Answer **all** the questions.

1 **(a)** Find the range of values of p for which the equation $px^2 + x + p(x+1) = 0$ has real and distinct roots. [3]

$$px^2 + x + p(x+1) = 0$$
$$px^2 + (1+p)x + p = 0$$

$$D = (1+p)^2 - 4(p)(p)$$
$$= -3p^2 + 2p + 1$$

[M1 cao] Correct expression for D

For real and distinct roots, $D > 0$

$$-3p^2 + 2p + 1 > 0$$ [M1] State $D > 0$

$$3p^2 - 2p - 1 < 0$$
$$(3p+1)(p-1) < 0$$

$$-\frac{1}{3} < p < 1$$ [A1]

(b) State the value(s) of p for which the curve $y = px^2 + x + p(x+1)$ is tangent to the x-axis. [1]

For the curve to be tangent to y-axis, $D = 0$

$$(3p+1)(p-1) = 0$$

$p=-\frac{1}{3}$, $p-1$ [A1]

2 Solve the following equations

(i) $2\log_3 x+\log_x 3=3$, [4]

$$2\log_3 x+\log_x 3=3$$
$$2\log_3 x+\frac{\log_3 3}{\log_3 x}=3$$

[M1 – Change of Base]

Let $y=\log_3 x$

Therefore, $2y+\frac{1}{y}=3$ [M1 – Substitution Method]

$$2y^2-3y+1=0$$
$$(2y-1)(y-1)=0$$

$y=\frac{1}{2}$ or $y=1$ [M1] Values of *y*

$\log_3 x=\frac{1}{2}$ $\log_3 x=1$

$x=\sqrt{3}$ $x-3$ [A1]

(ii) $3^{2y+1}-3^{y+2}+3=3^y$. [4]

$$3^{2y+1}-3^{y+2}+3=3^y$$
$$(3)\left(3^y\right)^2-(9)\left(3^y\right)+3-\left(3^y\right)=0$$

[M1] Break up the indices

$$3\left(3^y\right)^2-10\left(3^y\right)+3=0$$

Let $p=3^y$

$3p^2-10p+3=0$ [M1] Correct Quadratic Equation

$(3p-1)(p-3)=0$ [M1] Substitution and values of both p found

$p=\frac{1}{3}$ or $p=3$

$3^y=3^{-1}$ 3^y-3

$y=-1$, $y=1$ [A1]

3 In the expansion of $\left(x^2-\frac{m}{2x}\right)^{12}$, where m is a positive constant, the independent term of x is 126720.

(a) Show that $m-4$. [4]

General Term $=\binom{12}{r}\left(x^2\right)^{12-r}\left(-\frac{m}{2x}\right)^r$ [M1] General Term

$T_{r+1}=\binom{12}{r}\left(x^{24}\right)\left(x^{-2r}\right)\left(-\frac{m}{2}\right)^r\left(x^{-r}\right)$

$x^{24-2r-r}=x^0$

$24-3r$

$r-8$ [M1cao] – Correct value of r

$\binom{12}{8}\left(x^2\right)^{12-8}\left(-\frac{m}{2x}\right)^8=126720$

$(495)x^8\left(\frac{m^8}{256x^8}\right)=126720$

$m^8=65536$ [M1] Equate general term with $r-8$

$m=(65536)^{\frac{1}{8}}$ $m-4$ [AG1]

(b) **Hence**, find the coefficient of x^9 in the expansion of $\left(x^2-\frac{m}{2x}\right)^{12}\left(8x^9+5\right)$ [3]

$x^{24-3r} = x^9$

$24 - 3r = 9$

$r - 5$ [M1] Value of *r* found

Term with x^9 $= \binom{12}{5}(x^2)^{12-5}\left(-\frac{4}{2x}\right)^5$

$= (792)x^{14}\left(\frac{-1024}{32x^5}\right)$ $= -25344x^9$ [M1] Find term with x^9

$$\left(x^2 - \frac{m}{2x}\right)^{12}(8x^9+5) = (....-25344x^9 + 126720....)\ (8x^9+5)$$

$$= ...-126720x^9 + 1013760x^9 + ...$$

$= ...887040x^9$ Coefficient of x^9 is 887040 [A1]

4 **(a)** Express $\frac{-x-11}{(2x+1)(x-3)}$ in partial fraction. [3]

