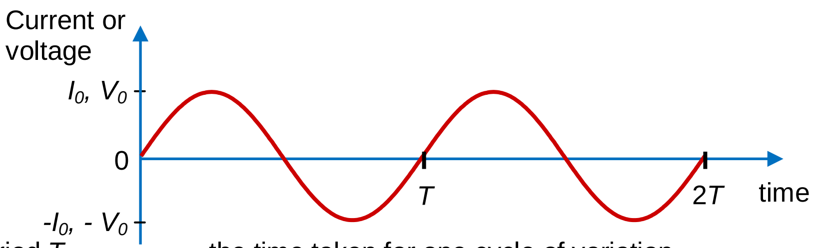
1 Basics of AC

An alternating current is a current whose *magnitude* and *direction* varies periodically. We focus on sinusoidal a.c. because any given non-sinusoidal current can be synthesised from a selection of sinusoidal alternating currents of appropriate weightings and frequencies.



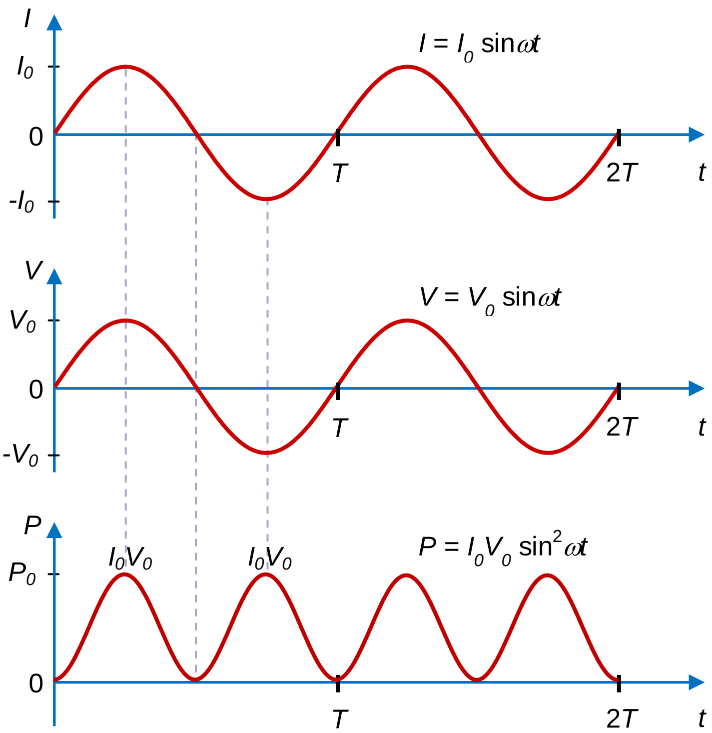
Period T	the time taken for one cycle of variation.
Frequency f	the number of cycles per unit time.
Peak value	the maximum magnitude of current I_0 or voltage V_0.
Peak to peak value	the magnitude of the interval between the positive peak and negative peak. $2I_0$ or $2V_0$.

An alternating current is a current whose *magnitude* and *direction* varies periodically. In contrast, a d.c. doesn't change direction.

2 Power & RMS value

The representative mean power

$P = IV$, hence if I and V both vary with time as shown (in phase for resistive load), the resultant power is given by the product of I and V moment by moment, resulting in a $\sin^2$ variation.



A typical a.c. has frequency 50 Hz, so the instantaneous power fluctuates at 100 Hz. For many applications such as lighting or heating, the fluctuation is not noticeable. What we observe is the mean effect, hence it is more useful to measure the mean power and regard it as *representative* or *effective* power of an a.c..

For a fluctuating power of an a.c., the mean power is more *useful* & *representative*.

The r.m.s. value for current and voltage

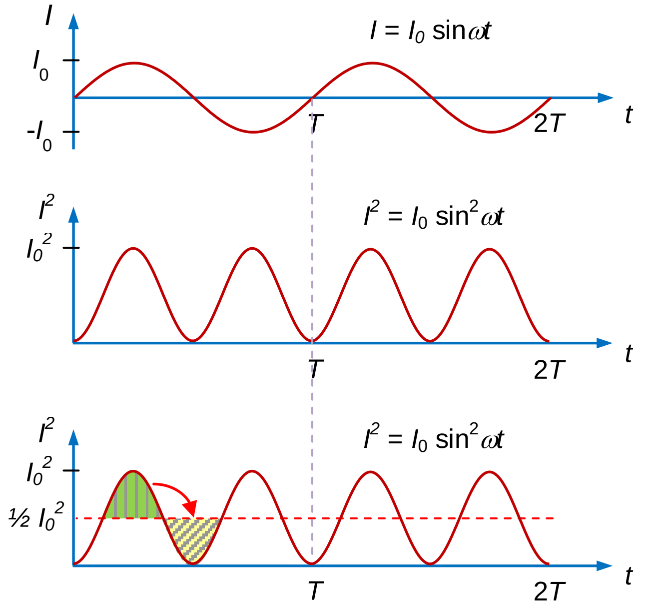
For sinusoidal a.c. current or voltage, is the mean value a useful representative value? No, because the mean is zero for any amplitude so it cannot tell us its effectiveness in producing useful work. What value then should we look at?

