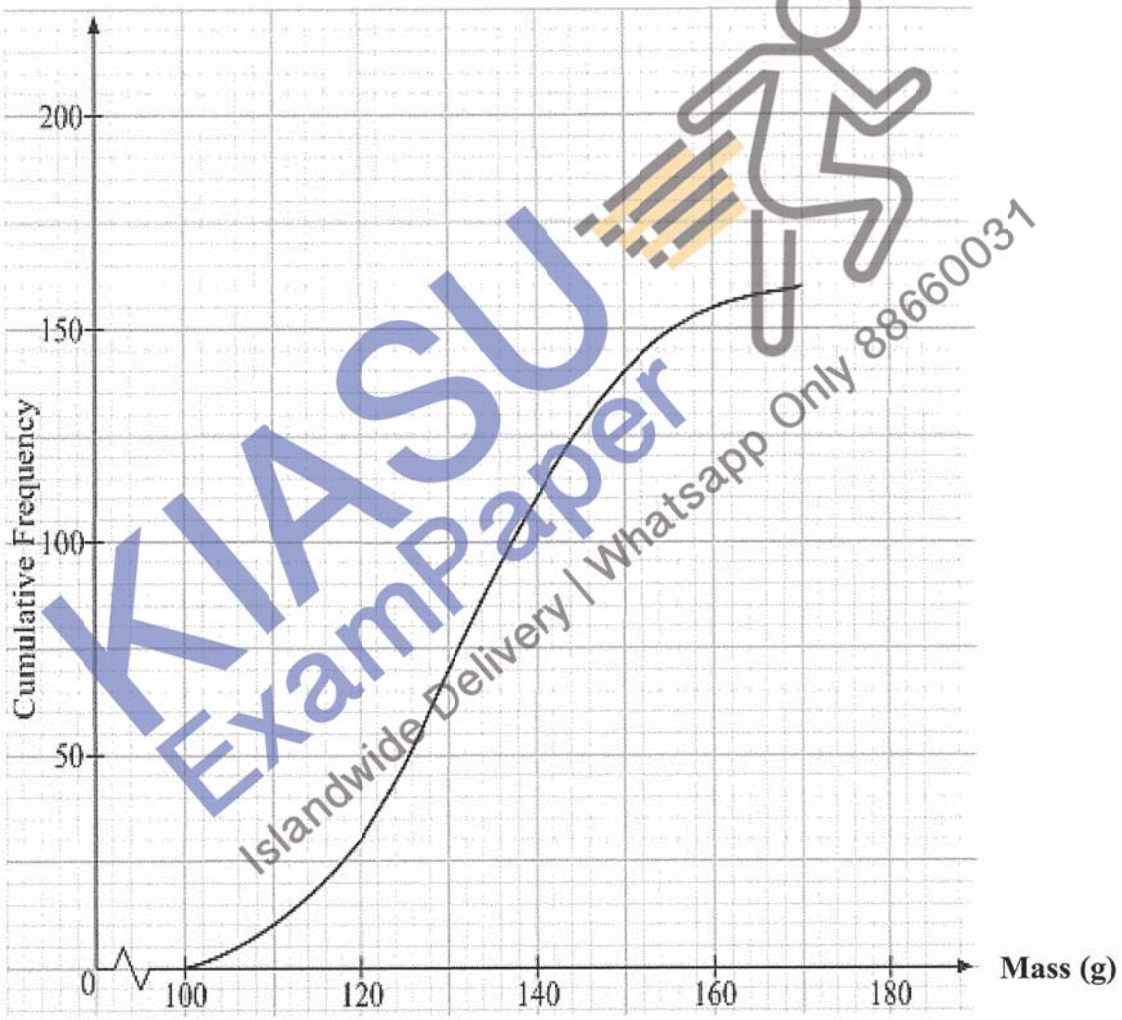
Answer

$\angle ABC = 90°$ (angles in a semicircle)
$\angle CBG = 180° - 90° = 90°$ (adj angles on a str line)
$\angle ADC = 90°$(angles in a semicircle)
$\angle CBG = \angle ADG$ } [A2]
$\angle BGC = \angle AGC$(common angle)

Hence, triangle BCG is similar to triangle DAG.

