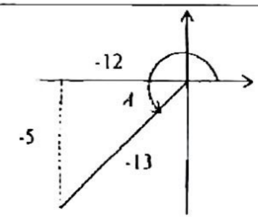
2024 NCHS Prelim AM Paper 2
Student Solutions

Qn	Solution
1(a)	$T_{r+1} = \binom{n}{r}(5x^2)^{n-r}\left(-\frac{1}{\sqrt{x}}\right)^r$
	$= \binom{n}{r}(5)^{n-r}(-1)^r x^{2(n-r)} x^{-\frac{1}{2}r}$
	$= \binom{n}{r}(5)^{n-r}(-1)^r x^{2n-\frac{5}{2}r}$
	Since there is a term independent of x,
	$x^{2n-\frac{5}{2}r} = x^0$
	$2n - \frac{5}{2}r = 0$
	$n = \frac{5}{4}r$
	Since both n and r are positive integers, Smallest $n = 5$, when $r = 4$.
1(b)	For the term $\left(5x^2 - \frac{1}{\sqrt{x}}\right)^n$,
	$T_{r+1} = \binom{n}{r}(5)^{n-r}(-1)^r x^{2n-\frac{5}{2}r}$
	For constant, $r = 4$,
	Constant $= \binom{5}{4}(5)^{5-4}(-1)^4$
	$= 25$
	For x^5,
	$x^{2n-\frac{5}{2}r} = x^5$
	$2n - \frac{5}{2}r = 5$
	$r = 2$
	Coefficient of $x^5 = \binom{5}{2}(5)^{5-2}(-1)^2$
	$= 1250$
	Term independent of x
	$= (1)(125) + \left(-\frac{1}{50}\right)(1250)$
	$= 0$
	Hence, there are no term independent of x.

2	Method 1
	$f'(x) = e^{5x} + 2\sin^2 x$
	$= e^{5x} + 2(\sin x)^2$
	$= e^{5x} + 1 - \cos 2x$
	$f''(x) = \frac{d}{dx}(f'(x))$
	$= \frac{d}{dx}(e^{5x} + 2(\sin x)^2)$
	$= 5e^{5x} + 4\sin x\cos x$
	$= 5e^{5x} + 2\sin 2x$
	$f(x) = \int f'(x)\,dx$
	$= \int e^{5x} + 1 - \cos 2x\,dx$
	$= \frac{e^{5x}}{5} + x - \frac{\sin 2x}{2} + c$
	Given
	$10f(x) + 3f'(x) - f''(x) + 7\sin 2x + 3\cos 2x = 10x + 43$
	$10\left(\frac{e^{5x}}{5} + x - \frac{\sin 2x}{2} + c\right) + 3(e^{5x} + 2\sin^2 x) - (5e^{5x} + 2\sin 2x) + 7\sin 2x + 3\cos 2x = 10x + 43$
	$2e^{5x} + 10x - 5\sin 2x + 10c + 3e^{5x} + 6\sin^2 x - 5e^{5x} - 2\sin 2x + 7\sin 2x + 3\cos 2x = 10x + 43$
	$10c + 6\sin^2 x + 3\cos 2x = 43$
	$10c + 3(1 - \cos 2x) + 3\cos 2x = 43$
	$c = 4$
	$\therefore f(x) = \frac{e^{5x}}{5} + x - \frac{\sin 2x}{2} + 4$
	Method 2
	$f'(x) = e^{5x} + 2\sin^2 x$
	$f''(x) = \frac{d}{dx}(f'(x))$
	$= \frac{d}{dx}(e^{5x} + 2(\sin x)^2)$
	$= 5e^{5x} + 4\sin x\cos x$
	$= 5e^{5x} + 2\sin 2x$
	Given
	$10f(x) + 3f'(x) - f''(x) + 7\sin 2x + 3\cos 2x = 10x + 43$
	$10f(x) + 3(e^{5x} + 2\sin^2 x) - (5e^{5x} + 2\sin 2x) + 7\sin 2x + 3\cos 2x = 10x + 43$
	$10f(x) + 3e^{5x} + 6\sin^2 x - 5e^{5x} - 2\sin 2x + 7\sin 2x + 3\cos 2x = 10x + 43$
	$10f(x) - 2e^{5x} + 5\sin 2x + 3(1 - \cos 2x) + \cos 2x = 10x + 43$
	$10f(x) - 2e^{5x} + 5\sin 2x + 3 - 3\cos 2x + \cos 2x = 10x + 43$
	$10f(x) = 2e^{5x} - 5\sin 2x + 10x + 40$
	$f(x) = \frac{e^{5x}}{5} + x - \frac{\sin 2x}{2} + 4$

