NAME:		( )
CLASS:	TEACHING GROUP:	MARKS /90

PEI HWA SECONDARY SCHOOL

PRELIMINARY EXAMINATION 2021

Secondary Four Express

ADDITIONAL MATHEMATICS 4049/02

Paper 2 31 August 2021

2 hour 15 minutes

Candidates answer on the question paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in the brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 90.

For Examiner's Use	
Category	Question No.
Correction tape	
Pencil written	
Arrows	
Units	
Others	

This question paper consists of 21 printed pages, inclusive of this cover page.

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1 Express $\frac{1-2x^2+x^3}{x^3+3x}$ in partial fractions. [5]

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2 In Tiger City, the number of Plaque Forming Units (PFU) of a new strain of virus, N, increases with time, t hours. The measured values of N and t are recorded in the following table:

t	2	4	6	8	10
N	66.4	163.3	401.7	988.1	2430.4

It is known that N and t are related by the equation $\frac{N}{N_0} = e^{bt}$, where N_0 and b are constants.

(i) Explain clearly how N_0 and b can be calculated when a straight line graph of $\ln N$ against t is drawn. [3]

(ii) On the following grid, draw a straight line graph of $\ln N$ against t to illustrate the data given in the table. [2]

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(iii) Use your graph to estimate the value of N_0 and of b. [2]

(iv) A state of emergency will be announced in Tiger City when the number of Plaque Forming Units (PFU) of virus reaches 5000. There are rumours online claiming that the announcement will happen after 13 hours. Using your graph, explain clearly if this is true. [2]

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(v) Due to a mutation of the virus, the World Health Organisation (WHO) updates the health advisory and suggests that N and t are now related by the equation $3\ln N - 150 = 6N_0 + 2bt$, where N_0 and b are constants.

The new straight line graph of $\ln N$ against t has a $\ln N$-intercept of 180.16.

Find the new value of N_0. [2]

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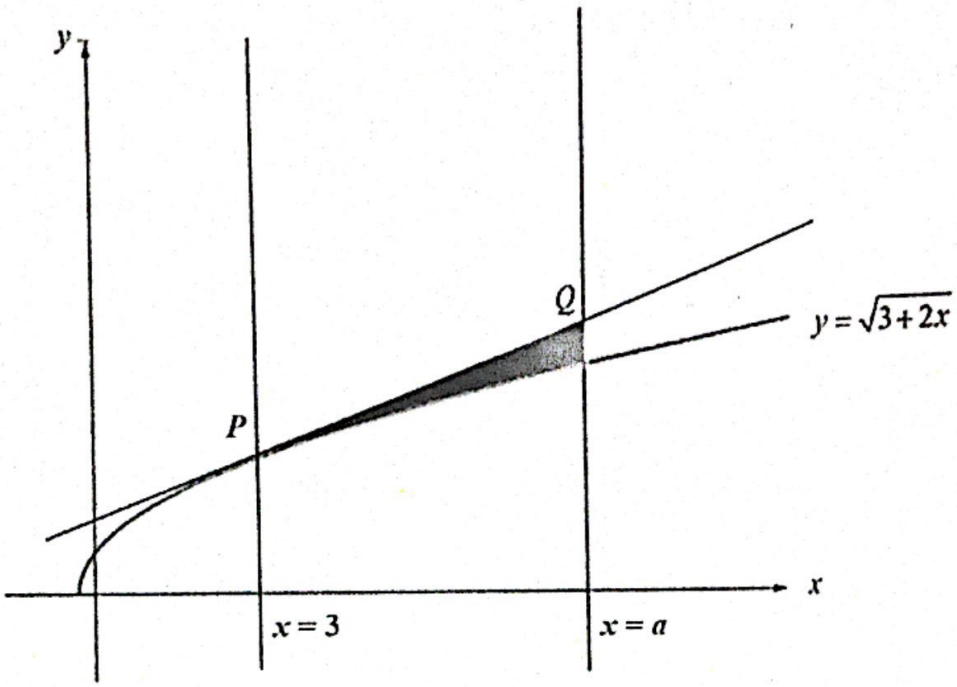
3 The diagram shows part of the curve $y = \sqrt{3+2x}$. PQ is a tangent to the curve at $x = 3$ and intersects the line $x = a$ at Q.



(i) Find the equation of the tangent. [4]

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(ii) The area bounded by PQ, the line $x = 3$ and the line $x = a$ is given as 24 units^2.

