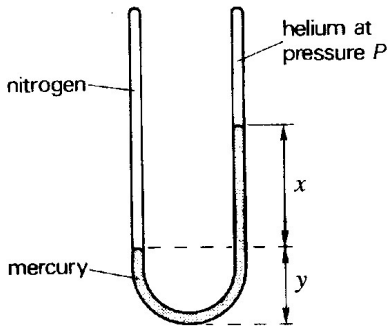
Forces Tutorial

Hooke's Law & Upthrust

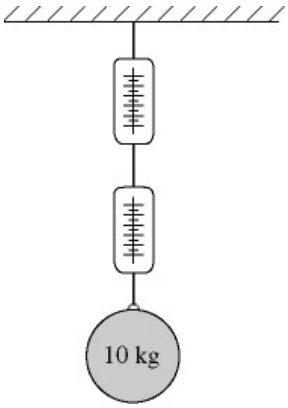
1. A sealed U-tube contains nitrogen in one arm and helium at pressure P in the other arm. The gases are separated by mercury of density ρ with dimensions as shown in the diagram. The acceleration of free fall is g.



What is the pressure of the nitrogen?

A p B $x\rho g$ C $p + x\rho g$ D $p + (x+y)\rho g$

2. A 10 kg mass is suspended from two spring scales that read in kilograms as shown, each of which has negligible weight.



Which statement is correct?

A The top scale will read zero, the lower scale will read 10 kg
B Each scale will read 10 kg
C Each scale will read 5 kg
D The lower scale will read zero, the top scale will read 10 kg

3. A boat with its cargo has a total mass of 1.2 x 10^7 kg. Taking the density of water to be 1000 kg m^{-3}, calculate

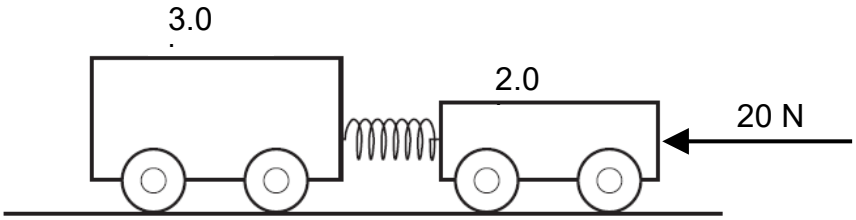
(a) the upthrust on the boat when it is in stable equilibrium, [1.18×10^8 N]
(b) the volume of the boat below the water line. [1.20×10^4 m^3]

4. A spring A of spring constant 6.0 N m^{-1} is connected in series with a spring B of force constant 3.0 N m^{-1}. One end of the combination is securely anchored and a force of 0.6 N is applied to the other end.

(a) By how much does each spring extend? [0.10 m, 0.20 m]

(b) What is the force constant of the combination? [2.0 N m^{-1}]

5. A light spring is attached between a 2.0 kg and 3.0 kg trolley as shown.



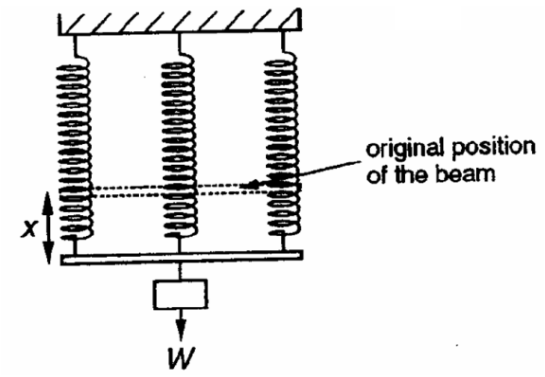
The spring constant of the spring is 4.0 N cm^{-1} and it obeys Hooke's law.

A 20 N force is applied horizontally to the 2.0 kg trolley. The trolleys move on a level floor.

Neglecting the effect of all resistive forces, what is the compression in the spring?

