BENDEMEER SECONDARY SCHOOL
2024 PRELIMINARY EXAMINATION
SECONDARY 4 EXPRESS / 5 NORMAL (ACADEMIC)

CANDIDATE NAME	MARKING SCHEME
CLASS	
INDEX NUMBER	

MATHEMATICS
Paper 1

4052/01
20 Aug 2024
2 hours 15 minutes

Candidates answer on the Question Paper.
No additional materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid/tape.
DO **NOT** WRITE ON ANY BARCODES.

Answer **all** questions.
The number of marks is given in brackets [ ] at the end of each question or part question.

If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
The total of the marks for this paper is 90.

The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142.

FOR EXAMINER'S USE
90

MATHEMATICAL FORMULAE

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle ABC} = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta\text{, where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta\text{, where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard Deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions.

1 Factorise each of the following completely.

(a) $4a^2 - b^2$

$= (2a + b)(2a - b)$ [B1]

Answer .... [1]

(b) $5x - 1 + 10x^2y - 2x$

$= (5x - 1) + 2xy(5x - 1)$ [M1]

$= (5x - 1)(1 + 2xy)$ [A1]

Answer .... [2]

2 **(a)** Simplify $(2x^3y)^3$.

$= 8x^9y^3$ [B1]

Answer .... [1]

(b) Use the laws of indices to solve the following equation for *x*. Show your working clearly.

$2^2 \times 5^3 + \frac{1}{125^x} = 5^4$.

$5^{-3x} = 5^4 - 4 \times 5^3$

$5^{-3x} = 5^3$ [M1]

$-3x = 3$

$x = -1$ [A1]

Answer *x* =.................................... [2]

3 **(a)** Express 1176 as a product of its prime factors.

$= 2^3 \times 3 \times 7^2$ [B1]

Answer .... [1]

(b) The number 1176*k* is a perfect cube.

Find the smallest positive integer value of *k*.

$1176k = 2^3 \times 3 \times 7^2 \times k = 2^3 \times 3^3 \times 7^3$

$\therefore k = 63$ [B1]

Answer *k* = ... [1]

(c) The highest common factor of two distinct integers, *n* and 1176, is 28.

Given that $500 < n < 1000$, find the smallest possible value of *n*.

$HCF = 28 = 2^2 \times 7$

$\therefore n = 2^2 \times 7 \times 19 = 532$ [B1]

Answer *n* = ... [1]

4 The volume of a right pyramid is 4.8×10^{-8} m^3 and its length of the square base is 1.5×10^{-3}m. Giving your answer in standard form, find its height.

$\frac{1}{3} \times (1.5 \times 10^{-3})^2 \times h = 4.8 \times 10^{-8}$ [M1]

$\therefore h = \frac{3 \times 4.8 \times 10^{-8}}{(1.5 \times 10^{-3})^2} = 6.4 \times 10^{-2}$ [A1]

Answer .... m [2]

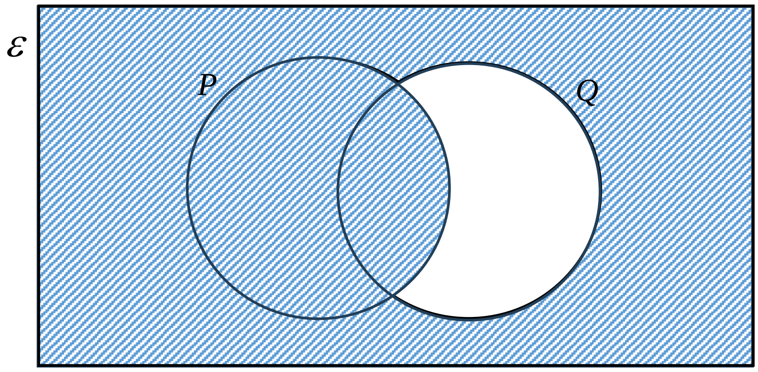
5 Benjamin invested $8000 at a rate of 3.3% per annum compounded monthly.
What is the value of his investment at the end of 6 months?

$A = 8000\left(1 + \frac{3.3}{12 \times 100}\right)^6$ [M1]

= $8132.91 [A1]

Answer $ .... [2]

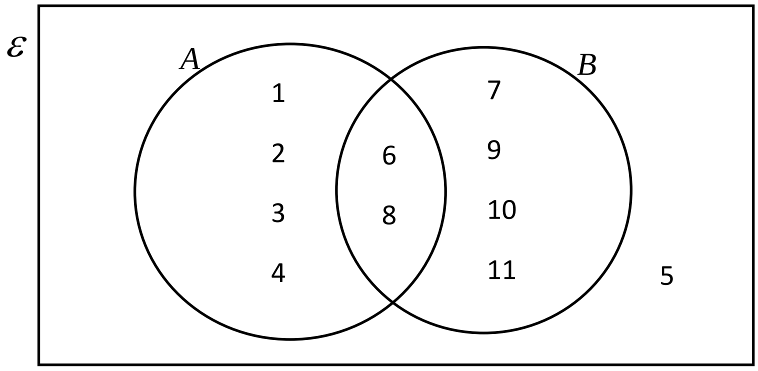
6 **(a)** In the Venn diagram below, shade the region $P \cup Q'$.



[1]

(b) A universal set ε and its subset A and B are given by
ε = {x: x is an integer and $0 < x < 12$},
A = {x: x is a factor of 24},
B = {x: $2x - 7 \geq 5$}.