$\frac{-x-11}{(2x+1)(x-3)} = \frac{A}{2x+1} + \frac{B}{x-3}$ [M1cao] Break up

$-x-11 = A(x-3) + B(2x+1)$

Substitute $x = -\frac{1}{2}$: $-\left(-\frac{1}{2}\right) - 11 = -3\frac{1}{2}A$ [M1]Substitution Method

$A - 3$

Substitute $x - 3$: $-3-11 = 7B$ $B = -2$

$\frac{-x-11}{(2x+1)(x-3)} = \frac{3}{2x+1} - \frac{2}{x-3}$ [A1] Both values of *A* and *B* correct

(b) **Hence**, evaluate $\int \frac{-2x-22}{(2x+1)(x-3)}\,\mathrm{d}x$. [3]

$$\int \frac{-2x-22}{(2x+1)(x-3)}\,\mathrm{d}x$$
$$= 2\left[\int \frac{3}{2x+1}\,\mathrm{d}x - \int \frac{2}{x-3}\,\mathrm{d}x\right]$$
[M1] Relate to $\frac{3}{2x+1} - \frac{2}{x-3}$
$$= 2\left[\frac{3}{2}\int \frac{2}{2x+1}\,\mathrm{d}x - 2\int \frac{1}{x-3}\,\mathrm{d}x\right]$$
[M1] $\frac{3}{2}$ and 2 seen
$$= 3\ln(2x+1) - 4\ln(x-3) + c$$
[A1]

5 The curve $y = \frac{\ln x}{2x^2}$, for $x > 0$ has a stationary point at point *A*.

Find

(i) $\frac{\mathrm{d}y}{\mathrm{d}x}$, [2]

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\left(\frac{1}{x}\right)(2x^2) - (4x)(\ln x)}{(2x^2)^2}$$
[M1] Use Product Rule
$$= \frac{2x - 4x\ln x}{4x^4}$$
$$= \frac{2x[1-2\ln x]}{4x^4}$$
$$= \frac{1-2\ln x}{2x^3}$$
[A1]

(ii) the x-coordinate of A, [2]

At stationary point, $\frac{dy}{dx}=0$,

$\frac{2x-4x\ln x}{4x^4}=0$ [M1] Equate $\frac{dy}{dx}=0$

$2x(1-2\ln x)=0$

$2x=0$ or $1-2\ln x=0$

$x=0$ $\ln x=\frac{1}{2}$

(Not required) $x=e^{\frac{1}{2}}=1.64872\approx 1.65$ (3 s.f.) [A1]

(iii) the nature of A. [3]

$$\frac{dy}{dx}=\frac{2x-4x\ln x}{4x^4}=\frac{1}{2}x^{-3}-\left(x^{-3}\right)(\ln x)=x^{-3}\left(\frac{1}{2}-\ln x\right)$$

$$\frac{d^2y}{dx^2}=-3x^{-4}\left(\frac{1}{2}-\ln x\right)+\left(-\frac{1}{x}\right)\left(x^{-3}\right)$$

[M1] Product Rule

Or by 1st derivative test

At A, $x=e^{\frac{1}{2}}$

[M1] Use 1st or second derivative correctly to determine its nature

$\frac{d^2y}{dx^2}=-\text{ve}$,

Since $\frac{d^2y}{dx^2}<0$, point A is a maximum point.

[A1] Conclusion and state $\frac{d^2y}{dx^2} < 0$

6 **(a)** Given that $5\sin x\cos x = 1$, find

(i) $\sin 2x$, [1]

$$\sin 2x = 2\sin x\cos x$$
$$= 2\left(\frac{1}{5}\right)$$
$$= \frac{2}{5}$$

[B1]

(ii) $\cos 4x$. [2]

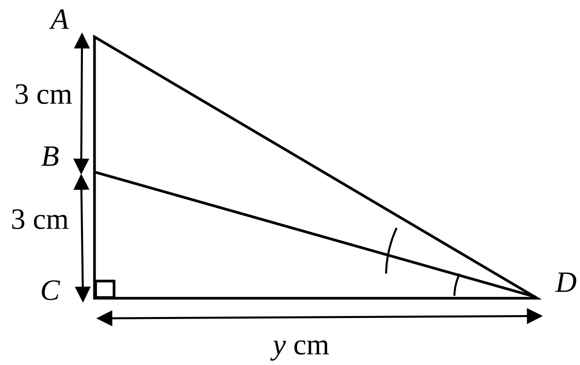
$$\cos 4x = 1 - 2\sin^2 2x$$
$$= 1 - 2\left(\frac{2}{5}\right)^2$$
$$= 1 - \frac{8}{25}$$
$$= \frac{17}{25}$$

[M1] Use correct double angle formulae

[A1]

(b) The diagram below shows a right angled triangle *ACD*.