A 0 cm **B** 3.0 cm **C** 5.0 cm **D** 6.0 cm

6. A beam, the weight of which may be neglected, is supported by three identical springs. When a weight W is hung from the middle of the beam, the extension of each spring is x. The middle spring and the weight are removed. What is the extension, in terms of x, when a weight of 2W is hung from the middle of the beam? (J03/I/22)
Ans: 3x



7. A toy which has a mass of 50 g works by the use of a compressed spring. The spring is compressed by 2.0 cm when a force of 20 N is exerted on it. A child releases the toy after compressing the spring by 3.0 cm. How high above the point of release can the toy jump?
[0.917 m]

Equilibrium of Forces and Turning Effects of Forces

8. A block is attached to a uniform beam using a light string. The beam is horizontal and in equilibrium on a pivot, as shown in Fig. 8.1.

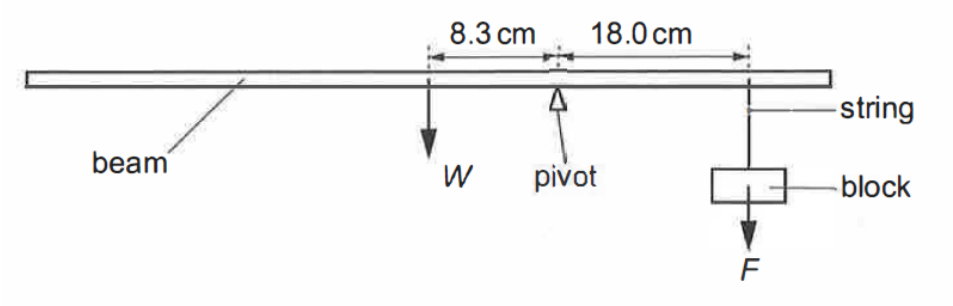


Fig.8.1 (not to scale)

The weight *W* of the beam acts at a distance of 8.3 cm from the pivot. The weight *F* of the block acts at a distance of 18.0 cm from the pivot.

The block is then submerged in water of density 1.0×10^3 kg m^{-3}. The pivot and the block are both moved so that the beam is once more in equilibrium, as shown in Fig .8.2.

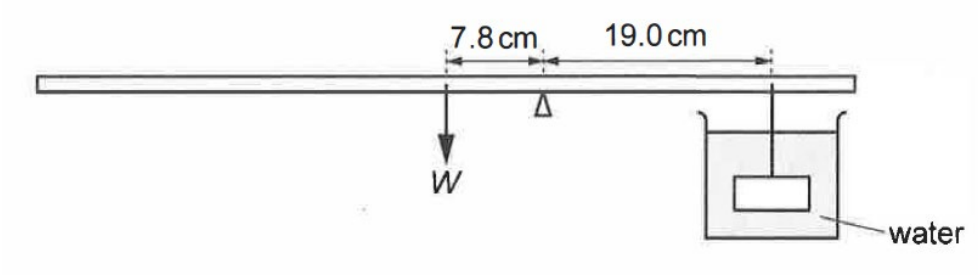


Fig.8.2 (not to scale)

The weight W of the beam acts at a distance of 7 .8 cm from the pivot.
The string supporting the block is at a distance of 19.0 cm from the pivot.

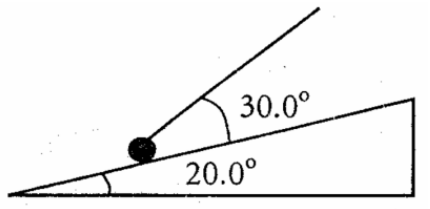
(a) Explain the origin of the force of upthrust acting on the block when it is submerged in the water. [2]

(b) The block has a volume of $27.8 cm^3$.

(i) Show that the upthrust acting on the block is 0.27 N. Explain your working. [2]
(ii) Determine *F*. [3]

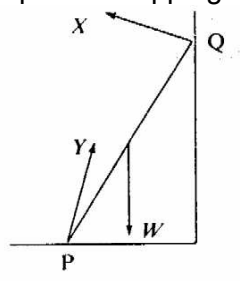
[Nov23/II/1]

9. A body of mass 2.00 kg rests on a smooth slope of angle 20.0^0 to the horizontal, supported by a string which makes an angle of 30.0^0 with the slope as shown.