3(a)	
	$\sin A = -\frac{5}{13}$
	$\cos A = -\frac{12}{13}$
	Method 1
	$\cos 3A = \cos(2A + A)$
	$= \cos 2A\cos A - \sin 2A\sin A$
	$= (1 - 2\sin^2 A)\cos A - (2\sin A\cos A)\sin A$
	$= \left(1 - 2\left(-\frac{5}{13}\right)^2\right)\left(-\frac{12}{13}\right) - \left(2\left(-\frac{5}{13}\right)\left(-\frac{12}{13}\right)\right)\left(-\frac{5}{13}\right)$
	$= -\frac{828}{2197}$
	Method 2
	$\cos 3A = \cos(2A + A)$
	$= \cos 2A\cos A - \sin 2A\sin A$
	$= (2\cos^2 A - 1)\cos A - (2\sin A\cos A)\sin A$
	$= \left(2\left(-\frac{12}{13}\right)^2 - 1\right)\left(-\frac{12}{13}\right) - \left(2\left(-\frac{5}{13}\right)\left(-\frac{12}{13}\right)\right)\left(-\frac{5}{13}\right)$
	$= -\frac{828}{2197}$
3(b)	$\tan A = \frac{2\tan\frac{A}{2}}{1 - \tan^2\frac{A}{2}}$
	$\frac{5}{12} = \frac{2\tan\frac{A}{2}}{1 - \tan^2\frac{A}{2}}$
	$5 - 5\tan^2\frac{A}{2} = 24\tan\frac{A}{2}$
	$0 = 5\tan^2\frac{A}{2} + 24\tan\frac{A}{2} - 5$
	$\tan\frac{A}{2} = \frac{-(24) \pm \sqrt{(24)^2 - 4(5)(-5)}}{2(5)}$
	$= -5$ or $\frac{1}{5}$
	Since $\pi < A < \frac{3\pi}{2}$,
	$\frac{\pi}{2} < \frac{A}{2} < \frac{3\pi}{4}$ (2nd Quadrant),
	$\therefore \tan\frac{A}{2} = -5$

4(a)

$$\text{Gradient} = \frac{0.88 - 2.69}{20 - 5} = -\frac{181}{1500}$$

$$Y - 2.69 = -\frac{181}{1500}(X - 5)$$

$$Y = -\frac{181}{1500}X + \frac{247}{75}$$

$$\ln y = -\frac{181}{1500}t + \frac{247}{75}$$

$$y = e^{-\frac{181}{1500}t + \frac{247}{75}}$$

$$\therefore k = -\frac{181}{1500},\ c = \frac{247}{75}$$

4(b)

Initial Temperature

When $t = 0$,

$$y = e^{\frac{247}{75}}$$

$$\text{Initial Temperature} = 25 + e^{\frac{247}{75}} = 51.9325°C$$

Temperature to drop to half

$$y = \frac{51.9325}{2} - 25 = 0.966245$$

When $y = 0.966245$,

$$\ln(0.966245) = -\frac{181}{1500}t + \frac{247}{75}$$

$$\therefore t = 27.577$$

$$= 27.6 \text{ min (to3sf)}$$

4(c)

$$e^{(k+0.2)t} = e^{5-c}$$

$$e^{kt}e^{0.2t} = \frac{e^5}{e^c}$$

$$e^{kt}e^{c} = \frac{e^5}{e^{0.2t}}$$

$$e^{kt+c} = e^{5-0.2t}$$

Tumbler Model

$$y = e^{5-0.2t}$$

$$\ln y = 5 - 0.2t$$

Step 1:

Draw a straight-line graph in (a), where the gradient of the straight line is -0.2 and the $\ln y$-intercept is 5.

Step 2:

The intersection between the 2 straight line graphs will be the timing where the temperatures are equivalent.

