Name:		Register No.:	Class:

CRESCENT GIRLS' SCHOOL
SECONDARY FOUR
PRELIMINARY EXAMINATION 2024

MATHEMATICS   **4052/02**
Paper 2   **20 Aug 2024**
                                                                         2 hours 15 minutes

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, register number and class on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 90.

For Examiner's Use

Question	1	2	3	4	5	6	7	8	9	10
Marks										

Table of Penalties		Question No.		
Presentation	−1			
Accuracy/ Units	−1	Parent's / Guardian's Signature	90

This document consists of **22** printed pages

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta \text{ , where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta \text{ , where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

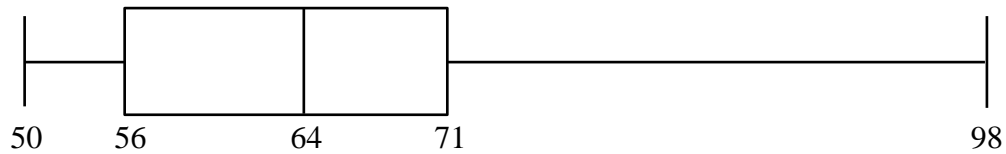$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

1 20 students of class 4A took a test. The marks are shown in the stem and leaf diagram.

5	0	2	3	3	5	7	8
6	0	1	2	6	7	7	9
7	0	*k*	5	4	7		
8							
9	8						

Key: 5|2 means 52 marks

The marks are also represented in the box-and-whisker plot.

(a) Find the value of *k*.

Answer *k* = …………………… [1]

(b) Would you prefer to use the mean or the median mark to describe the performance of class 4A? Give a reason for your answer.

Answer ……………………………………………………………………………

…………………………………………………………………………… [2]

(c) Class 4B took the same test as class 4A. A student from 4B was subsequently transferred to 4A and a new box-and-whisker plot for 4A was then drawn. It is given that the plot remained unchanged.

How many marks did the transfer student obtain in the test?

Answer ……………………marks [1]

2 **(a)** Given that $p = \sqrt{\dfrac{q}{2} - r^2} + 3$, express r in terms of p and q.

Answer …………………………… [2]

(b) Simplify $\dfrac{\left(-x^2 y^3\right)^4}{3} \div \dfrac{2xy^0}{5xy^2}$.

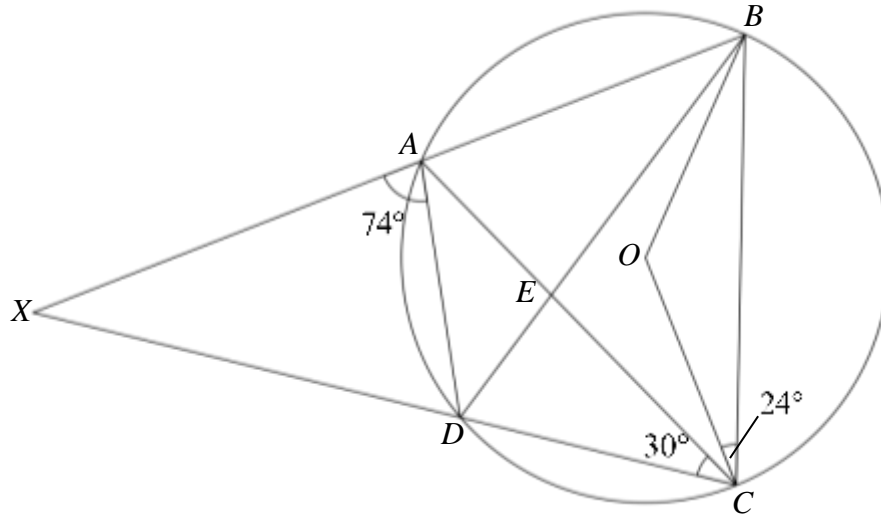*Answer* …………………… [4]

(c) **(i)** Factorise $9x^2 - 4$.

Answer …………………………… [1]

(ii) Hence, express $\dfrac{3x}{9x^2-4} - \dfrac{2}{3x-2}$ as a single fraction in its simplest form.