[2]

4 The cumulative frequency curve shows the distribution of the masses of 160 chicks from Dairy Farm.



(a) From the curve, find

(i) the median mass,

132g [B1]

Answer ..... [1]

(ii) the interquartile range,

20g [B1]

Answer ..... [1]

(b) Given that 62.5% of the chicks have a mass of more than x g, find the value of x.

37.5% × 160 = 60 chicks have a mass of less than or equals to x g. [M1]

From the curve, $x = 128$ [A1]

Answer ..... [2]

(c) Chicks that have mass less than 120g are classified as malnourished. Two chicks are selected at random. Find the probability that at least 1 chick selected is malnourished.

Number of malnourished chicks = 30. [M1]

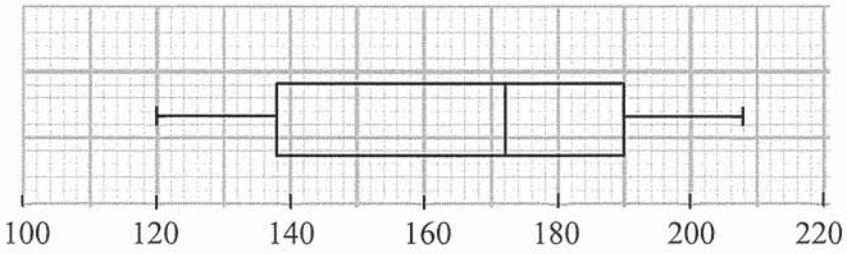
P(at least 1 malnourished) $= \frac{30}{160} \times \frac{29}{159} + \frac{30}{160} \times \frac{130}{159} + \frac{130}{160} \times \frac{30}{159} = \frac{289}{848}$ [A1] or

$1 - P(\text{both not})$

$= 1 - \frac{130}{160} \times \frac{129}{159} = \frac{289}{848}$

Answer ..... [2]

(d) The box-and-whisker plot below shows the distribution of the weight of the chicks one month later.



Make two comparisons between the distributions of the weights of the chicks in the one month.

Answer Since the median weight is higher one month later (172g>132g), the chicks are generally heavier after one month. [A1] Since the interquartile range is higher one month later(52g>20g), the weight of the chicks is less consistent after one month. [A1] [Values stated – M1]

.....

..... [3]

5 At a bowling alley, Patrick rolls a bowling ball towards the pins. The ball travels 18 metres at an average speed of x metres per second.

(a) Write an expression, in terms of x, for the time taken, in seconds, for the ball to reach the pins.

Time taken $= \frac{18}{x}$ s [B1]

Answer ..... s [1]

(b) Patrick then picks up a second bowling ball and rolls it. The average speed of the second ball is 0.5 metres per second greater than the average speed of the first ball.

(i) Write an expression, in terms of x, for the time taken, in seconds, for the second ball to reach the pins.

Time taken $= \frac{18}{x+0.5}$ s [B1]

Answer ..... s [1]

(ii) There is a 54-seconds time gap between Patrick rolling the second ball and when the first ball hit the pins. The total time taken when Patrick rolls off his first ball to the time when the second ball hits the pins is 1 minute.
Write down an equation in x, and show that it simplifies to $2x^2 - 11x - 3 = 0$.

Answer

$\frac{18}{x} + \frac{18}{x+0.5} + 54 = 60$ [M1]

$\frac{18}{x} + \frac{18}{x+0.5} = 6$

$18(x+0.5) + 18x = 6x(x+0.5)$

$18x + 9 + 18x = 6x^2 + 3x$

$6x^2 - 33x - 9 = 0$

$2x^2 - 11x - 3 = 0$ [A1]

[2]

(c) **(i)** Solve the equation $2x^2 - 11x - 3 = 0$, giving both answers correct to two decimal places.

$$x = \frac{-(-11) \pm \sqrt{(-11)^2 - 4(2)(-3)}}{2(2)}$$ [M1]

$x = 5.7603$

$x = 5.76(2d.p)$ [A1]

or

$x = -0.26039$

$x = -0.26(2d.p.)$ [A1]

Answer $x =$ ..... or ..... [3]

(ii) Explain why one of the solutions in **(c)(i)** is rejected.