Find the tension T in the string. Also, determine the normal reaction force N acting on the body. [T = 7.75 N, N = 14.6 N]

10. A ladder PQ, resting on a rough floor against a rough wall, is just on the point of slipping.

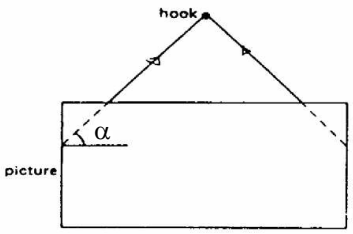


(a) Explain the origin of the forces X, Y, W.

(b) Draw a vector diagram to show these forces in equilibrium.

(c) If the weight of the ladder is 200 N and Y is 150 N in the direction 70° from the floor, determine the magnitude of force X. [78.2 N]

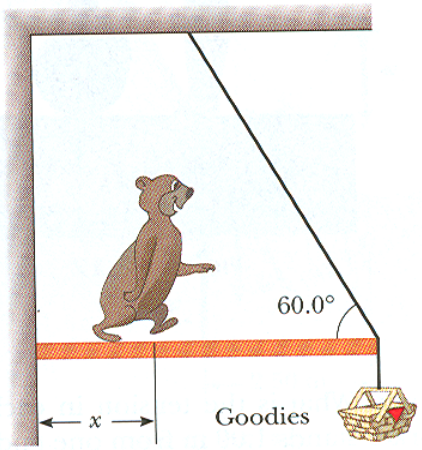
11. A picture of weight 5 N is suspended from a hook on a wall by a chord that has a breaking strength of 25 N.



(a) Calculate the angle α that the chord makes with the horizontal when the tension in the chord is 25 N. [5.74^0]

(b) Is this value of α the maximum or minimum possible value?

12. A hungry bear of weight 700 N walks out on a beam in an attempt to retrieve some goodies hanging at the end. The beam is uniform, of length 6.00 m and weight 200 N. The goodies weigh 80 N.



(a) Draw a free-body diagram for the beam.

(b) When the bear is at x = 1.00 m, find the tension in the wire and the components of the reaction force at the hinge. [343 N, 172 N, 683 N]

(c) If the wire can withstand a maximum tension of 900 N, what is the maximum distance the bear can walk before the wire breaks? [5.14 m]

Data Analysis

13. The Safti Link Bridge is a 100 m long by 14.5 m wide dual carriageway with pedestrian footpath linking the Safti Military Institute and the training ground across the Pan-Island Expressway (PIE). The side view of the bridge is shown in Fig. 13.1.

A tower is sunk into the ground and the roadway is supported by many cables. One end of each cable is attached to the roadway while the other end is attached to the tower. As you travel along the bridge, you can see the cables along the centre divider of the roadway. As there are a total of ten cables, each cable is supporting one-tenth of roadway. Segment **A** is a typical section of one-tenth of the roadway. The cable attached to segment **A** is supporting the mass of segment **A** as well as the load of the traffic on it. Having many cables is a much better system than that of a traditional suspension bridge which relies on the immerse strength of one pair of cables taking the entire load. Simplified data concerning the bridge are given below:

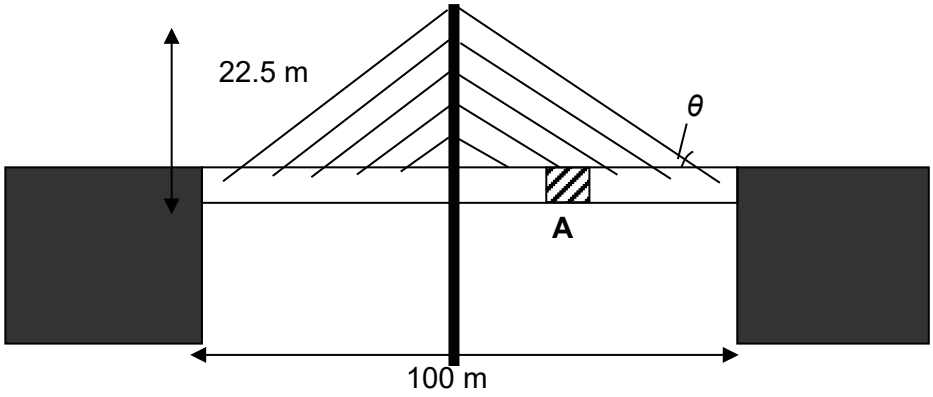


Fig. 13.1

Length of bridge supported by the tower	= 100 m
Height of tower above roadway	= 22.5 m
Total mass of all the cables	= 6.8×10^4 kg
Mass of roadway	= 3.5×10^5 kg
Maximum mass of load of traffic on a normal day (assume uniform distribution)	= 1.3×10^5 kg
Maximum mass of load of traffic during military exercise (assume uniform distribution)	= 2.9×10^5 kg
Horizontal distance between cables	= 10 m
Vertical distance between cables	= 4.5 m
Fundamental frequency of the bridge	= 1.12 Hz

(a) What reason does the paragraph give for the construction with many cables? [2]

(b) When a civil engineer designs the tower, he needs to consider the

maximum total mass which the tower may need to support. Calculate that maximum total mass. [2]

(c) Calculate the mass of 10 m of the roadway and the maximum mass of traffic which the 10 m of roadway may have to support. [2]

(d) Calculate the angle, θ, (in degree) between a cable and the horizontal. [2]

(e) Draw a force diagram of a fully laden 10 m section of road at segment A given that the following forces act at the segment.

W : Weight of 10 m of roadway
N : Force exerted by traffic on 10 m of roadway
T : Tension in cable
R : Net force exerted by the rest of the roadway [4]

(f) By considering the conditions necessary for translational equilibrium, state the equations relating *W*, *N*, *T* and *R*. [2]

(g) Hence, or otherwise, calculate the tension in a cable when the bridge is fully laden. (The tension in all cables is assumed to be the same.) [2]

(h) When the civil engineer designs the bridge, he needs to consider the possibility of vibration of the bridge.

Under what condition will the bridge vibrate with maximum amplitude? Suggest two possible external sources that will give rise to this condition. [4]

(i) Calculate the deflection of the bridge. [2]

Given that the deflection, d, of the bridge is governed by

$$d = \frac{5PL^3}{384EI}$$

where *P* is the loading which is equal to 4.7 x10^6 N.

L is the length of the bridge.

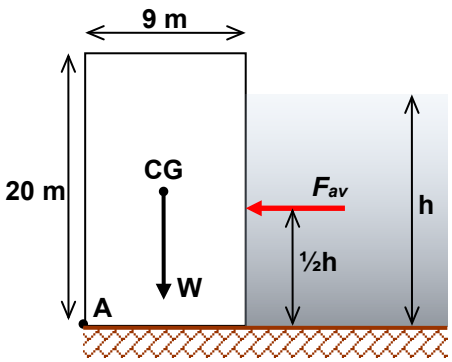
E is the Modulus of Elasticity which is equal to 8 x 10^{11} N m^{-2}

I is the Moment of Inertia which is equal to 15 m^4.

Challenging Questions

14. A beaker of water has a piece of ice floating in it. By applying appropriate principles, determine if the level of the water in the beaker rises, falls or remains the same after the ice melts?

15. A dam is constructed out of a rectangular block of concrete with dimensions as shown in the figure on the right. The densities of concrete and water may be taken to be 2200 kg m^{-3} and 1000 kg m^{-3} respectively.



(a) Assuming the force due to the water pressure F_{av} to act at mid depth as shown in the figure, determine the depth of water which can be held by the dam before it topples about point A.

(b) Explain if the value obtained in part (a) is an over-estimate or under-estimate.

(c) Considering that water pressure acts normally on a surface, suggest a more practical design of the dam.