Answer …………………… [3]

3 *A*, *B*, *C* and *D* are points on the circumference of a circle with centre *O*. *BA* produced and *CD* produced meet at *X*. Angle $BCO = 24°$, angle $ACD = 30°$ and angle $XAD = 74°$.

(a) Stating the reasons clearly, find

(i) angle *ABD*,

Answer ……………………° [1]

(ii) angle *BAC*,

Answer ……………………° [2]

(iii) angle *ECO*.

Answer …………………….° [2]

(b) *W* is a point on the same side of the major segment formed by the chord *AD*. Given that angle *AWD* = 45° , state with reason whether the point *W* lies on the circumference of the circle, outside the circle or inside the circle.

[2]

4   In the diagram, $\overrightarrow{OP} = \mathbf{p}$, $\overrightarrow{OQ} = \mathbf{q}$, $\overrightarrow{OP} = \dfrac{2}{3}\overrightarrow{OM}$, $\overrightarrow{OQ} = \dfrac{1}{3}\overrightarrow{OS}$, $\overrightarrow{OQ} = \overrightarrow{SN}$ and $SM = 3SR$.
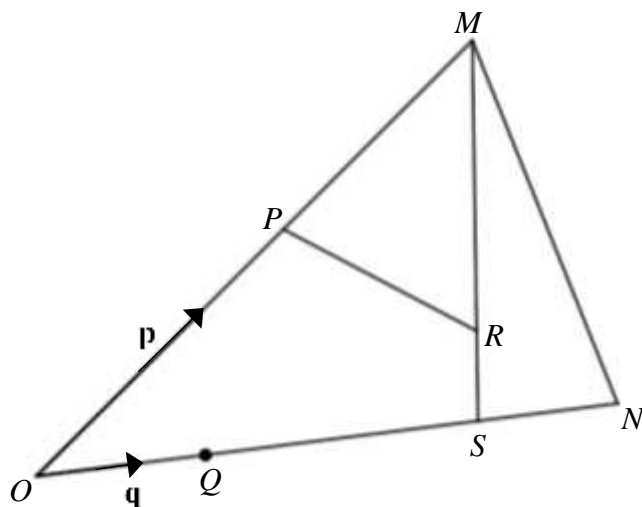
(a)   Express, as simply as possible, in terms of $\mathbf{p}$ and $\mathbf{q}$,

(i) $\overrightarrow{MS}$,

Answer $\overrightarrow{MS} = $ ………………… [1]

(ii) $\overrightarrow{PN}$,

Answer $\overrightarrow{PN} = $ ………………… [2]

(iii) $\overrightarrow{PR}$.

Answer $\overrightarrow{PR}$ = …………………… [3]

(b) Write down two facts about *P*, *R* and *N*.

Answer ……………………………………………………………………………

…………………………………………………………………………………………

………………………………………………………………………………… [2]

(c) Find the numerical value of $\dfrac{\text{Area of } \triangle MPR}{\text{Area of } \triangle MON}$.

Answer $\dfrac{\text{Area of } \triangle MPR}{\text{Area of } \triangle MON} =$ ………………… [2]

(d) The coordinates of *A* and *B* are $(6,5)$ and $(8,1)$ respectively.

(i) Express $\overrightarrow{AB}$ as a column vector.

Answer $\overrightarrow{AB} =$ …………………… [1]

(ii) Find $\left|\overrightarrow{AB}\right|$.

Answer $\left|\overrightarrow{AB}\right| =$ …………………… [2]

5 A bag contains 42 red, 15 blue and *y* orange sweets. One sweet is picked at random from the bag. The probability of picking up an orange sweet is $\dfrac{6}{25}$.

(a) Show that the value of *y* is 18.

