BEDOK SOUTH SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2023

4E5N

CANDIDATE NAME	
CLASS	
REGISTER NUMBER	

ADDITIONAL MATHEMATICS

Paper 2

4049 / 02

23 August 2023

2 hours 15 minutes

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on ***all*** the work you hand in.
Write in ***dark blue*** or ***black***.
You may use a HB ***pencil*** for any ***diagrams*** or ***graphs***.
Do ***not*** use staples, paper clips, highlighters, glue or correction fluid.

Answer ***all*** the questions.
The use of an ***approved scientific calculator*** is expected, where appropriate.
If the degree of accuracy is not specified in the question and if the answer is not exact, give non-exact numerical answers correct to ***3 significant figures***, or ***1 decimal place*** in the case of angles in degrees.

For π, use your calculator value or 3.142.
You are reminded of the need for ***clear presentation*** in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
If working is needed for any question it must be shown in the ***space*** below the question.
Omission of essential working will result in loss of marks.
The total number of marks for this paper is **90**.

Setter : Mr Loh Jia Perng

For Examiner's Use		
Student Check & Sign	Marks before deduction	
Parent's Signature	TOTAL	90

This paper consists of **18** printed pages.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}\ b + \binom{n}{2} a^{n-2}\ b^2 + \ldots + \binom{n}{r} a^{n-r}\ b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n\,!}{r\,!\,(n-r)\,!} = \frac{n\,(n-1)\ldots(n-r+1)}{r\,!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for $\triangle ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\Delta = \frac{1}{2} ab \sin C$$

Answer all questions.

1. The cubic polynomial $f(x)$ is such that the coefficient of x^3 is 2 and the roots of the equation $f(x)=0$ are 2, $-\frac{1}{2}$ and k. Given that $f(x)$ has a remainder of -6 when divided by $x-1$. [3]

Find the value of k.

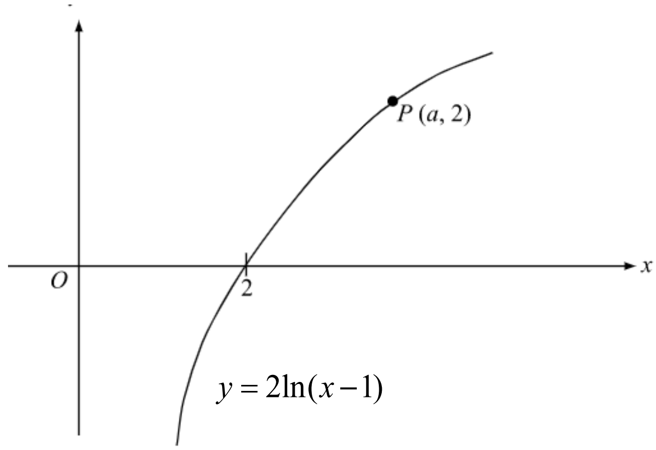
2. A merry-go-round horse ascends and descends as it travels along a circular path. The distance between the bottom of the horse and the ground, d cm, can be modelled by the equation, $d = 3 - 2\cos kt$, where t is the time in minutes after the merry-go-round ride starts and k is a constant.

Each merry-go-round ride lasts 5 minutes and the horse ascends 10 times on each ride before coming to rest.

(a) Find the value of k in radians per minute. [2]

(b) For how long, during one merry-go-round ride, is d > 2? [4]

3. The diagram shows part of the curve $y = 2\ln(x-1)$. The point P $(a, 2)$ lies on the curve.



(a) Find the value of a, in terms of e. [2]

(b) Show that $\int_0^2 x \, \mathrm{d}y = 2e$. Hence explain why $\int_2^{1+e} 2\ln(x-1) \, \mathrm{d}x = 2$. [6]

4.

(a) Prove that $\dfrac{\sec^2 x}{1+\sec^2 x} = \dfrac{2}{\cos 2x + 3}$. [3]

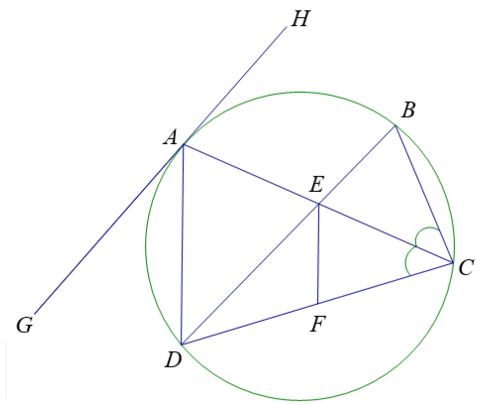
(b) Hence solve the equation $\dfrac{\sec^2 2x}{1+\sec^2 2x} = \dfrac{4}{7}$ for $0 \le x \le 3$. State the number of solutions for $-2\pi \le x \le 2\pi$. [4]

5. **(a)** Sketch the graph of the equation $y = e^{-2x}$. [2]

(b) The gradient function of a curve is given by $\frac{dy}{dx} = e^{2x} + e^{-2x}$. Explain why the function is an increasing function. [2]

(c) Find the equation of the function, given that y-intercept is 3. [3]

6.



