2023 Dynamics Tutorial - Suggested Solutions for Discussion Questions

Part 1: Newton's Laws, Inertia, Force, Momentum, Impulse

D1

D

Constant velocity implies a = 0, F_{net} = 0. Hence, the first part of the graph should be zero.
Constant deceleration implies a = constant, F_{net} = constant, so the last part must be constant.

Comment:

Students tend to get confused or distracted by the term "deceleration" and leap to the conclusion that the graph must be sloping downwards which is actually how the velocity is changing and not the acceleration or net force.

D2

D

Constant force implies uniform acceleration.
Applying equations of motion: $v^2 = u^2 + 2as$. The railway carriage starts from rest, so u = 0
Thus $v = \sqrt{(2as)}$
$\Rightarrow p = m\sqrt{(2as)}$, i.e. $p \propto \sqrt{s}$
Note: Slight error in the answer option.
$\lim_{s\to 0} \frac{dp}{ds} = 0$ (i.e. limit of the gradient) should approach infinity as s approaches zero.

Problem-solving skills:

A repeat of skills taught in kinematics regarding the use of an equation relating the 2 axes is deployed here.

Common mistake:

Students may choose A, thinking they are tested on force = rate of change of momentum.

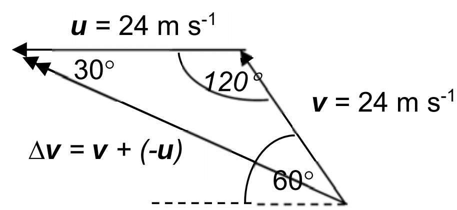
Extension:

Try to plot momentum-time graph.

D3

A

$\boldsymbol{u} + \Delta\boldsymbol{v} = \boldsymbol{v}$
$\Rightarrow \Delta\boldsymbol{v} = \boldsymbol{v} + (-\boldsymbol{u})$



Using the sine rule, $\frac{\Delta v}{\sin 120^\circ} = \frac{24}{\sin 30^\circ}$

$\Delta v = 48 \sin 120^\circ$

Alternatively, use the cosine rule.

By Newton's 2nd Law, or impulse-momentum relationship

$$\langle F \rangle = \frac{\Delta p}{\Delta t} = \frac{m\Delta v}{\Delta t} = \frac{(0.11)(48\sin 120^\circ)}{0.025} = 180 \text{ N}$$

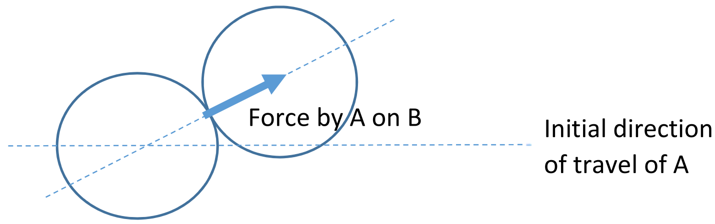
Comment:

Students who are not as strong in their maths or vectors can choose to solve the components separately. Taking rightwards and upwards as positive:

- *horizontally:* $\Delta v_x = -24 \cos 60^o - 24 = -36$ m s^{-1};
- *vertically:* $\Delta v_y = 24 \sin 60^o = 20.8$ m s^{-1};
- *total change in momentum is* $\Delta p = m\ \Delta v = m\sqrt{36^2 + 20.8^2} = 0.11 \times 41.6 = 4.57$ *N s;*
- *average force is* $\langle F \rangle = \Delta p/\Delta t = 4.57 / 0.025 = 183$ N = 180 N *(2 s.f.)*

D4

At the instant of collision, if the collision is not head-on (line joining center of A and B is not along the initial direction of travel of A) the contact force between them will have a component perpendicular to the initial direction of travel of A. This means that there will be a change in momentum of sphere B in the perpendicular direction and the initial velocity of sphere B will no longer be along the direction of travel of A.