[2]

(b) Two sweets are drawn at random one after another, from the bag, without replacement. Calculate the probability that

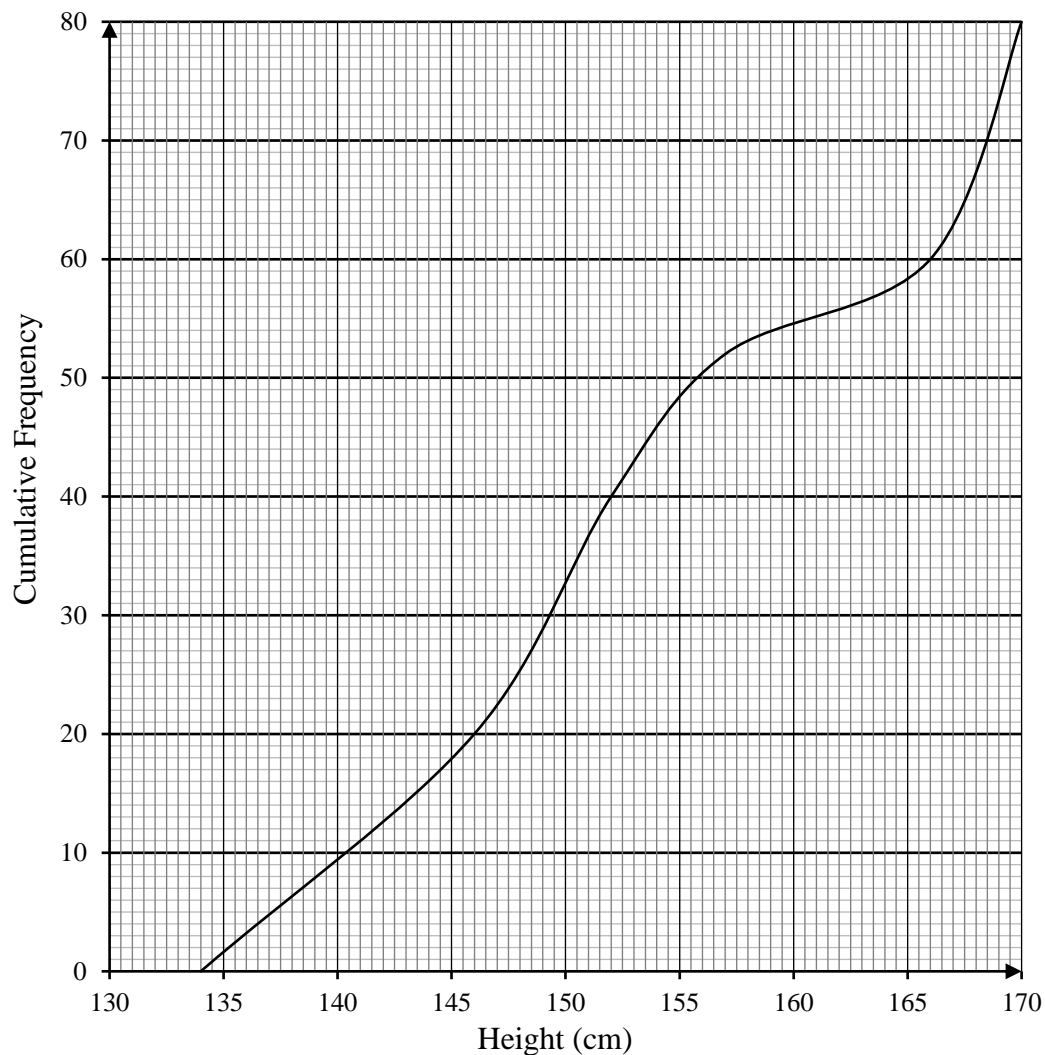**(i)** both sweets are of the same colour,

Answer ………………… [2]

(ii) one sweet drawn is red and the other is blue,

Answer ………………… [2]

(iii) at least one sweet drawn is orange.

Answer ………………… [2]

6 The cumulative frequency graph shows the distribution of the heights, in centimetres, of 80 girls.

(a) Use your graph to estimate

(i) the median height,

Answer .………………… cm [1]

(ii) the interquartile range,

Answer .………………… cm [2]

(iii) 35% of the girls are taller than *h* cm. Find the value of *h*.