It is given that $\angle ADB = \beta$, $\angle BDC = \alpha$, $AB = BC = 3$ cm and $CD = y$ cm.



Express $\tan\beta$ in terms of y. [4]

$\tan\alpha = \dfrac{3}{y}$ and $\tan(\alpha+\beta) = \dfrac{6}{y}$ [M1 cao] $\tan(\alpha+\beta) = \dfrac{6}{y}$

Since $\tan(\alpha+\beta) = \dfrac{\tan\alpha + \tan\beta}{1-\tan\alpha\tan\beta}$

$$\frac{6}{y} = \frac{\frac{3}{y} + \tan\beta}{1-\left(\frac{3}{y}\right)\tan\beta}$$

[M1] Apply Addition Formulae

Let $m = \tan\beta$

$$\frac{6}{y} = \frac{\frac{3}{y} + m}{1-\left(\frac{3}{y}\right)m}$$

$$\frac{6}{y} = \frac{3+my}{y-3m}$$

$$6y - 18m = 3y + my^2$$

[M1] Correct Quadratic Equation

$$my^2 + 18m = 6y - 3y$$

$$m = \frac{3y}{y^2+18}$$

Therefore, $\tan\beta = \dfrac{3y}{y^2+18}$. [A1]

7 A curve has the equation $y = \dfrac{x-2}{3x+2}$.

(a) Explain why the curve $y = \dfrac{x-2}{3x+2}$ has no turning point. [2]

$$y = \frac{x-2}{3x+2}$$

$$\frac{dy}{dx} = \frac{(1)(3x+2)-(3)(x-2)}{(3x+2)^2}$$

$$\frac{dy}{dx} = \frac{(1)(3x+2)-(3)(x-2)}{(3x+2)^2}$$

$$\frac{dy}{dx} = \frac{8}{(3x+2)^2}$$

[M1 cao] Correct $\frac{dy}{dx}$

Since $8 > 0$ and $(3x+2)^2 > 0$, therefore $\frac{dy}{dx} > 0$ and $\frac{dy}{dx} \neq 0$. [A1] Correct Explaination
y has no stationary point.

(b) Given that *y* is increasing at a rate of 0.4 units per second at the instant when $y = \frac{1}{11}$, find the rate of change of *x* at that instant. [3]

Given $\frac{dy}{dt} = 0.4$ units per second

By Chain Rule,

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

[M1] Form Chain Rule

When $y = \frac{1}{11}$, $\frac{1}{11} = \frac{x-2}{3x+2}$,

$3x + 2 = 11x - 22$,

$x = 3$ [M1] Value of $x = 3$

$$\frac{8}{(3(3)+2)^2} = (0.4) \times \frac{dt}{dx}$$

$$\frac{dt}{dx} = 0.165289$$

$\frac{dx}{dt} = 6.05$ units per second (3 s.f.) [A1]

(c) A curve is such that $\frac{d^2y}{dx^2} = 24x^2$ and has a turning point $(-1,-6)$.

Find the equation of the curve. [4]

$\frac{d^2y}{dx^2} = 24x^2$

$\frac{dy}{dx} = \int 24x^2 \ dx$

$= \frac{24x^3}{3} + c$ [M1] Find $\frac{dy}{dx}$

At turning point $(-1,-6)$, $\frac{dy}{dx} = 0$

$\frac{24(-1)^3}{3} + c = 0$

$c = 8$ [M1] Value of *c*

Therefore, $\frac{dy}{dx} = 8x^3 + 8$

$y = \int 8x^3 + 8 \ dx$

$y = \frac{8x^4}{4} + 8x + d$ [M1] Integrate $\frac{dy}{dx}$ to find *y*

At $(-1,-6)$,

$-6 = \frac{8(-1)^4}{4} + 8(-1) + d$

$d = 0$

Therefore, $y = 2x^4 + 8x$ [A1]

8 The expression $f(x) = ax^3 + x^2 - bx + 3a$ has a factor of $(x+3)$ and leaves a reminder of -4 when divided by $(x-1)$.