Consider the rate of heating in a resistor, $P = I^2R$ (note P and I^2 are time varying). The representative mean power $<P>$ is $= <I^2>R$, where $<I^2>$ is the *mean square current*. For the *same* resistance R, if a constant current I_c gives a power P_c which is $= <P>$, then we can take I_c to be a good representative or effective current of the a.c. since I_c essentially produces the same outcome. As $P_c = <P>$, $I_c^2 = <I^2>$ and so $I_c = \sqrt{<I^2>}$. This constant I_c that is representative of the effectiveness of the a.c. is formally known as *root mean square* or *r.m.s* current. A similar consideration using $P = V^2/R$ leads to the r.m.s. voltage $\sqrt{<V^2>}$. Hence, the r.m.s. value of an alternating current is defined as

> A constant current which produces the same effective rate of heating (mean power) as the alternating current for the same resistance.

How is the r.m.s. value calculated?
Looking at the origin of r.m.s. value, it is not difficult to see that it is calculated by firstly squaring the a.c. current or voltage, then finding the mean value over time and finally taking square root of that mean.

For sinusoidal a.c., a graphical way to find the r.m.s. value is as follows:



The second graph shows the result of squaring. The third graph shows the exploitation of the symmetry to get a mean value of ½ I_0^2. Finally, taking the square root of that yields r.m.s. value of $I_0/\sqrt{2}$.

Hence, for **sinusoidal** a.c. and resistive load:

Instantaneous Power	Peak Power	Mean Power
$P = IV$ $= (I_0\sin\omega t)(V_0\sin\omega t)$ $= I_0V_0\sin^2\omega t$ where P varies with t	$P_0 = I_0V_0$ $= I_0^2R$ $= V_0^2/R$	$<P> = I_{rms}V_{rms} = I_0V_0/2$ $= I_{rms}^2R = I_0^2R/2$ $= V_{rms}^2/R = V_0^2/2R$ $<P> = P_0/2$

R.M.S. values are *effective* or *representative* constant values of the a.c. current or voltage.

The r.m.s. value of an alternating current/voltage is defined as a constant current/voltage which produces the same effective rate of heating (mean power) as the alternating current/voltage for the same resistance.

Steps to calculate rms value for fluctuating a.c.:
1 square the a.c.
2 mean
3 square root

For **sinusoidal** a.c. only:

$I_{rms} = \dfrac{I_0}{\sqrt{2}}$

$V_{rms} = \dfrac{V_0}{\sqrt{2}}$

Power for a.c., resistive load:
- varies with time
- peak value is obtained from using peak current and voltage
- mean value is obtained using r.m.s. values

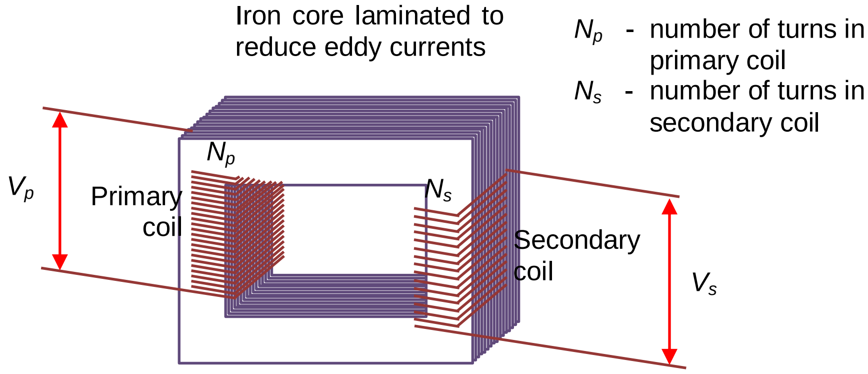
$<P> = P_0/2$

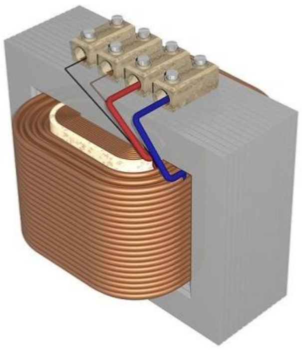
3 AC in Transformers

A transformer is a device that converts an alternating voltage to another alternating voltage of different magnitude using electromagnetic induction (Faraday's law). It consists of 2 coils, a primary coil and a secondary coil, wound on an iron core. The main function of the iron core is to drastically increase the flux linkage between the primary and secondary coils and thereby improve the efficiency of conversion. It is useful here to visualise *flux* as the total number of field lines given by *BA* where *B* is the number of lines per unit area.