Answer ………………… cm   [2]

(b) The cumulative frequency graph of the heights of 80 boys was drawn on the same axes.
The curve representing the heights of the boys lies on the right side of the curve representing the heights of the girls.
The curve representing the heights of the boys is steeper than the curve representing the heights of the girls.
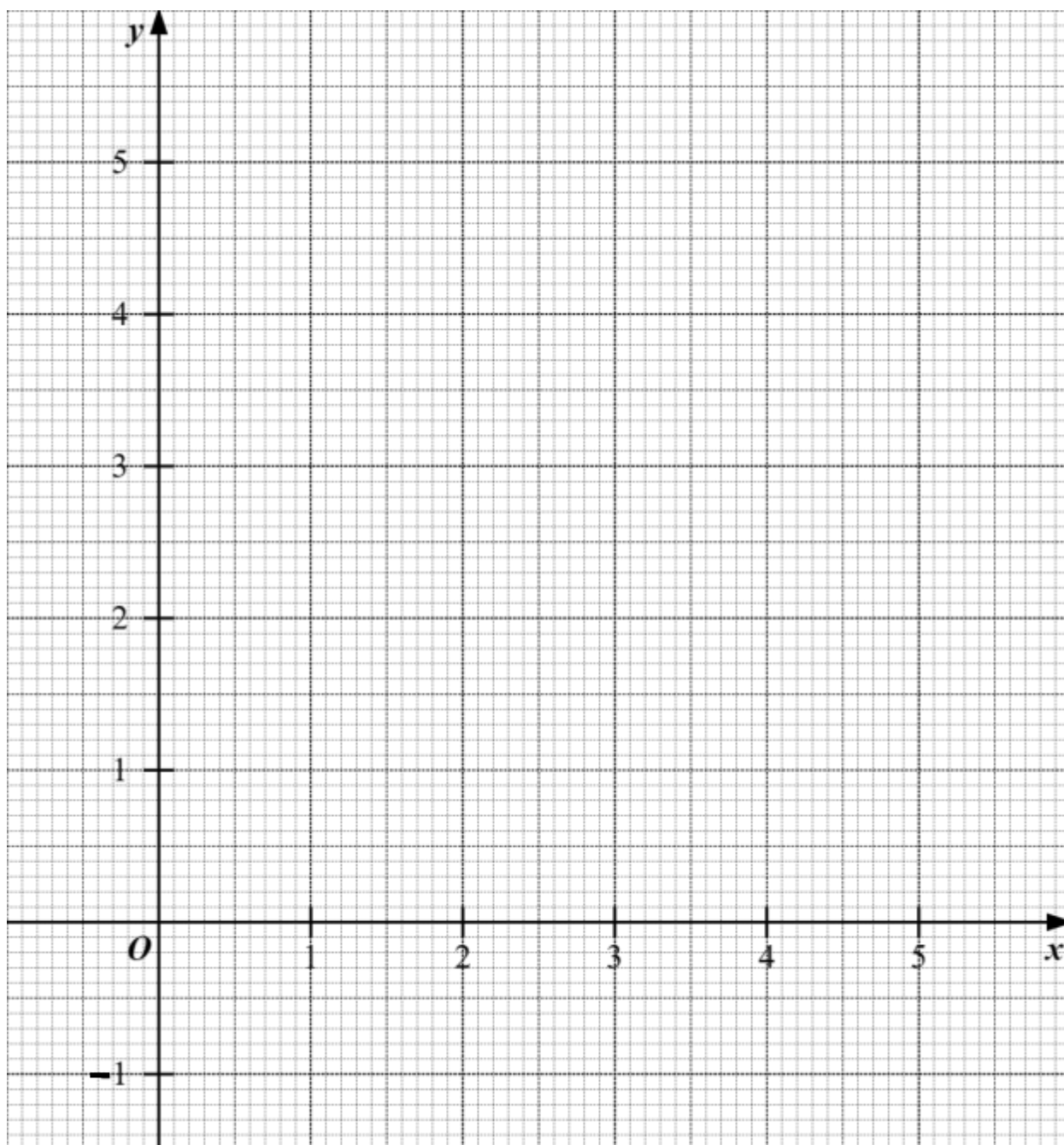Compare the median and interquartile range of the heights of the boys and the heights of the girls.

Answer ………………………………………………………………………………

………………………………………………………………………………………

………………………………………………………………………………………

……………………………………………………………………………   [2]

7 The following is the table of values for $y = 2x + \dfrac{5}{x} - 6$.

x	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
y	5	1	0.3	0.5	1	1.7	2.4	3.3	4.1	p

(a) Calculate the value of p.

Answer $p =$ ………………… [1]

(b) On the grid, draw the graph of $y = 2x + \dfrac{5}{x} - 6$ for $0.5 \leq x \leq 5$. [2]

(c) By drawing a tangent, find the gradient at the point where $x = 1$.