D5

At terminal velocity,

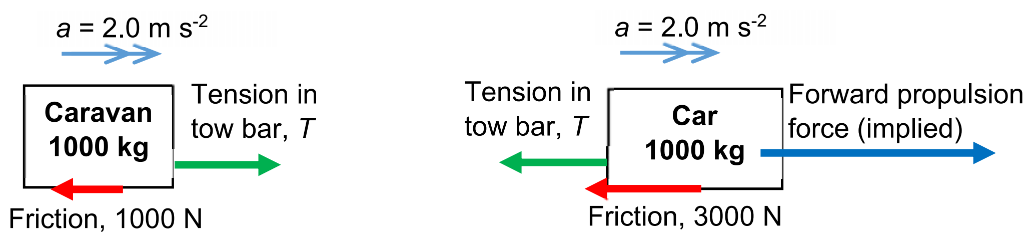
$$(F_{net} = 0)$$
$$mg = R$$
$$(3.0)(9.81) = 0.60v$$
$$v = 49.1 \text{ m s}^{-1}$$

At $v = 12 \text{ m s}^{-1}$,

$$(F_{net} = ma)$$
$$mg - R = ma$$
$$(3.0)(9.81) - 0.60(12) = 3.0a$$
$$a = 7.41 \text{ m s}^{-2}$$

D6

a)

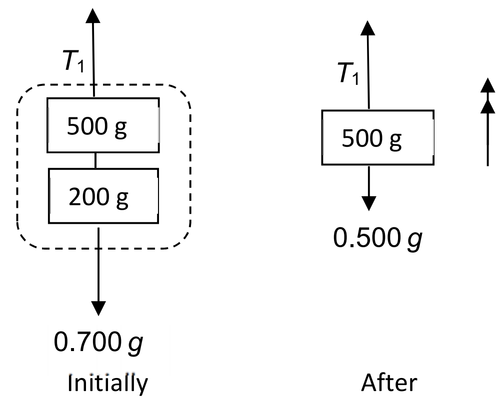
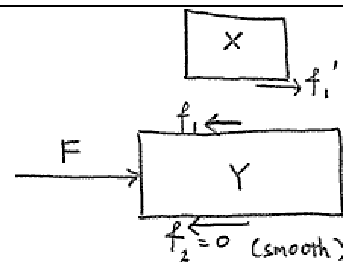
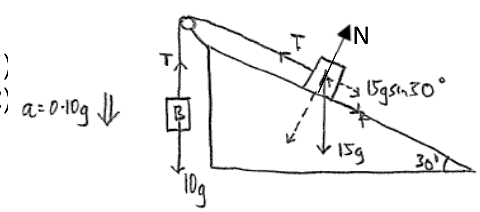


- As the light tow bar is under tension, it exerts forces of equal magnitude on the caravan as well as the car. The tension in the tow bar pulls the caravan forward and the car back.
- Applying Newton's second law on the caravan, taking rightwards as positive,
  $F_{net} = ma$
  $T - 1000 = (1000)\ (2.0)$
  $T = 3000$ N
- The force by the tow bar on the caravan is 3000 N to the right.
- Hence the force by the tow bar on the car is **3000 N to the left**.

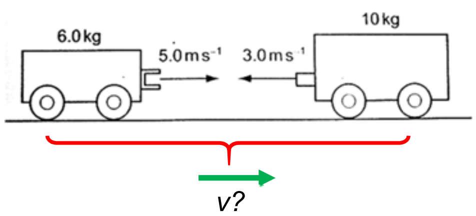
b) When the vehicles are moving at constant speed, net force on each of them must be zero.

Hence, referring to caravan again, the force by the tow bar on the caravan is equal in magnitude to the friction on the caravan. Hence, the force by the tow bar on the caravan is 1000 N to the right.

By Newton's third law, the force by the tow bar on the car is **1000 N to the left**.