5(a)

Consider ΔP,

$$\text{hyp} = \sqrt{h^2 + h^2} = \sqrt{2}h$$

Consider ΔZ,

$$\text{Side} = \frac{80 - \sqrt{2}h}{2}$$

$$= 2\left(\frac{80 - \sqrt{2}h}{2}\right)^2$$

$$= \frac{(80 - \sqrt{2}h)^2}{2}$$

$$\text{base} = \text{hyp} = \frac{80 - \sqrt{2}h}{\sqrt{2}}$$

$\therefore$ Total exterior area, A

$= 4 \times \text{base} \times \text{height} + \text{base} \times \text{base}$

$$= 4h\left(\frac{80 - \sqrt{2}h}{\sqrt{2}}\right) + \left(\frac{80 - \sqrt{2}h}{\sqrt{2}}\right)^2$$

$$= \frac{320}{\sqrt{2}}h - 4h^2 + \frac{6400 - 160\sqrt{2}h + 2h^2}{2}$$

$$= \frac{320\sqrt{2}}{2}h - 4h^2 + 3200 - 80\sqrt{2}h + h^2$$

$$= 3200 + 80\sqrt{2}h - 3h^2$$

Method 2

Consider ΔZ,

$$\text{Side} = \frac{80 - \sqrt{2}h}{2}$$

$\therefore$ Total exterior area, A

$= 80 \times 80 - 4 \times \text{small triangle} - 4 \times \text{big triangle}$

$$= 6400 - 4 \times \left(\frac{1}{2}\right)(\sqrt{2}h)^2 - 4 \times \left(\frac{1}{2}\right)\left(\frac{80 - \sqrt{2}h}{2}\right)^2$$

$$= 6400 - 4h^2 - \frac{6400 - 160\sqrt{2}h + 2h^2}{2}$$

$$= 6400 - 4h^2 - 3200 + 80\sqrt{2}h - h^2$$

$$= 3200 + 80\sqrt{2}h - 3h^2$$

5(b)

When the total exterior area is a maximum, $\frac{dA}{dh} = 0$.

$$A = 3200 + 80\sqrt{2}h - 3h^2$$

$$\frac{dA}{dh} = 80\sqrt{2} - 6h$$

$$0 = 80\sqrt{2} - 6h$$

$$h = \frac{80\sqrt{2}}{6} = \frac{40\sqrt{2}}{3} \text{ cm}$$

$$\frac{d^2A}{dh^2} = -6\ (< 0)$$

$\therefore A$ is maximum.

5(c)

When $h = \frac{40\sqrt{2}}{3}$,

$$A = 3200 + 80\sqrt{2}\left(\frac{40\sqrt{2}}{3}\right) - 3\left(\frac{40\sqrt{2}}{3}\right)^2$$

$$= 4266\frac{2}{3} \text{ cm}^2$$

6

$$y = 3 - 5\cos 2x$$

Point N

When $y = 0$,

$$3 - 5\cos 2x = 0$$

$$\cos 2x = \frac{3}{5}$$

$$\alpha = \cos^{-1}\frac{3}{5} = 0.927295$$

$2x$ lies in 1^{st} (rej)/ 4^{th} quad.

$$2x = 5.35589$$

$$x = 2.67795$$

Point P

$$y = 3 - 5\cos 2x$$

$$\frac{dy}{dx} = -5(-\sin 2x)(2) = 10\sin 2x$$

When $\frac{dy}{dx} = -5$,

$$10\sin 2x = -5$$

$$\sin 2x = -\frac{1}{2}$$

$$\alpha = \sin^{-1}\frac{1}{2} = \frac{\pi}{6}$$

$2x$ lies in 3^{rd}/ 4^{th} (rej) quad.

$$2x = \frac{7\pi}{6}$$

$$x = \frac{7\pi}{12}$$

When $x = \frac{7\pi}{12}$,

$$y = 3 - 5\cos 2\left(\frac{7\pi}{12}\right) = 7.330127$$

$$\therefore P\left(\frac{7\pi}{12}, 7.330127\right)$$

Equation PQ, Point Q

$$y - 7.330127 = -5\left(x - \frac{7\pi}{12}\right)$$

$$y = -5x + 16.4931$$

When $y = 0$,

$$-5x + 16.4931 = 0$$

$$x = 3.29862$$

$$\therefore Q(3.29862, 0)$$

Area below the curve from x_P to x_N

$$\text{area} = \int_{\frac{7\pi}{12}}^{2.67795} 3 - 5\cos 2x \, dx$$

$$= \left[3x - \frac{5\sin 2x}{2}\right]_{\frac{7\pi}{12}}^{2.67795}$$

$$= 10.0338 - 6.74779$$

$$= 3.28601 \text{ units}^2$$

$\therefore$ shaded area

$=$ area of triangle $-$ area below curve

$$= \frac{1}{2}(7.330127)\left(3.29862 - \frac{7\pi}{12}\right) - 3.28601$$

$$= 5.37307 - 3.28601$$

$$= 2.0871$$

$$= 2.09 \text{ units}^2$$

(ii) To prove $(x + 2y)$ is a factor:

Method 1

Let $f(x) = 4x^3 + x^2y - 11xy^2 + 6y^3$.