Answer gradient = …………………… [2]

(d) Showing your working clearly, explain how your graph shows that there is no solution to the equation $2x + \dfrac{5}{x} = 5$.

[2]

(e) **(i)** Draw the graph of $y = \dfrac{1}{2}x + 1$ on the grid in part **(b)** for $0.5 \le x \le 5$.

[1]

(ii) Write down the x coordinates of the points at which the two graphs intersect.

Answer $x =$ ……………… or ………………… [1]

(iii) Given that *a*, *b* and *c* are integers, find the equation, in the form $ax^2 + bx + c = 0$, which is satisfied by the values of *x* found in **(e)(ii)**.

Answer …………………… [2]

8 A chocolate sculpture consists of a solid hemisphere of radius 6 cm which is mounted on a frustum as shown in Diagram 2. The frustum is created by cutting a solid right circular cone of radius 10 cm into two parts horizontally, as shown in Diagram 1. The height of the frustrum is given by x cm.
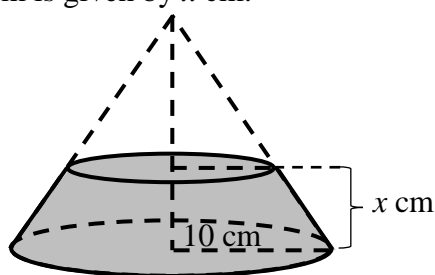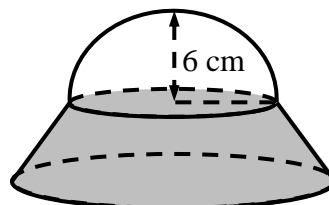

Diagram 1   Diagram 2

(a) The height of the cone is 8 cm before it is cut to obtain the frustum. Show that the value of x is 3.2.
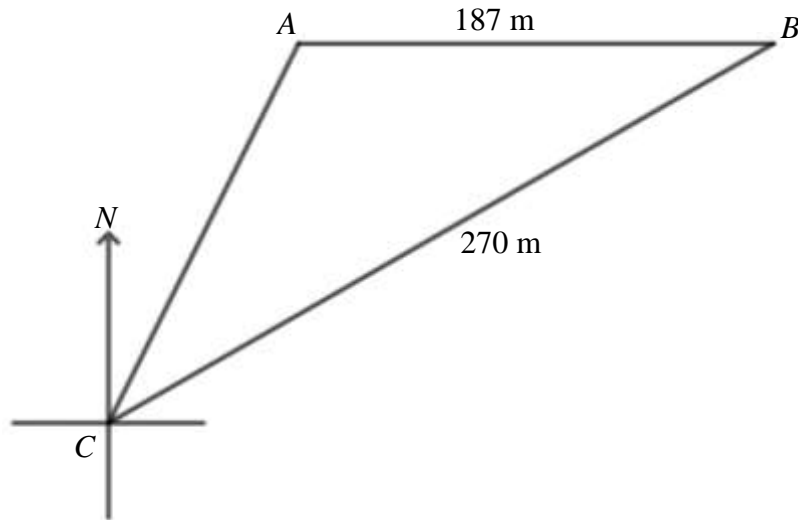
[2]

(b) Find the volume of the chocolate sculpture, giving your answer to 3 significant figures.

Answer ………………… cm^3 [3]

(c) Find the total surface area of the chocolate sculpture, giving your answer to 3 significant figures.

Answer ………………… cm^2 [4]

9 The diagram shows a triangular field *ABC* on horizontal ground. *B* is due east of *A* and is on a bearing of $061°$ from *C*. *AB* = 187 m and *BC* = 270 m.

(a) Show that *AC* is approximately 139.82 m.

[3]

(b) Find the area of the field *ABC*, giving your answer to 4 significant figures.

Answer ………………… m^2 [2]

(c) A drone is hovering vertically above *B*. The angle of depression of *A* from the drone is $12°$. Calculate the height of the drone above *B*.

Answer ………………… m   [2]

(d) Alex moves in a straight line from *B* to *C* and stops at *P* where $AP = 183$ m.
He claims that there are two such positions of *P*.
Determine, with clear calculations, if Alex is right.

