Marking Scheme AM P2 (4049/02) Setters : J Joseph(1,2,4,5 and 6) Ng SL (3,8,9 and 10)

Qn	Answer	Marks	Partial Marks	Guidance
1	$2\sin^4 x + 7\left(1-\sin^2 x\right) = 4$			
	$\sin^2 x = \frac{1}{2}$			
	Basic angle $= 45^\circ$			
	$x = 45^\circ, 135^\circ, 225^\circ, 315^\circ$			
2(a)	$120 = 150e^{-50k}$			
	$e^{50k} = \frac{150}{120}$ $k = \frac{\ln\left(\frac{150}{120}\right)}{50}$ $= 0.00446287$			
	$M = 150e^{-120\times 0.00446287}$ $= 87.8g$ (3 s.f.)			
2(b)	When M $= \frac{2}{3}\times 150 = 100\,g$ $100 = 150e^{-0.00446287t}$ $e^{0.00446287t} = \frac{150}{100}$ $t = \frac{\ln\left(\frac{150}{100}\right)}{0.00446287}$ $= 90.9$ hours (3 s.f.)			
3(a)	$\frac{dy}{dx} = \frac{2e^{2x}(x-2) - e^{2x}}{(x-2)^2}$			
3(b)	$\frac{2e^{2x}(x-2) - e^{2x}}{(x-2)^2} = 0$ $e^{2x}(2x-4-1) = 0$ $x = \frac{5}{2} = 2.5$ $y = 2e^5$ $\left(2.5,\ \ 2e^5\right)$			
3(c)	(2.5, $2e^5$) is a minimum point.			

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4(a)	A$(a,0)$ and B$(0,b)$ gradient of AB $=-\dfrac{b}{a}$ gradient of perpendicular bisector $=\dfrac{a}{b}$ midpoint of AB is $=\left(\dfrac{a}{2},\dfrac{b}{2}\right)$ gradient of perpendicular bisector $=\dfrac{\frac{b}{2}+7}{\frac{a}{2}+3}$ $\dfrac{a}{b}=\dfrac{b+14}{a+6}$ $a^2+6a=b^2+14b$ (shown)			
4(b)	$a=2b$ $(2b)^2+6(2b)=b^2+14b$ $3b^2-2b=0$ $b(3b-2)=0$ $b=\dfrac{2}{3}$ $a=\dfrac{4}{3}$			
5(a)	$2g=-10,\quad 2f=-4$ centre$(5,2)$ $r=\sqrt{(-5)^2+(-2)^2-25}$ $=2$ units			
5(b)	The centre of the circle is 2 units above the *x*-axis and the radius of the circle is 2 units. Hence the *x*-axis is a tangent to the circle.			
5(c)	When $x=3$ $y=2$ $(3,2)$ and $(5,2)$ gradient $=0$ Equation of tangent is $x=3$ $\therefore$ P$(3,0)$			

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6(a)	$PT = 12\cos\theta$ $SR = 5\sin\theta$ $d = 12\cos\theta + 5\sin\theta$			
6(b)	$R = \sqrt{12^2 + 5^2}$ $= 13$ $\alpha = \tan^{-1}\left(\frac{5}{12}\right)$ $= 22.6^\circ$ $d = 13\cos\left(\theta - 22.6^\circ\right)$			
6(c)	$13\cos\left(\theta - 22.6^\circ\right) = 10$ $\theta = 62.3^\circ$			
7(a)	$f(-1) = 0 \rightarrow -a + b = 5$ $f(-3) = -60 \rightarrow -3a + b = 3$ $a = 1,\ b = 6$			
7(b)	$x^3 - 4x^2 + x + 6 = 0$ $(x+1)\left(x^2 - 5x + 6\right) = 0$ $x = -1,\ 2,\ 3$			
7(c)	$\left(3^x\right)^2 + 1 = 4\left(3^x\right) - \frac{6}{3^x}$ $\left(3^x\right)^3 - 4\left(3^x\right)^2 + 3^x + 6 = 0$ The equation can be solved by taking each solution in part b 3^x.			