(a) Find the value of *a* and of *b*. [4]

By Factor Theorem,

$f(-3) = a(-3)^3 + (-3)^2 - b(-3) + 3a$

$-27a + 9 + 3b + 3a = 0$

$3b - 24a = -9$ ----------------- (1) [M1] Use Remainder/Factor Theorem

By Remainder Theorem,

$f(1) = -4$

$a + 1 - b + 3a = -4$

$b = 4a + 5$ -------------------- (2) [M1] Either Equation (1) and (2)

Substitute (1) into (2)

$3(4a+5) - 24a = -9$

$a - 2$ [A1] Value of *a*

When $a - 2$, $b = 4(2) + 5 = 13$ [A1] Value of *b*

(b) **Using part (a),** solve $f(x)=ax^3+x^2-bx+3a$. [3]

By Synthetic Division **or** Long Division [M1] Use either method correctly

$f(x)=2x^3+x^2-13x+6$

$f(x)=(x+3)(2x^2-5x+2)$

$f(x)=(x+3)(x-2)(2x-1)$ [M1] Factorised correctly

When f(x) =0

$x=-3,\ 2,\ \frac{1}{2}$ [A1]

(c) **Hence**, solve $f(x)=ae^{3x}+e^{2x}-be^x+3a$. [2]

$e^x=-3$, (no solution) , $e^x=2$, $x=\ln 2$, $x=0.693$, $e^x=\frac{1}{2}$, $x=\ln\frac{1}{2}$, $x=-0.693$ [M1] Equate to e^x

[A1] Both answer correct including reject -3

9 **(i)** Differentiate $4xe^{3x}$ with respect to x. [2]

$$\frac{\mathrm{d}}{\mathrm{d}x}4xe^{3x} = 4e^{3x} + 3e^{3x}(4x)$$

[M1] Differentiate exponential correctly/Product Rule

$$= 4e^{3x} + 12xe^{3x}$$

[B1]

(ii) **Hence**, evaluate $\int_0^3 6xe^{3x}\,\mathrm{d}x$**,** giving your answer in **exact form**. [4]

$$\int_0^3 4e^{3x}\,\mathrm{d}x + \int_0^3 12xe^{3x}\,\mathrm{d}x = \left[4xe^{3x}\right]_0^3$$

[M1]Form equation

$$\left[4\left(\frac{1}{3}\right)e^{3x}\right]_0^3 + 2\int_0^3 6xe^{3x}\,\mathrm{d}x = \left[4xe^{3x}\right]_0^3$$

[M1] Integrate $\int_0^3 4e^{3x}\,\mathrm{d}x$ correctly

$$\left[\frac{4}{3}e^9 - \frac{4}{3}\right] + 2\int_0^3 6xe^{3x}\,\mathrm{d}x = \left[12e^9 - 0\right]$$

[M1] Substitute limits correctly

$$\int_0^3 6xe^{3x}\,\mathrm{d}x = \left[12e^9 - \frac{4}{3}e^9 + \frac{4}{3}\right] \div 2$$

$$= 5\frac{1}{3}e^9 + \frac{2}{3}$$

[A1] Exact answer only

10 A circle passes through the points $P(-2,8)$ and $Q(-4,4)$.

(i) Find the equation of the perpendicular bisector of PQ. [3]

Gradient of PQ $= \dfrac{8-4}{-2-(-4)} = 2$

Gradient of perpendicular bisector of PQ $= -\dfrac{1}{2}$ [A1 cao] Gradient $-\dfrac{1}{2}$

Midpoint of PQ $= \left(\dfrac{-4+(-2)}{2}, \dfrac{4+8}{2}\right) = (-3,6)$ [A1 cao] Midpoint

$$y = mx + c$$
$$6 = (-3)\left(-\frac{1}{2}\right) + c \qquad c = 4\frac{1}{2}$$

Equation of perpendicular bisector of PQ : $y = -\dfrac{1}{2}x + 4\dfrac{1}{2}$ or $2y = -x + 9$ [A1]