[4]

10   Electronic Road Pricing System (ERP) is used in managing road congestion. Based on a pay-as-you-use principle, motorists are charged when they use priced roads during peak hours. The table below shows the time periods and respective ERP rates in a certain year.

Time Period	ERP Rates			
	Clementi Avenue 2 into AYE (City) Gantry 53	Clementi Avenue 6 into AYE (City) Gantry 52	AYE to City before Alexandra Road Gantry 36	PIE into CTE Gantry 42
8.30 am – 8.35 am	$2.50	$2.50	$1.50	$4.50
8.35 am – 8.55 am	$3.00	$3.00	$2.00	$5.00
8.55 am – 9.00 am	$2.50	$2.50	$1.50	$4.50
9.00 am – 9.05 am	$2.00	$2.00	$1.00	$4.00
9.05 am – 9.25 am	$2.00	$2.00	$1.00	$4.00
9.25 am – 9.30 am	$1.50	$1.50	$0.50	$3.50

Adapted from: https://onemotoring.lta.gov.sg/content/dam/onemotoring/Driving/pdf/24Jun24/Cars-24_June_2024.pdf

(a)   Ms Teng travelled through the Clementi Avenue 6 into AYE (City) ERP gantry (52) at 9.05 am and then passed the AYE to City before Alexandra Road gantry (36) at 9.15 am. The distance-time graph is shown.
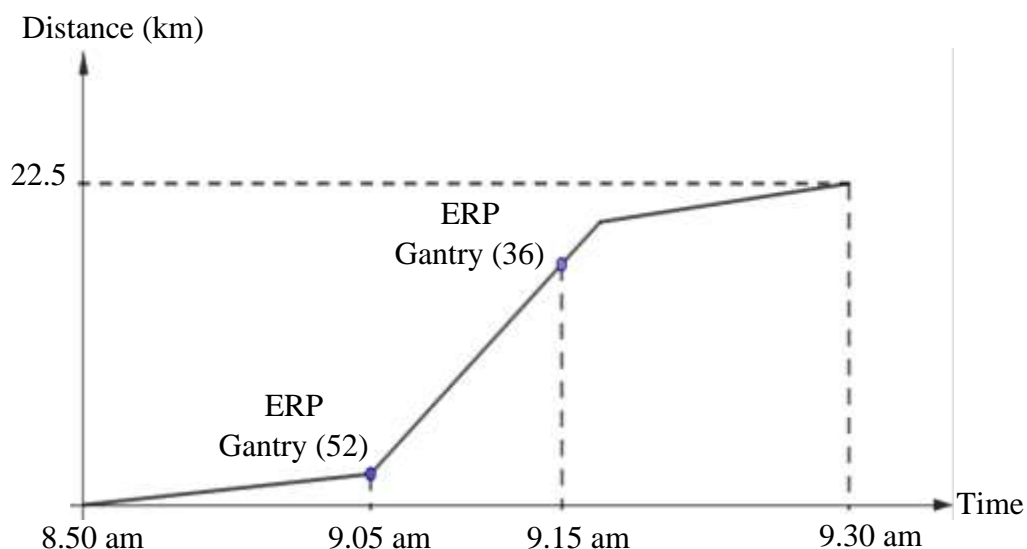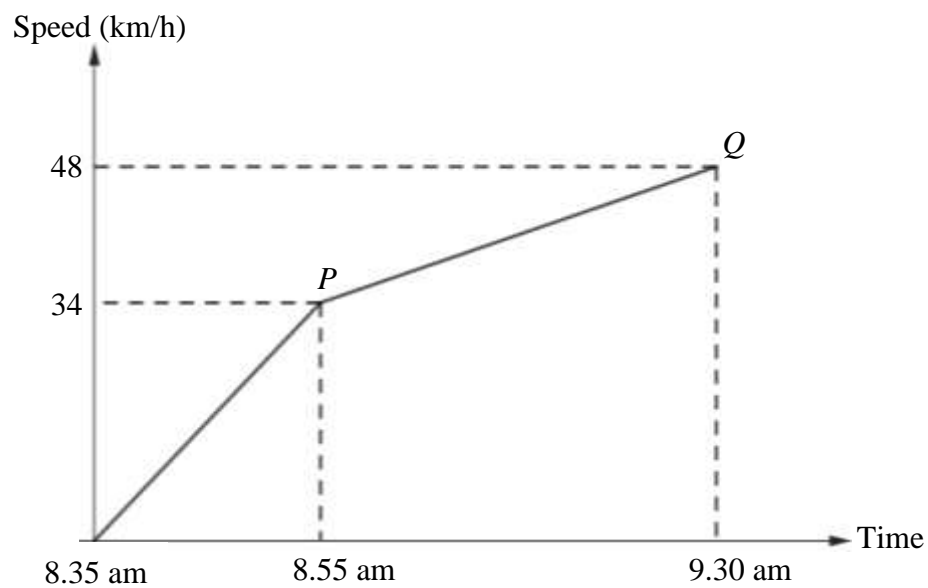