The diagram shows a circle passing through points A, B, C and D. GAH is a tangent to the circle at point A. $\angle ECF = \angle ECB$. E and F are the midpoints of AC and DC respectively.

(a) Prove that $\angle DAG = \angle ECB$. [2]

(b) Prove that $\triangle ECF$ is similar to $\triangle BCE$. [4]

(c) Show that $EC \times BE = \frac{1}{2} AD \times BC$. [2]

7. Using a suitable substitution in the form $y = a^x$, where a is a constant, show that the equation $2^{1+3x} + 2^2 = 7\left(4^x\right) + 5\left(2^x\right)$ can be expressed as $2y^3 - 7y^2 - 5y + 4 = 0$.

Hence, given that x is an integer, solve the equation $2^{1+3x} + 2^2 = 7\left(4^x\right) + 5\left(2^x\right)$. [8]

8. Two particles A and B, each moving in a straight line and in the same direction, passes point O at the same instant. The velocity of particle A, t seconds after passing O, is given by $V_A = t^2 - 4t + 7$ m/s.

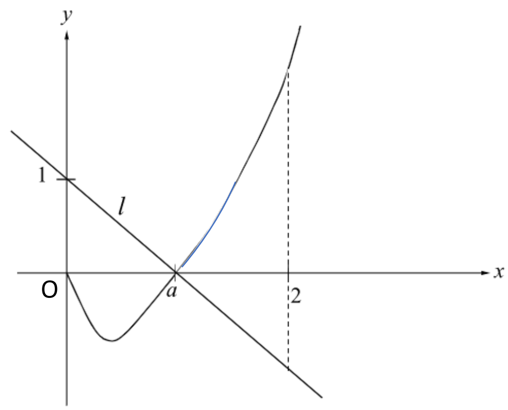
Particle B passes O with a velocity of 23 m/s and moves with an acceleration, $a_B = 6t - 16$ m/s^2, where t is the time in seconds after passing O.

(a) Calculate the range of values of t for which the velocity of the particle A is greater than that of particle B. [6]

(b) Is there any overtaking between the two particles after they pass point O? Explain your reason clearly. [5]

9. **(a)** Differentiate $x^2 \ln x - x$ with respect to x. [1]

(b) The diagram shows the line l and part of the curve $y = 2x \ln x$. Both graphs intersect the x-axis at a. Line l cuts the y-axis at 1.



(i) Find the value of a. [3]

(ii) Find the equation of line l. [2]

(iii) Determine the exact area of the region bounded by the curve, the line $x = 2$ and the line l. [4]

10.

(a) Express $\frac{2}{r^2+r}$ as a partial fraction. [3]

(b) Hence, state the **exact** value of $\frac{2}{1\times 2}+\frac{2}{2\times 3}+\cdots\cdots+\frac{2}{98\times 99}+\frac{2}{99\times 100}$. [2]

(c) Show that $\int_1^3 \frac{3}{r^2 + r}\, \mathrm{d}r = \ln \frac{27}{8}$. [5]

11. The amount of active ingredients in a lotion brand L, y units, is dependent on the time, t years after the lotion is manufactured. The variables y and t are related by the equation $y = ae^{bt}$, where a and b are constants. Some values of y and t are shown in the table below.

t (years)	1	2	3	4	5	6
y (units)	950.6	303.2	221.4	30.8	9.8	3.1

(a) Plot a graph of ln y against t. [4]

(b) Using your graph, find the value of a and of b. [2]

(c) There was an error in the measurements. Identify the error and suggest, with reasons, the correct amount of active ingredients. [2]

(d) The amount of active ingredients in another lotion brand M is given by the equation

$$y = \frac{e^3}{\sqrt{e^t}}.$$

By adding a suitable straight line to your graph, find the time after which brand M contains a higher level of active ingredients. [4]