The centre, *C*, of the circle lies on the line $2x+y=6$.
Find the

(ii) coordinate of *C*, [3]

Solve $2y=-x+9$ and $2x+y=6$ simultaneously [M1] Solve simultaneously

$2(6-2x)=-x+9$

$x=1$ [A1] Correct value for *x*

$2(1)+y=6$

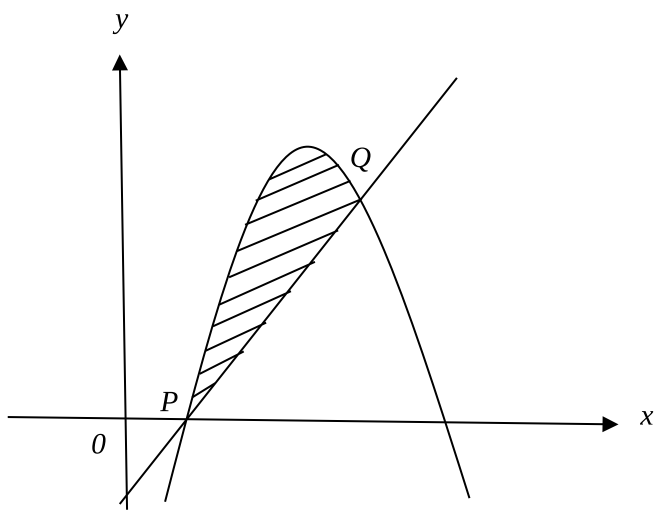
$y=4$ $C(1,4)$ [A1] Correct value for *y*

(iii) equation of the circle. [2]

$r = \sqrt{3^2 + 4^2} = 5$ units [M1] Radius

Equation of circle : $(x-1)^2 + (y-4)^2 = 5^2$ or $x^2 + y^2 - 2x - 8y - 8 = 0$ [A1]

11 In the diagram, the curve $y = -x^2 + 5x - 4$ cuts the line $y = x - 1$ at two points *P* and *Q*.



Calculate the area of the shaded region bounded between the curve and line. [6]

Equate $y = x - 1$ with the curve $y = -x^2 + 5x - 4$

$x - 1 = -x^2 + 5x - 4$ [M1] Use substitution method / Equate both *y*

$x^2 - 4x + 3 = 0$

$(x-1)(x-3)=0$

$x=1, x=3$ [M1] Either both *x* or both *y* values are correct

When $x=1, \quad y=0$

When $x=3, \quad y=2$

The coordinates are $P(1,0)$ and $Q(3,2)$ [A1] Coordinate form for *P* and *Q*

[Continue your working here for Question 11]

Area of shaded region

$=\int_1^3 -x^2+5x-4 \text{ dx} - \text{Area of Triangle}$ [M1] Integration curve from 3 to 1 correctly

$=\left[-\frac{x^3}{3}+\frac{5x^2}{2}-4x\right]_1^3 - \frac{1}{2}(2)(2)$

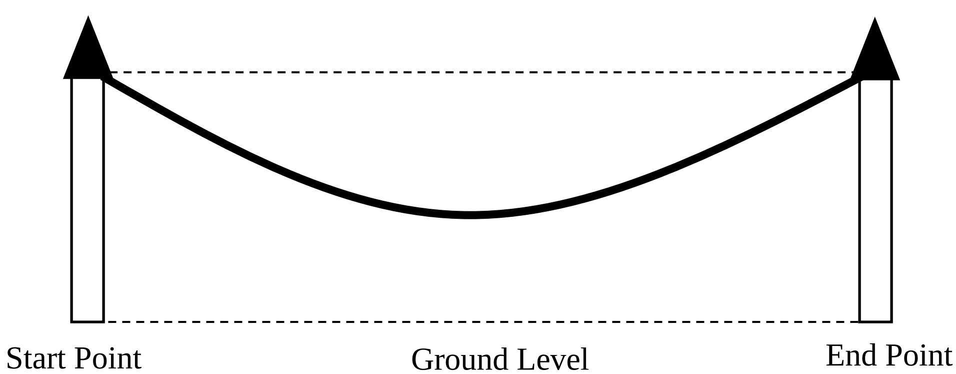
[M1] Area of triangle

$=\left[(-9+22.5-12)-\left(-\frac{1}{3}+\frac{5}{2}-4\right)\right]-2$

$=1.5-\left(-\frac{11}{6}\right)-2$

$=1\frac{1}{3}$ units^2 [A1]

12 The diagram below shows a zip line found in a particular Outward Bound School.
The height, h m, of a participant above the ground in a section of the zip line is given by $h = \frac{2}{3}x^2 - 4x + 12$, where x m is the participant's horizontal distance from the start point.