Calculate

(i)   the total ERP charges paid by Ms Teng,

Answer $ …………………… [1]

(ii) the average speed travelled by Ms Teng throughout the journey.

Answer ……………… km/h [2]

(b) The next day, Ms Teng travelled using alternative high traffic routes so as to avoid all the ERP charges. Her speed-time graph is shown below.

Given that the distance travelled is the area under the speed-time graph, calculate the total distance travelled,

Answer ………………… km [2]

(c) While Ms Teng saved on ERP charges travelling on the alternative routes, she also valued her time and the cost of petrol consumed. The value of Ms Teng's time can be modelled by the following equation:

Value of time = time saved in minutes × $0.15

Given that the petrol consumption rate of her car is 8.7 litres per 100 kilometres and the cost of petrol is $2.76 per litre, determine the route that Ms Teng would prefer in future. Justify your answer with clear workings. [5]

END OF PAPER

Answer Key

1(a)	2
1(b)	I will use the median mark as there is an outlier, 98 marks, and the median is less affected by the outlier than the mean.
1(c)	64
2(a)	$r = \pm\sqrt{\dfrac{q}{2} - (p-3)^2}$
2(b)	$\dfrac{5x^8 y^{14}}{6}$
2(c)(i)	$(3x-2)(3x+2)$
2(c)(ii)	$-\dfrac{3x+4}{(3x-2)(3x+2)}$ or $\dfrac{-3x-4}{(3x-2)(3x+2)}$ or $\dfrac{3x+4}{(-3x+2)(3x+2)}$
3(a)(i)	30°
3(a)(ii)	66°
3(a)(iii)	20°
3(b)	*W* must lie inside the circle
4(a)(i)	$3\mathbf{q} - \dfrac{3}{2}\mathbf{p}$
4(a)(ii)	$4\mathbf{q} - \mathbf{p}$
4(a)(iii)	$2\mathbf{q} - \dfrac{1}{2}\mathbf{p}$
4(b)	*P*, *R* and *N* are collinear. $PN = 2PR$ or $PR = \dfrac{1}{2}PN$
4(c)	$\dfrac{1}{6}$
4(d)(i)	$\begin{pmatrix} 2 \\ -4 \end{pmatrix}$
4(d)(ii)	4.47 units
5(b)(i)	$\dfrac{373}{925}$
5(b)(ii)	$\dfrac{42}{185}$
5(b)(iii)	$\dfrac{393}{925}$
6(a)(i)	152 cm
6(a)(ii)	20 cm
6(a)(iii)	$h = 157$
6(b)	Median height of the boys is higher than the girls. Interquartile range of the height of the boys is smaller than the girls.
7(a)	5
7(c)	−3
7(d)	Draw $y = -1$ No solution as curve does not intersect the line $y = -1$

Crescent Girls' School   2024 Prelim S4 Math P2   [Turn Over

7(e)(ii)	$x = 0.9$ or 3.75
7(e)(iii)	$3x^2 - 14x + 10 = 0$
8(b)	1110 cm^3
8(c)	798 cm^2
9(b)	12240 m^2
9(c)	39.7 m
9(d)	He is wrong. There is only one such position of P, instead of two.
10(a)(i)	\$3
10(a)(ii)	$33\frac{3}{4}$ or 33.75 km/h
10(b)	29.6 km
10(c)	Ms Teng will choose the ERP route in future