D7	**B** Initially, the system is in equilibrium. $T_1 = 0.700\,g$ Immediately after the thread is cut, Applying Newton's 2nd Law, $\sum F = ma$ $T_1 - 0.500\,g = 0.500\,a$ $a = 2/5\,g$ $= 0.4\,g$ 
	<u>Note:</u> *Questions like these come up quite regularly in exams. An object (in this case the block of 500 g) is initially at rest, when suddenly one of the forces "disappears". The new net force on the object <u>immediately after</u> the force disappears is given by* $F_{net,new} = -F_{disappeared}$*. The force that disappears is the tension in the lower string, which is equal to the weight of the block of 200 g.*
D8	Applying Newton's second law For X & Y: $F = (m + 3m)\,a = 4m\,a$ $a = F/4m$ For X alone: $f_1' = m\,a$ $= m\,(F/4m)$ $= \mathbf{F/4}$ 
	<u>Note:</u> *The "m" that occurs in Newton's second law is in general the mass of the object that is being accelerated. However, in the first part of this working, the mass as indicated in Newton's second law is actually equal to 4m.*
D9	**A** Consider the blocks separately. B: $10\,g - T = 10\,(0.10\,g)$ -------------- (1) A: $T - 15\,g \sin 30° - f = 15\,(0.10\,g)$ -------------- (2) $10\,g - 15\,g \sin 30° - f = (10 + 15)\,0.10\,g$ $f = 0$ N 
	<u>Note:</u> *This is a difficult question for most students. The pulley confuses them. Most students would choose downwards as positive and then get stuck at forming the equation for the block on the slope.*

Part 2: Conservation of Linear Momentum/Collisions

D10	**C**  $\sum p_i$ = 6.0 (5.0) + 10(-3) = 0 N s $\sum p_f$ = 0 N s For inelastic collision, (6.0 + 10) v = 0 Final velocity = 0 m s^{-1} Consider the 6.0 kg trolley, $\langle F \rangle = \frac{\Delta p}{\Delta t} = \frac{\lvert 0 - 6.0(5.0) \rvert}{0.20} = 150\text{ N}$
D11	Let initial velocity of bullet be u, velocity of bullet after emerging from 1.2 kg block be v. Before hitting 1.2 kg block, $\sum p_i$ = 0.0035 u ------------------- (1) After bullet emerging from 1.2 kg block, $\sum p_f$ = 1.2(0.63) + 0.0035 v ------------------ (2) After bullet stuck in 1.8 kg block, $\sum p_f'$ = 1.2(0.63) + (1.8 + 0.0035)(1.4) = 3.2809 ------------- (3) (a) By the principle of conservation of linear momentum, (2) = (3): 0.0035 v = 2.5249 v = 721 m s^{-1} (b) By the principle of conservation of linear momentum, (1) =(3): 0.0035 u = 3.2809 u = 937 m s^{-1}
	<u>Note</u>: *You are strongly encouraged to draw "before" and "after" diagrams to apply COLM.*

D12

a) (i) refer to lecture notes

(ii) refer to lecture notes

b) Elastic: total kinetic energy is conserved
Head-on: The motions of the molecules after the collision will be along the same straight line of motions before the collision.

c) For elastic collision,

relative speed of separation = Relative speed of approach = $1.88 \times 10^3 + 405 = 2285$ m s^{-1}.

After the collision,

(ii) Applying the principle of conservation of linear momentum,
taking rightwards as positive,
$2.00u\,(1.88 \times 10^3) + 32.0u\,(-405) = 2.00u\,v_H + 32.0\,u\,v_O$
$v_H + 16\,v_O = -4600$ --------------------------(1)

(iii) Relative speed of approach = relative speed of separation,
taking rightwards as positive
$u_H - u_O = v_O - v_H$
$v_O - v_H = 2285$ -----------------------(2)

(1) + (2): $17\,v_O = -2315$
$v_O = -136$ m s^{-1}
Sub into (2): $v_H = -2420$ m s^{-1}

Hence, the velocity of the oxygen molecule is 136 m s^{-1} to the left and the velocity of the hydrogen molecule is 2420 m s^{-1} to the left.