Answer.....x=-0.26 is rejected because x is a speed and speed cannot be negative.....

.....[1]

(d) How much longer, in seconds, does the first ball take to hit the pins than the second ball?

Time

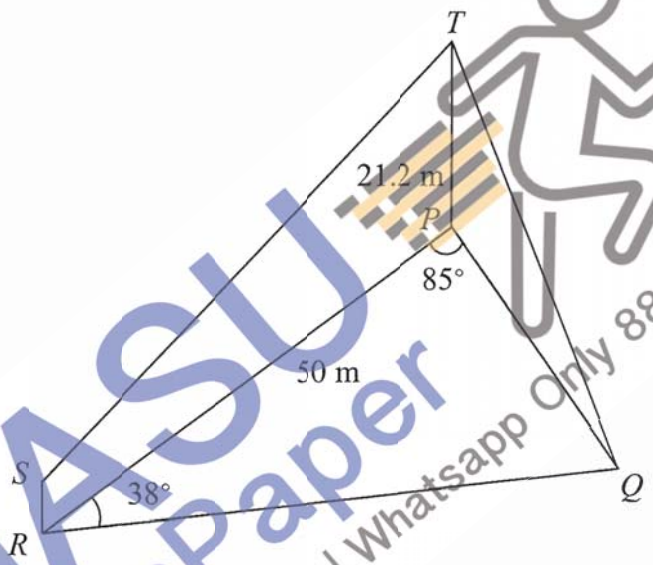
$$= \frac{18}{5.7603} - \frac{18}{5.7603 + 0.5}$$ [M1]

$= 0.24957$

$= 0.250s$ (3s.f.) [A1]

Answer ..... s [2]

6 P, Q and R are three points on a horizontal ground. It is given that $PR = 50$ m, $\angle RPQ = 85°$, $\angle PRQ = 38°$. The height of the vertical tower TP is 21.2 m. S is the top of a vertical statue standing at R.



(a) Find

(i) the distance PQ,

$\angle PQR = 180° - 85° - 38° = 57°$ [M1]

$\frac{PQ}{\sin 38°} = \frac{50}{\sin 57°}$

$PQ = 36.7046 = 36.7$ m (3.s.f) [A1]

Answer $PQ =$ ..... m [2]

(ii) the area of triangle PQR,

Area of triangle PQR $= \frac{1}{2}(50)(36.7046)\sin(85°) = 914 \text{m}^2$ (3.s.f) [A1]

[M1]

Answer ..... m^2 [2]

(iii) the greatest angle of elevation of the tower when viewed from a man walking along RQ.

Let the shortest distance from P to RQ be x and the greatest angle of elevation be θ.

$\sin 38° = \frac{x}{50}$ [M1]

$x = 30.783$ [M1]

$\tan\theta = \frac{TP}{x}$ [M1]

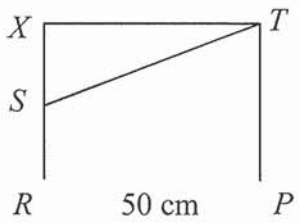
$\tan\theta = \frac{21.2}{30.783}$

$\theta = \tan^{-1}(\frac{21.2}{30.783})$

$\theta = 34.6°$ (1.d.p) [A1]

Answer ..... m [4]

(b) If the angle of depression of S from T is 20°, find the height of the statue.



$\tan 20° = \frac{XS}{50}$ [M1]

$XS = 18.198$

$XR = TP$

$SR = TP - XS$

$= 21.2 - 18.198$ [M1]

$= 3.002$

$= 3.00$ m (3.s.f) [A1]

Hence, the height of the statue is 2.99 m.