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8 (a)	$12 = 8(2) - c(2)^3$ $c = \frac{1}{2}$			
8(b)	$\frac{dv}{dt} = a = 8 - \frac{3}{2}t^2$ $= 8 - \frac{3}{2}(2)^2$ $= 2\ m/s^2$			
8(c)	$8t - \frac{1}{2}t^3 = 0$ $t\left(8 - \frac{1}{2}t^2\right) = 0$ $t^2 = 16$ $t = 4$			
	$s = \int\left(8t - \frac{1}{2}t^3\right)dt$ $= \frac{8t^2}{2} - \frac{t^4}{8} + c$ $s = 0, t = 0 \Rightarrow c = 0$ $s = 4t^2 - \frac{1}{8}t^4$ $t = 4,$ $s = 4(4)^2 - \frac{1}{8}(4)^4$ $= 32$ m $t = 5,$ $s = 4(5)^2 - \frac{1}{8}(5)^4$ $= 21.875$m Distance travelled = $= 32 + 32 - 21.875 = 42.125$ Average speed = 42.125 ÷ 5 = 8.425 m/s			

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9(a)	$\frac{dy}{dx}=-\frac{1}{2}(10-6x)^{-\frac{1}{2}}(-6)$ $=3(10-6x)^{-\frac{1}{2}}$ $x=-1,\quad \frac{dy}{dx}=\frac{3}{4},$ $y=-4$ Gradient of normal = $-\frac{4}{3}$ Equation of normal : $y+4=-\frac{4}{3}(x+1)$ $3y=-4x-16$			
9(b)	At C, $y = 0$, $x = -4$ $At\quad A,\ y=0,\ x=\frac{5}{3}$ Area of triangle = $\frac{1}{2}\times3\times4=6$ unit^2 Area under curve = $-\int_{-1}^{\frac{5}{3}}-\sqrt{10-6x}\ dx$ $=\left[\frac{(10-6x)^{\frac{3}{2}}}{\frac{3}{2}\times(-6)}\right]_{-1}^{\frac{5}{3}}=\left[0-\frac{4^3}{9}\right]$ Area of shaded region = $6+\frac{4^3}{9}$ $=13\frac{1}{9}$ or 13.1 unit^2			

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10 (a) (i)	$\frac{2x^2+3x}{x^2+3x+2}=2-\frac{3x+4}{x^2+3x+2}$ $\frac{3x+4}{x^2+3x+2}=\frac{A}{x+2}+\frac{B}{x+1}$ $3x+4=A(1+x)+B(x+2)$ $Let\ x=-1,\ B=1$ $Let\ x=-2,\ A=2$ $\frac{2x^2+3x}{x^2+3x+2}=2-\frac{2}{x+2}-\frac{1}{x+1}$			
10 (a) (ii)	$\int_1^3 \left(2-\frac{2}{x+2}-\frac{1}{x+1}\right)dx$ $=\left[2x-2\ln(x+2)-\ln(x+1)\right]_1^3$ $=\left[6-2\ln 5-\ln 4\right]-\left[2-2\ln 3-\ln 2\right]$ $=2.29$			
10 (b)	$\frac{\mathrm{d}y}{\mathrm{d}x}=\ln(x^2+3x+2)+\frac{x}{x^2+3x+2}\times(2x+3)$ $=\ln(x^2+3x+2)+\frac{2x^2+3x}{2+x-x^2}$			
10 (c)	$\int_1^3\left[\ln(x^2+3x+2)+\frac{2x^2+3x}{x^2+3x+2}\right]\mathrm{d}x$ $=\left[x\ln(x^2+3x+2)\right]_1^3$ $\int_1^3 \ln(x^2+3x+2)dx$ $=\left[x\ln(x^2+3x+2)\right]_1^3-\int_1^3\left[\frac{2x^2+3x}{x^2+3x+2}\right]\mathrm{d}x$ $=3\ln 20-\ln 6-2.2852$ $=4.91$			