(i) Express the function in the form $h = a(x-b)^2 + c$. [2]

$h=\frac{2}{3}x^2-4x+12$

$h=\frac{2}{3}\left[x^2-6x+18\right]$

$h=\frac{2}{3}\left[x^2-6x+\left(-\frac{6}{2}\right)^2+18-\left(-\frac{6}{2}\right)^2\right]$ [M1 cao] Factorised correctly

$h=\frac{2}{3}\left[(x-3)^2+9\right]$

$h=\frac{2}{3}(x-3)^2+6$ [A1]

(ii) Find the participant's lowest height from the ground. [1]

$h=\frac{2}{3}(x-3)^2+6$

Minimum Point : $(3,6)$

The minimum height is 6 m. [B1]

(iii) If the participant is 9 m above the ground, find the 2 possible horizontal distance of the participant from the start point. [2]

$9=\frac{2}{3}(x-3)^2+6$ [M1]

$\frac{2}{3}(x-3)^2=3$

$(x-3)^2=\frac{9}{2}$

$x-3=\pm\frac{3}{\sqrt{2}}$

$x=3+\frac{3}{\sqrt{2}}$ or 5.12 m and $x=3-\frac{3}{\sqrt{2}}$ or 0.879 m [A1]

(iv) One of the participant, Charlene claimed that at an angle of elevation of $55°$ from the lowest point along the zip line, she was able to see an owl standing at the top of the end point.

Verify, with reason, the validity of her claim. [2]
State one assumption made in your calculation.

$\tan\theta = \frac{6}{3}$

$\theta = 63.4°$ (1 decimal place)

Her claim is not true. The angle of elevation should be $63.4°$. [B1]

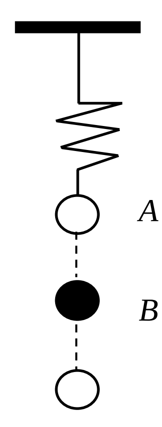
Assumption (or any reasonable answer)

(1) Assume the zip line is fully straighten.
(2) Assume that Charlene is not moving/ stationary on the zip line [B1]

13 A steel ball is attached to a spring is then pulled to position *C* from its equilibrium position *B* and released. The steel ball will oscillate up and down between *A* to *C*. It is assumed that there is no loss in energy or momentum of the steel ball.

Given that *C* is the initial position of the steel ball, the change in position, *h* cm, of the steel ball from *B* over time, *t* seconds, can be modelled as

$$h = -20\cos\left(\frac{\pi}{2}t\right).$$



C

(a) Find the vertical distance between *A* and *B*. [1]

Amplitude = 20
Hence, vertical distance between *A* and *B* is 20 cm. [B1]

(b) Find the time taken for the steel ball to move from *C* to *A*. [2]

Period = $\dfrac{2\pi}{\frac{\pi}{2}}$ = 4 [M1]
Period

Half of one period of motion is from *C* to *A*.
Therefore, the steel ball takes 2 seconds to move from *C* to *A*. [A1]

(c) Find the vertical distance of the steel ball from *B* at $t = 5.5$. [1]

$$h = -20\cos\left(\frac{\pi}{2} \times 5.5\right)$$

$$h = 14.1 \text{ cm}$$

(d) Find the two earliest times when the mass is 2 cm above *B*. [2]

$$2 = -20\cos\left(\frac{\pi}{2}t\right)$$

$$\cos\left(\frac{\pi}{2}t\right) = -\frac{1}{10}$$ [M1]

$$\cos\left(\frac{\pi}{2}t\right) = -\frac{1}{10}$$

Basic Angle $\alpha = \cos^{-1}\left(\frac{1}{10}\right)$ = 1.4706 rad

$\frac{\pi}{2}t$ lies in 2nd and 3rd quadrant

$$\frac{\pi}{2}t = \pi - 1.4706, \pi + 1.4706$$

$$t = 1.06\,\text{s},\ 2.94\,\text{s}$$ [A1]

End of Paper