Answer ..... m [3]

7 **(a)** Given that $\overrightarrow{RS} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$ and the position vector of R is $\begin{pmatrix} 3 \\ -5 \end{pmatrix}$, find

(i) $\left|\overrightarrow{RS}\right|$,

$\left|\overrightarrow{RS}\right| = \sqrt{(-1)^2 + 2^2} = 2.24$ units (3s.f.) [B1]

Answer ..... units [1]

(ii) the coordinates of S.

$\overrightarrow{OS} = \overrightarrow{OR} + \overrightarrow{RS}$

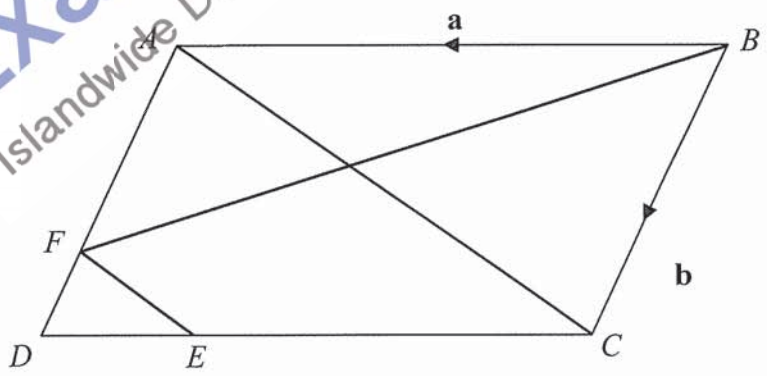
$\overrightarrow{OS} = \begin{pmatrix} 3 \\ -5 \end{pmatrix} + \begin{pmatrix} -1 \\ 2 \end{pmatrix}$ [M1]

$\overrightarrow{OS} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$

Hence coordinates of S is (2,-3) [A1]

Answer (..... , ...) [2]

(b)



$ABCD$ is a parallelogram. $\overrightarrow{AF} = 2\overrightarrow{FD}$, $\overrightarrow{ED} = \frac{1}{3}\overrightarrow{CD}$, $\overrightarrow{BA} = \mathbf{a}$ and $\overrightarrow{BC} = \mathbf{b}$.

(i) Express, as simply as possible, in terms of **a** and/or **b**,

(a) $\overrightarrow{EC}$,

$\overrightarrow{EC} = -\frac{2}{3}\mathbf{a}$ [B1]

Answer ..... [1]

(b) $\overrightarrow{DF}$,

$\overrightarrow{DF} = -\frac{1}{3}\mathbf{b}$ [B1]

Answer ..... [1]

(c) $\overrightarrow{CA}$,

$\overrightarrow{CA} = \overrightarrow{CD} + \overrightarrow{DA}$ [B1]

$\overrightarrow{CA} = \mathbf{a} - \mathbf{b}$

Answer ..... [1]

(d) Are $\overrightarrow{FE}$ and $\overrightarrow{CA}$ parallel vectors? Explain your answer with working.

Answer

$\overrightarrow{FE} = \overrightarrow{FD} + \overrightarrow{DE}$

$\overrightarrow{FE} = \frac{1}{3}\mathbf{b} - \frac{1}{3}\mathbf{a}$

$\overrightarrow{FE} = -\frac{1}{3}(\mathbf{a} - \mathbf{b})$

$\overrightarrow{FE} = -\frac{1}{3}\overrightarrow{CA}$ [M1]

Hence, they are parallel vectors. [A1]

[2]

(ii) Calculate the value of

(a) $\dfrac{\text{area of } \Delta DEF}{\text{area of } \Delta DCA}$,

$\dfrac{\text{Area of } \Delta DEF}{\text{Area of } \Delta DCA} = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$ [B1]

Answer ..... [1]