A transformer converts an a.c. voltage to another a.c. voltage of different magnitude using electromagnetic induction.

The iron core *channels* and greatly *magnifies* the magnetic flux linking the coils.





A different kind of core where the primary and secondary coils are wound one over the other around the central column. This has less *flux leakage* and better efficiency than the above core.

For ideal transformer:

$$\boxed{\frac{N_p}{N_s} = \frac{V_p}{V_s}}$$

Also, Power in = Power out

$$\boxed{I_p V_p = I_s V_s}$$

Therefore,

$$\boxed{\frac{V_p}{V_s} = \frac{I_s}{I_p}}$$

The turns ratio N_p/N_s is = V_p/V_s. Having a turns ratio > 1 steps up the voltage and steps down the current by the same factor N_p/N_s.

For ideal transformer, $I_pV_p = I_sV_s$. If say 90% efficient, then $0.9\ I_pV_p = I_sV_s$

Real transformers have efficiency of about 94% to 99% and some reasons for the lost power are:

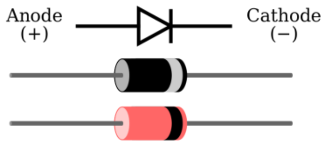
1 Heating (I^2R) in the windings.
2 Induced currents (eddy currents) in the core which also lead to heating.
3 Flux leakage (not all the flux from the primary coil are channelled to the secondary coil).

One important use of transformers is to step up the a.c. voltage before transmitting electrical power over long distance transmission lines and then step down to smaller voltages near to users. This is to exploit the fact that when stepping up the voltage, the current is correspondingly reduced, thereby reducing the loss due to heating in the transmission lines.

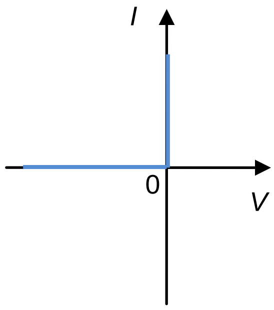
4 Rectification - AC to DC

Some devices (e.g. computer) need d.c. to work. The process of converting a.c. to d.c. is called rectification and the circuit that does it is called a rectifier. Diodes are typically used in rectifiers. They are devices which behave like an infinite resistance in forward bias and zero resistance in reverse bias, thus allow current to flow during forward bias but not reverse bias.

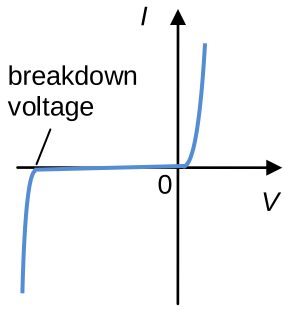
Picture to the right shows the symbol for a diode as well as how semiconductor diodes may look like.



The I-V characteristics of an ideal and semiconductor diode are as shown below:

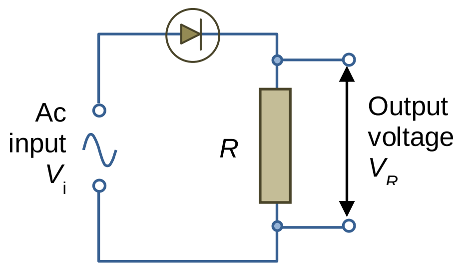


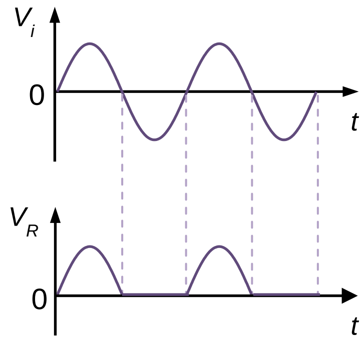
Ideal diode



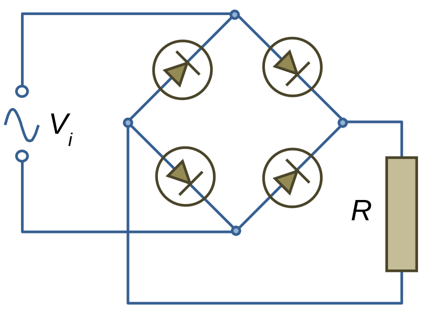
Real diode

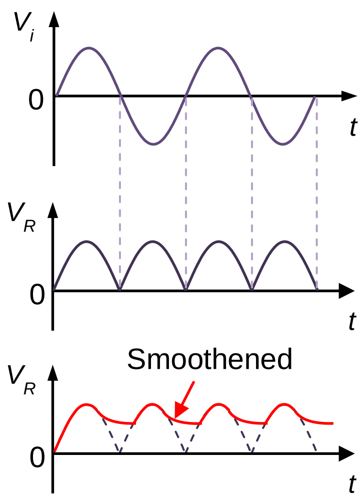
A half wave rectifier circuit, its input and corresponding output voltages are shown below.