<u>Note:</u>
It is simpler to use relative speeds to solve the simultaneous equations, rather than the conservation of total kinetic energy.

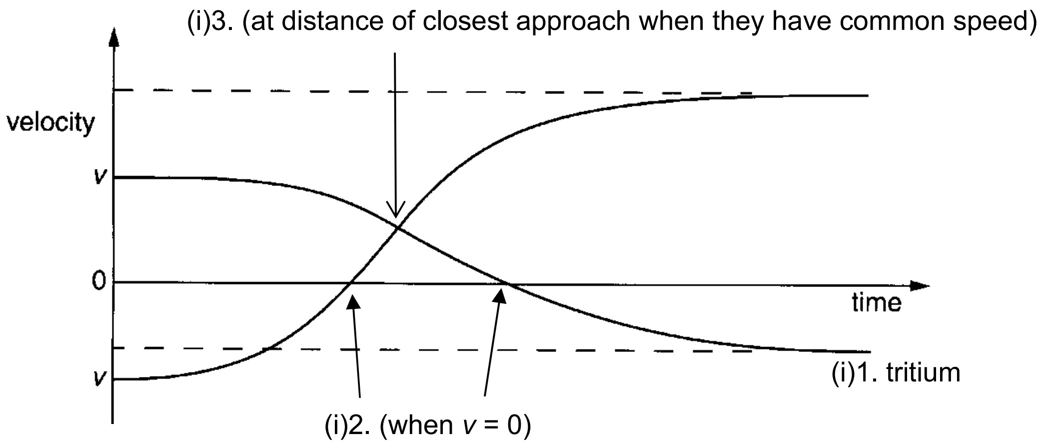
D13	**a)**

In this case, total momentum before collision = $3mv - 2mv = mv$ to the right.
By the principle of conservation of linear momentum (PCLM), the total momentum in a closed system is conserved.
Thus at any instant, the total momentum of both the nuclei together is mv.
If both the nuclei were to stop at the same instant, total momentum would be zero and the principle would be violated.

b) By Principle of conservation of linear momentum,

$$(3m + 2m)\,v' = mv$$
$$v' = 0.20\,v$$

c)



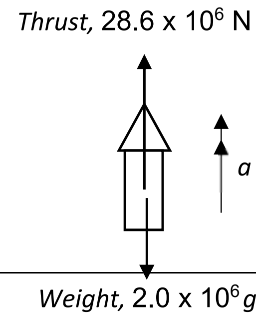
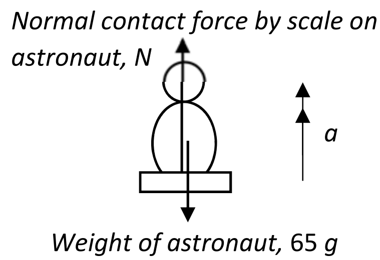
(ii)

By the principle of conservation of linear momentum, taking rightwards as positive,
$m_T\, u_T + m_D\, u_D = m_T\, v_T + m_D\, v_D$
$(3m)\, v + (2m)(-v) = (3m)v_T + (2m)v_D$
$v = 3v_T + 2v_D$ ----- (1)

For an elastic collision, relative speed of separation equals relative speed of approach,
$u_T - u_D = v_D - v_T$
$v - (-v) = v_D - v_T$
$2v = v_D - v_T$ ----- (2)

Solving (1) and (2), we find $v_T = -0.60v$ and $v_D = 1.40v$.

Hence, the final speed of the tritium atom is **0.60*v*** (directed to the left), while the final speed of the deuterium atom is **1.40*v*** (directed to the right).