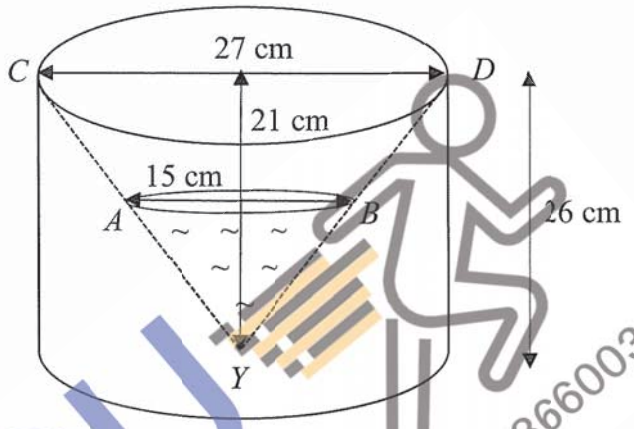
(b) $\dfrac{\text{area of } \Delta DEF}{\text{area of parallelogram } ABCD}$.

Area of ΔDCA = Area of ΔCAB

$\dfrac{\text{Area of } \Delta\text{DEF}}{\text{Area of ABCD}} = \dfrac{1}{9 \times 2} = \dfrac{1}{18}$ [B1]

Answer ..... [1]

8



The figure shows a solid cylinder of height 26 cm with the cone CYD removed from it. The diameter and height of the big cone CYD is 27 cm and 21 cm respectively. Water is then poured into the container forming a small cone AYB with diameter of 15 cm as shown in the figure.

(a) Show that the height of the cone ABY is $\frac{35}{3}$ cm.

Answer

Let h be the height of the small cone. By similar triangles,

$$\frac{h}{21} = \frac{15}{27}$$

$$h = \frac{35}{3} \qquad \text{[B1]}$$

[1]

(b) Calculate the length of YB.

$$YD = \sqrt{21^2 + \left(\frac{27}{2}\right)^2} \qquad YD > 0$$

$$YD = 24.96497$$

$$YD = 25.0 \qquad \text{[M1]}$$

$$\frac{YB}{15} = \frac{24.96497}{27}$$

$$YB = 13.8694$$

$$YB = 13.9 \quad \text{(3 s.f.)} \quad \text{[A1]}$$

OR

$$YB = \sqrt{7.5^2 + \left(\frac{35}{3}\right)^2} \qquad \text{[M1]}$$

$$YB = 13.9 \qquad \text{[A1]}$$

Answer YB = ... cm [2]

(c) The water is then emptied from the solid container. Find

(i) the total surface area of the solid container,

$YD = 24.96497$ [M1]

Total SA

$= \pi(13.5)(24.96497) + \pi(13.5)^2 + 2\pi(13.5)(26)$ [M1]

$= 3840 \text{ cm}^2 (3 \text{ s.f.})$ [A1]

Answer ..... cm^2 [3]

(ii) the volume of the solid container.

Total Volume

$= \pi(13.5)^2(26) - \frac{1}{3}\pi(13.5)^2(21)$ [M1]

$= 10900 \text{ cm}^3$ (3.s.f) [A1]

Answer ..... cm^3 [2]

9 Energy is released during the breaking of molecular bonds. The amount of energy (y joules) varies with the temperature (x °C) such that $y = 3(2^x)$. The table below shows $y = 3(2^x)$ for $-2 \le x \le 2$.

Temperature (x °C)	–2	–1	–0.5	0.5	1	1.5	2
Energy released (y joules)	0.75	1.5	2.12	p	6	8.49	12

(a) Find the value of p.

$p = 4.24$ [B1]

Answer $p =$ ..... [1]

(b) Using a scale of 2 cm to represent 0.5 degree Celsius, draw a horizontal x-axis for $-2 \le x \le 2$.

Using a scale of 1 cm to represent 1 joule on the y-axis, draw a vertical y-axis for $0 \le y \le 12$. [3]

On your axes, plot the points given in the table and join them with a smooth curve. Shape 1m, points 1m, scale 1m.

(c) Use your graph to estimate the amount of energy, in joules, that is released when the temperature is 0 degree Celsius.

[1]

Answer ..... J

(d) **(i)** By drawing a tangent, find the gradient of the curve at $x = 1$.

4.16 [A1] (Accept 3.3 to 5)

Tangent line [B1]

Answer ..... [2]

(ii) State briefly what this gradient represent.

Answer .....