Extra beyond syllabus material:
Below is a full-wave rectifier circuit which produces a full wave rectified d.c. with twice the power of the half rectified d.c.. By adding a capacitor in the circuit, the output voltage can be smoothened for a steadier d.c..





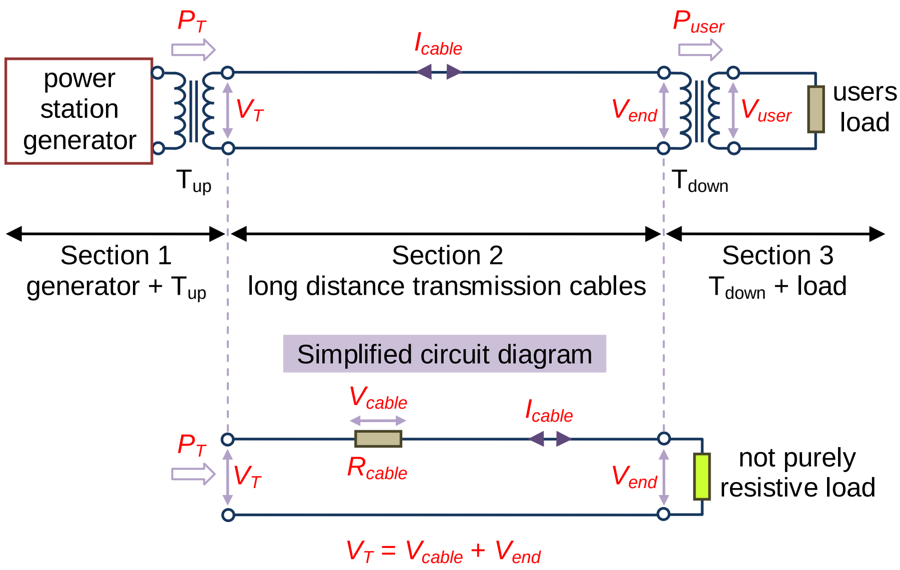
A rectifier circuit converts a.c. to d.c. with the help of a diode.

A diode allows current to flow only when forward biased. It acts as infinite resistance when reverse biased.

4 High Voltage Power Transmission

Transformers are essential in achieving efficient distribution of a.c. electrical power to users. We will see that by transmitting a given power at higher voltage through the long distance cables, power loss in the cables due to joule heating is reduced.

Diagram below shows a transformer T_{up} used to step up the voltage before transmission and another transformer T_{down} to step down the voltage for the users.



The simplified diagram focuses on section 2. It shows a power source that has power P_T to be transmitted. We assume the transformers are ideal. Note also that even if the electrical load at the users' side is purely resistive, the presence of T_{down} does not allow us to treat the overall load at the end of the cables as a simple resistor. Thus the formula $V = I^2R$ cannot be applied to the overall load.

For the *same* total power P_T to be transmitted, we want to compare the difference when using different transmitting voltages V_T. The transformer T_{down} will be adjusted to always maintain the same V_{user}.

$$V_T = V_{cable} + V_{end}$$

$$I_{cable}V_T = I_{cable}V_{cable} + I_{cable}V_{end}$$

$$P_T = P_{cable} + P_{load} \quad \text{------ (eq 4.1)}$$

P_{load} is a value that is not constant in reality because the demand for power by users fluctuates through the day. Power loss in the cables P_{cable} however is due to a known fixed cable resistance R_{cable}. From eq. 4.1, we know how much power P_{load} is available to users if we find $P_T - P_{cable}$.

$$P_{cable} = I^2_{cable}R_{cable} \quad \text{------ (eq. 4.2)}$$

and $\quad P_T = I_{cable}V_T$

so $\quad P_{cable} = (P_T/V_T)^2R_{cable} \quad$ ------ (eq. 4.3)

In Eq. 4.3, P_T and R_{cable} are fixed so $P_{cable} \propto 1/V_T^2$. This means that the higher V_T is stepped up, the lower the power loss in the cables and a larger portion of P_T will be available to end users! For a transformer, we learnt that when the voltage is stepped up, the secondary current drops. This means a smaller I_{cable} and smaller power loss $P_{cable} = I^2_{cable}R_{cable}$.

In eq. 4.2 if we wanted to use $P_{cable} = I_{cable}V_{cable}$ or V^2_{cable}/R_{cable} we will end up using $V_{cable} = I_{cable}R_{cable}$ and hence eq. 4.2 again.

For a transformer, when the voltage is stepped up, the secondary current drops. This means a smaller I_{cable} and smaller power loss $P_{cable} = I^2_{cable}R_{cable}$. Thus more power to end users.