D14	**(i)**	For the rocket, taking upwards as positive, $(28.6 \times 10^6) - (2.0 \times 10^6)(9.81) = (2 \times 10^6)a$ a = **4.49 m s^{-2} upwards** 
	(ii)	(ii) By Newton's 2nd Law, $F = m\,a$ $N - 65\,g = 65\,a$ $N = 65\,(a + g)$  Note that the above working only shows normal contact force on man by scale. Thus, it is necessary to apply Newton's third law. By Newton's 3rd Law, the force exerted on scale by man (apparent weight) is 65 (a +g) downwards. The force exerted downwards is (65) (4.49 + 9.81) = 929.5 N downwards. The scale will give a reading in kilograms; it will read (929.5) / (9.81) = 94.8 kg.
	(iii)	For the rocket, the thrust can be shown to be given by $F = \frac{dm}{dt} u_{rel}$ Hence, $(2.0 \times 10^4)\, v = 2.86 \times 10^7$ $v = 1.43 \times 10^3$ ms^{-1}

D15	a) The gases and the car are an isolated system, hence according to the principle of conservation of linear momentum, the total momentum of the system must remain constant all the time. As the rocket expels the exhaust gas backwards, there is an increase in momentum of these gases along their direction of travel. Therefore, there should be an increase in momentum of the car (i.e. car, rocket engine and remaining fuel) in the opposite direction (i.e. forward direction of the car) as the total momentum of the system is conserved. Since there is a change in momentum of the car, from Newton's second law, there must be a forward force acting on the car. b)(i)1. Gradient of the tangent to the velocity-time graph at t = 2.0 s represents the acceleration at that time. Acceleration = (15.2 – 2.8) / (4.60 – 0.10) = 2.76 m s^{-2} b)(i)2. $\Sigma F = m\,a$ $4.6 - F_D = (0.440)(2.76) \Rightarrow F_D = 3.4$ N b)(ii) The gradient of the velocity-time graph decreases with increase of speed. Hence net force decreases with increase of speed. Net force = Driving force – Resistive force With the driving force kept at a constant, the resistive force must increase with increase of speed. c) Estimate the value of the gradient of the velocity-time graph at t = 0 s. From the v-t graph, acceleration at t = 0, a_o = (11.2 – 0.0) / (1.8 – 0.0) = 6.2 m s^{-2}. At t = 0 s, there is no resistive force. $\Sigma F = m\,a$ $4.6 = m_o\,a_o \Rightarrow m_o = 4.6 / 6.2 = 0.74$ kg Upward thrust = 4.6 N Initial weight of car = $m_o g$ = (0.74)(9.81) = 7.28 N Thus, initial weight of the car > initial upward thrust and the rocket cannot be launched. Note: *It is hard to estimate the gradient at t = 0 accurately.* *The argument as to whether the rocket could be launched or not should be based on a comparison between weight of rocket and upward thrust.*

Learning Outcomes	Discussion Question
(a) state and apply each of Newton's laws of motion.	D3, D14
(b) show an understanding that mass is the property of a body which resists change in motion (inertia).	
(c) describe and use the concept of weight as the force experienced by a mass in a gravitational field.	D6, D9, PTT10
(d) define and use linear momentum as the product of mass and velocity.	D2, D10, PTT4
(e) define and use impulse as the product of force and time of impact.	PTT2, PTT9
(f) relate resultant force to the rate of change of momentum.	D1, D3, D10
(g) recall and solve problems using the relationship F = ma, appreciating that resultant force and acceleration are always in the same direction.	D1, D4, D5, D6, D7, D8, D9, PTT1, PTT3, PTT7, PTT8, PTT10
(h) state the principle of conservation of momentum.	
(i) apply the principle of conservation of momentum to solve simple problems including inelastic and (perfectly) elastic interactions between two bodies in one dimension. (Knowledge of the concept of coefficient of restitution is not required.)	D4,D15, D10, D11, D12, D13, PTT6
(j) show an understanding that, for a (perfectly) elastic collision between two bodies, the relative speed of approach is equal to the relative speed of separation.	D12, D13, PTT5
(k) show an understanding that, whilst the momentum of a closed system is always conserved in interactions between bodies, some change in kinetic energy usually takes place.	