Show that $a = 9$. [3]

(iii) Find the area of the shaded region bounded by PQ, the curve and the line $x = a$. [3]

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4 (a) A curve has equation $y-(k+1)=\frac{2}{x}$ and a line has equation $y+1=kx$, where k is a constant. Find the set of values of k for which the curve meets the line. [5]

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(b) The height above the ground, in metres, of the rooftop of a building is modelled as $P(x) = -x^2 + 6x + 11$, where x is the horizontal distance from the main office. The rooftop in the neighbouring school is modelled by $R(x) = 17 - x$. Find the values of x, in metres, for which the rooftop of the building is above that of the school. [3]

(c) The curve with equation $y = (a+1)x^2 + bx + 2 + b$, where a and b are constants, is always above the x-axis. Write down two conditions which apply to a and b. [2]

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5 **(a)** Express $\frac{2x}{2x-3}$ in the form $a+\frac{b}{2x-3}$ where a and b are constants.

Hence, find $\int \frac{2x}{2x-3}\,dx$. [4]

(b) Differentiate $\frac{x\ln(2x-3)}{2}$ with respect to x. [3]

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(c) Using the results from **(a)** and **(b)**, find $\int \frac{2\ln(2x-3)}{3} dx$. [3]

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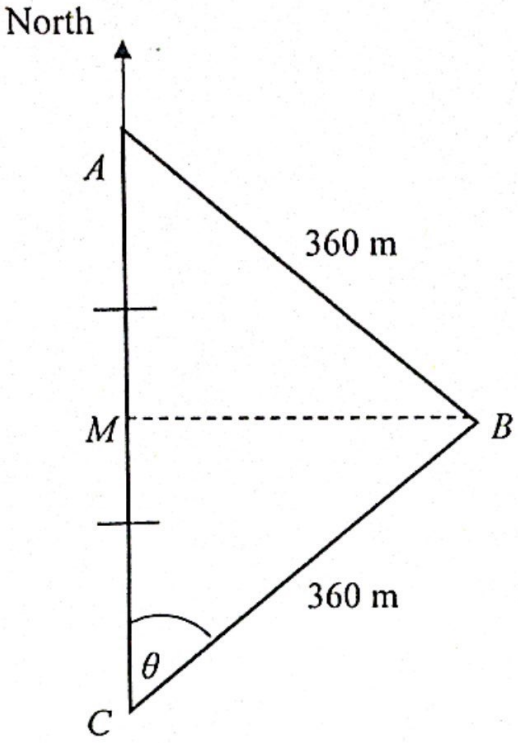
6 The following diagram shows the pathways in a nature reserve. The bearing of rest point B from C is $\theta°$. Rest points M and C are both due south of A.
Distance of pathways $AB = BC = 360$ m. Rest point M is the midpoint of pathway AC.



(i) A guide brings her visitors along a trail using pathways AB, BM, MC and CB. Show that T m, the distance of the trail, can be expressed in the form $T = 720 + 360\sin\theta + 360\cos\theta$. [3]

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(ii) Express T in the form $p + R\sin(\theta + \alpha)$, where $R > 0$ and α is an acute angle. [4]

(iii) Given that the distance of the trail in **(i)** is 1.2 km, find the value of θ. [2]

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7 The equation of a curve is $y = e^{3x-2x^2}$.

(a) Find expressions for $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. [5]

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(b) Find the exact value of the coordinates of the stationary point. [4]

(c) Find the nature of the stationary point. [2]

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8 A circle, C_1, has a diameter AB where A is the point $(-1, 1)$ and the tangent at B is the line $3y - 4x + 43 = 0$.

(i) Find the equation of the diameter AB and hence the coordinates of B. [4]

(ii) Find the equation of the circle, C_1. [3]

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A second circle, C_2, has equation $x^2 + y^2 - x + 4y = 12$.

(iii) Find the coordinates of the centre and the radius of C_2, [3]

(iv) Explain if the circle C_2 lies entirely within the circle C_1. [2]

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9 **(a)** Use the substitution of $U = 3^x$ to solve the equation $2(3^{2x}) - 3^{x+1} = 5$. [4]

(b) Solve the equation $\log_2 x - \log_4 2 = \log_x 8$. [4]

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(c) Explain why $\lg(x+1)+\lg(x-2)=\lg(x-5)$ has no real solution. [4]

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