$f(-2y) = 4(-2y)^3 + (-2y)^2y - 11(-2y)y^2 + 6y^3$

$= -32y^3 + 4y^3 + 22y^3 + 6y^3$

$= 0$

Since $f(-2y)$, by factor theorem, $(x + 2y)$ is a factor.

Method 2

$$x + 2y \overline{) 4x^3 + x^2y - 11xy^2 + 6y^3} \quad \text{quotient } 4x^2 - 7yx + 3y^2$$

... $3y^2x + 6y^3$

$-(3y^2x + 6y^3)$

0

Since the remainder is 0, $(x + 2y)$ is a factor.

Hence factorise:

Method 1 – Compare coefficients

$f(x) = (x + 2y)(4x^2 + bx ... y^2)$

Comparing coefficients of,

$x^{...}$ ... $-7y$

Method ... Division

As shown earlier

$\therefore f(x) = (x + 2y)(4x^2 - 7yx + 3y^2)$

$= (x + 2y)(x - y)(4x - 3y)$

7(b)

$4^{p+1} + 2^{p+1} = 44 - 48(2^{-p})$

$2^{2(p+1)} + 2^{p+1} = 44 - \frac{48}{2^p}$

$2^{2p}\,2^2 + 2^p\,2 = 44 - \frac{48}{2^p}$

Let $u = 2^p$,

$4u^2 + 2u = 44 - \frac{48}{u}$

$(\times u)$

$4u^3 + 2u^2 - 44u + 48 = 0$

From (a),

$48 = 6y^3$

$y^3 = 8$

$y = 2$

From (a), $x = u$

From (a),

$f(x) = (x + 2y)(x - y)(4x - 3y)$

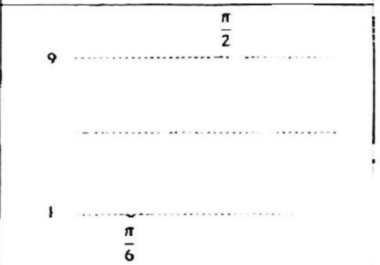
$f(x) = (x + 4)(x - 2)(4x - 6) = 0$

$x = -4$	$x = 2$	$x = \frac{3}{2}$
$2^p = -4$ (rej)	$2^p = 2$ $\therefore p = 1$	$2^p = \frac{3}{2}$

$\therefore p = \frac{\lg\frac{3}{2}}{\lg 2}$

$= 0.585$ (to3sf)

8(a)



$y = a\sin bx + c$

c	$\frac{1}{2}$ period	Amplitude
$= \frac{9 + 1}{2}$ $= 5$	$= \frac{\pi}{2} - \frac{\pi}{6}$ period $= \frac{2\pi}{3}$ $\frac{2\pi}{b} = \frac{2\pi}{3}$ $b = 3$	$= 9 - 5$ $= 4$ $a = -4$

8(b)(i)

$a = \frac{dv}{dt}$

$= 6t - 5$

When $t = 4$,

$a = 6(4) - 5$

$= 19\ m/s^2$

8(b)(ii)

For $0 \le t \le 5$

$v = 3t^2 - 5t + 7$

Displacement expression:

$s = \int 3t^2 - 5t + 7\ dt$

$= \frac{3t^3}{3} - \frac{5t^2}{2} + 7t + c$

When $t = 0, s = 0$,

$c = 0$

$\therefore s = \frac{3t^3}{3} - \frac{5t^2}{2} + 7t$

Turning point:

When $v = 0$,

$3t^2 - 5t + 7 = 0$

$b^2 - 4ac = (-5)^2 - 4(3)(7)$

$= -59\ (< 0)$

Since $b^2 - 4ac < 0$, there is no turning point.