It represents the rate of increase in the energy released when x=1. [B1]

..... [1]

(e) By drawing a suitable straight line on the same axes, solve the equation $2^{x+1} = \frac{2x+8}{3}$ for $-2 \le x \le 2$.

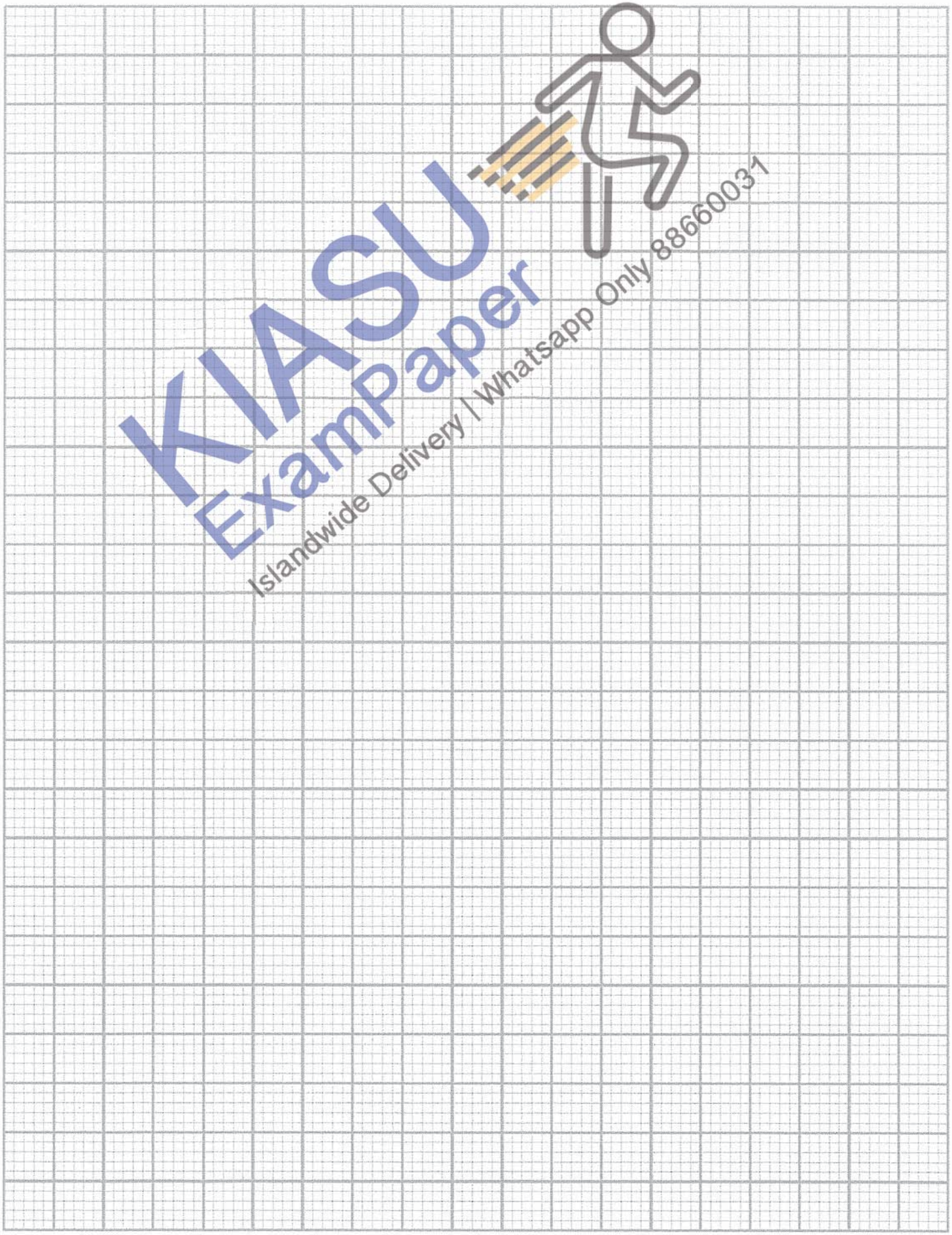
$2^{x+1} = \frac{2x+8}{3}$

$2^x(2) = \frac{2x+8}{3}$ Draw $y=x+4$ [M1] $x=0.624$ (accept 0.499 to 0.75)

$3(2^x) = x+4$ [M1]

$y = x+4$

Answer ..... [3]



10 (a) Jane planned a trip to Seoul, South Korea in November 2021 using the Vaccinated Travel Lane.