When $t = 0$, $s = 0$ m

$t = 5$, $s = 97.5$ m

For $t > 5$,

$v = -4t + 77$

Displacement expression:

$s = \int -4t + 77\ dt$

$= -\frac{4t^2}{2} + 77t + d$

When $t = 5, s = 97.5$,

$97.5 = -2(5)^2 + 77(5) + d$

$d = -\frac{475}{2}$

$\therefore s = -2t^2 + 77t - \frac{475}{2}$

Turning point:

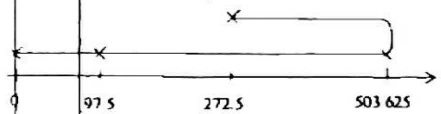
When $v = 0$,

$-4t + 77 = 0$

$t = \frac{77}{4} = 19\frac{1}{4}$

When $t = 19\frac{1}{4}$, $s = 503.625$ m

$t = 30$, $s = 272.5$ m



$\therefore$ total distance

$= 503.625 + (503.625 - 272.5)$

$= 734.75$ m

9(a)

Consider ΔBCD, using Pythagoras theorem,

$BD = \sqrt{8^2 + 15^2} = 17$

Consider ΔBXC,

$CX = 8\cos\theta$

$BX = 8\sin\theta$

Consider ΔCYD,

$CY = 15\sin\theta$

$YD = 15\cos\theta$

Total lengths

$= AB + 8 + 15 + AD + BD$

$= (CY - CX) + 8 + 15 + (BX + YD) + BD$

$= 15\sin\theta - 8\cos\theta + 8 + 15 + 8\sin\theta + 15\cos\theta + 17$

$= 40 + 23\sin\theta + 7\cos\theta$ (shown)

9(b)

$P = 40 + 23\sin\theta + 7\cos\theta$

$= 40 + R\sin(\theta + \alpha)$

$R = \sqrt{23^2 + 7^2}$

$= \sqrt{578}$

$= 24.04163 = 24.0$ (to 3sf)

	$\alpha = \tan^{-1}\frac{7}{23} = 16.9275°$ $= 16.9°$ (to 1 dp) $\therefore P = 40 + 24.0\sin(\theta + 16.9°)$
9(c)	When $P = 60$, $40 + 24.04163\sin(\theta + 16.9275°)$ $= 60$ $\sin(\theta + 16.9275°)$ $= 0.83189$ $\alpha = \sin^{-1} 0.83189$ $= 56.2934°$ $(\theta + 16.9275°)$ lies in 1^{st} / 2^{nd} (rej) quad. $\theta + 16.9275° = 56.2934°$ $\theta = 39.3659°$ $= 39.4°$ (to 1 dp)
9(d)	$\max(R\sin(\theta + \alpha))^2 = (\sqrt{578})^2 = 578$ $\therefore \min \frac{1}{40 + (R\sin(\theta + \alpha))^2}$ $= \frac{1}{40 + 578} = \frac{1}{618}$ $\theta + 16.9275° = 90°$ $\theta = 73.0725°$ $= 73.1°$ (to 1 dp)
10(a)	Since E is the midpoint of AD (given), F is the midpoint of DB (given), By midpoint theorem, $FE = \frac{1}{2}AB$
10(b) (i)	$\angle ABE$ $= \angle EDF$ (alternate segment theorem) From (a), by midpoint theorem, $EF \parallel AB$ $\angle DEF$ $= \angle BAE$ (corresponding angles, $EF \parallel AB$)
	$\therefore \Delta ABE$ is similar to ΔEDF (AA similari
10(b) (ii)	From (i), $\frac{AB}{ED}$ $= \frac{BE}{DF}$ (corresponding sides of similar Δ $AB \times DF = BE \times ED$ $AB \times \frac{1}{2}AB = BE \times ED$ $AB^2 = 2 \times BE \times ED$ Since $\angle DBA = 90°$ (tangent $\perp$ radius), Using Pythagoras' theorem, $AD^2 - BD^2 = 2 \times BE \times ED$
10(c)	<u>Method 1</u> Let $\angle GBE$ be α, $\angle GBA = \alpha$ (BG bisects $\angle EBA$) $\angle GEB$ $= \alpha$ (alternate segment theorem) $\angle DEB = 90°$ (rt $\angle$ in a semicircle) $\angle GHD$ $= 180°$ $- (90° + \alpha)$ ($\angle$ in opp segment) $= 90° - \alpha$ $= 90° - \angle GBE$ <u>Method 2</u> Let $\angle GBE$ be α, $\angle GBA = \alpha$ (BG bisects $\angle EBA$) $\angle DBA = 90°$ (tangent $\perp$ radius) $\angle GBD = 90° - \alpha$ $\angle GHD$ $= \angle GBD$ ($\angle$ in the same segment) $= 90° - \alpha$ $= 90° - \angle GBE$