The table below shows the prices of a plane ticket from Singapore to Seoul in November 2021 on a particular airline. For example, the cost of a flight departing Singapore for Seoul on 4 November and a flight returning from Seoul to Singapore on 10 November is $990. Assume that all flights depart at 10 am and the flight time is 6 hours.

	Return 10 Nov (Thurs)	Return 11 Nov (Fri)	Return 12 Nov (Sat)
Depart 4 Nov (Fri)	$990	$1105	$1105
Depart 5 Nov (Sat)	$1263	$1303	$1263
Depart 6 Nov (Sun)	$1303	$1263	$990

Travellers to South Korea have to take 5 different tests. The table below shows the cost of 5 different tests.

	Test	Where & When	Price
1	Pre-Departure PCR Test	Singapore, within 48 hours of departure	$91
2	On-Arrival PCR Test	Incheon Airport, upon arrival	$106 (Monday to Saturday) $212 (Sunday)
3	Self-Administered ART for those staying 8 days or more	South Korea, day 7 of arrival	$5
4	Pre-Departure ART	MOH approved hospitals within Seoul	$90
5	Supervised Self-administered ART	Quick Test Centre, within 24 hours of arrival in Singapore	$15

Jane wanted to spend 6 nights in Seoul.

(i) Which dates should Jane depart Singapore for Seoul and return to Singapore from Seoul? Justify your answer.

Answer She should depart on 4th November and return on 10th November [A1] as that is the cheapest flight for 6 nights in Seoul and the on-arrival PCR test is cheaper on Friday than on Sunday. The assumption is that she wants to save as much cost as possible and the dates travelled do not matter. [A1] [Accept other reasonable answers and explanation.]

.....

..... [2]

(ii) What would be the total cost incurred for the flight and all the PCR and ART tests throughout the trip if she travels on the dates in **(a)(i)**?

Total cost = \$990 + \$91 + \$106 + \$90 + \$15 = \$1292 [M1- correct total test cost. A1]

Answer \$ ..... [2]

(b) The tables below give information that Jane can use to work out her total expenses for her **7 days and 6 nights** trip. She intended to purchase a Tourist Transportation Card to cover her public transportation costs for all 7 days.

The costs are all given in South Korean won (₩).

Estimated living cost in South Korea

Hotel Accommodation	₩ 70 000 per night
Leisure (including sightseeing and theme park admission)	₩ 57 000 per day
Food & Beverages	₩ 60 000 per day

Tourist Transportation Card (for all public transportation)	
Card Duration	Price
1-day pass	₩ 15 000
3-day pass	₩ 30 500
5-day pass	₩ 44 500

*All costs are subjected to a 7.5% goods & service tax.

The exchange rate at a money exchange between South Korean won and Singapore dollars is given below.

		Singapore dollars (S$)	
Currency	Unit	Buying	Selling
South Korean won	1000	0.909	1.099

Suggest a sensible budget in Singapore dollars for Jane's trip excluding air tickets and all the PCR and ART tests.

Answer

Cheapest Transport Pass combination = 44 500 + 15 000 + 15 000 = 74 500 won [M1]

Total expenses without GST in won
$= 70000 \times 6 + 7 \times (57000 + 60000) + 74500$ [M1]
$= 1313500$ won

Total expenses with GST in won
1313500×1.075 [M1]
$= 1412012.50$ won

Total expenses with GST in SGD
$= \frac{1412012.50}{1000} \times 1.099$ [M1]
$= \$1551.80$

Hence, she should budget about $1600 for her expenses in South Korea
[A1 – accept any reasonable budget above $1551.80.]

[5]