(i) Write all the elements of ε in the Venn diagram below.



All sections correct [B2]

2 to 3 sections correct [B1]

[2]

(ii) Another number is included in the universal set ε.
This number is in the region $A \cap B$.
Write down a possible value of this number.

12 *or* 24 [B1]

Answer ………………………………………… [1]

7 The expression $2x^2 + 8x + 9$ is equivalent to $2(x + a)^2 + b$.

(a) Find the value of a and the value of b.

$= 2(x^2 + 4x) + 9$

$= 2[(x + 2)^2 - 2^2] + 9$

$= 2(x + 2)^2 - 8 + 9$

$= 2(x + 2)^2 + 1$

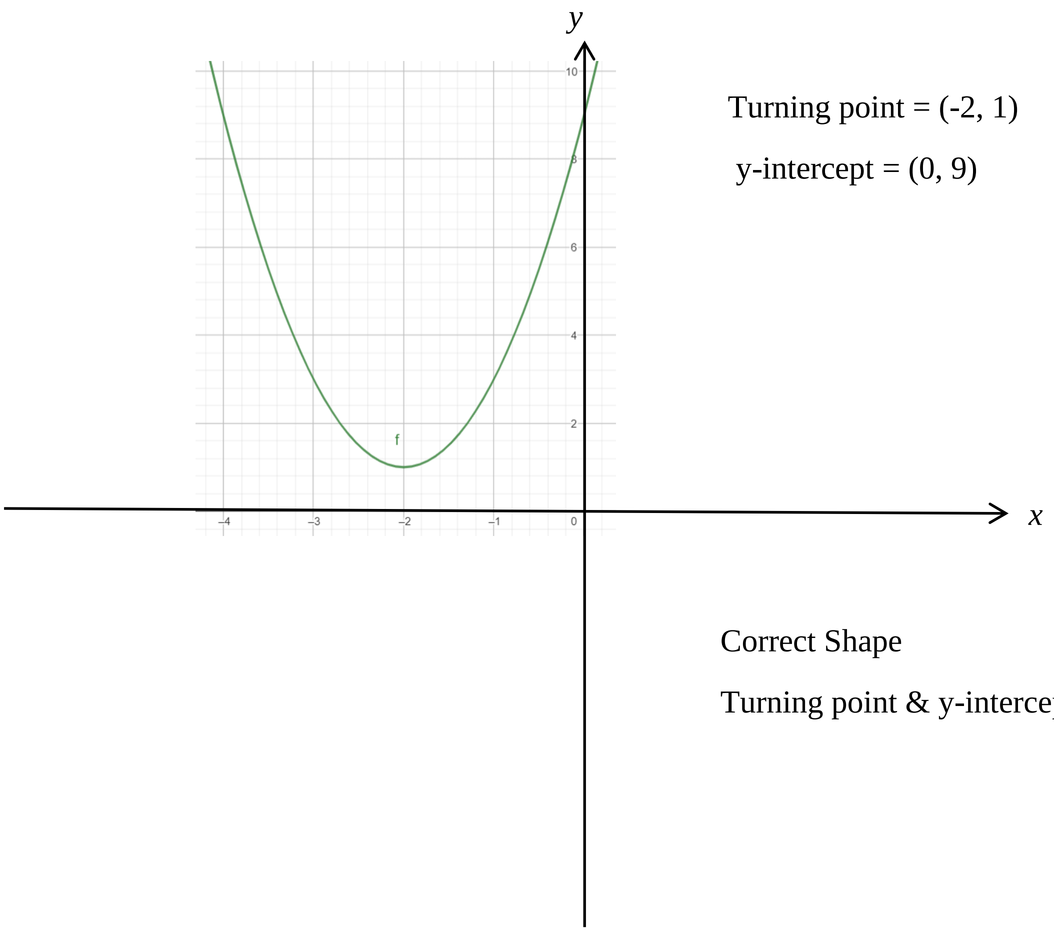
Hence, $a = 2$ [B1]

And $b = 1$ [B1]

Answer a = ... [1]

b = ... [1]

(b) Sketch the curve $y = 2x^2 + 8x + 9$ on the given axes below, clearly showing the y-intercept and the turning point.



Turning point = (-2, 1)

y-intercept = (0, 9)

Correct Shape [B1]

Turning point & y-intercept [B1]

8 Given that the formula of y is inversely proportional to the square of $(x + 2)$ and the value of $y = \frac{1}{3}$ when $x = 7$.

(a) Express y in terms of x.

$$y = \frac{k}{(x+2)^2}$$

$$\frac{1}{3} = \frac{k}{(7+2)^2}$$

$k = 27$ [M1]

$\therefore\ y = \frac{27}{(x+2)^2}$ [A1]

Answer y = ... [2]

(b) Hence, or otherwise, make x the subject of the formula.

$(x+2)^2 = \frac{27}{y}$ [M1]

$$x + 2 = \pm\sqrt{\frac{27}{y}}$$

$\therefore\ x = -2 \pm \sqrt{\frac{27}{y}}$ [A1]

Answer x = ... [2]

9 Explain why $(2n+3)^2-(4n+3)(n-6)$ is a multiple of 3 for all integer values of n.

Answer

$= 4n^2+12n+9-(4n^2+3n-24n-18)$ [M1]

$= 33n+27$

$= 3(11n+9)$ [A1]

Hence, it is a multiple of 3 for all integer values of *n*.

....

.... [2]

10 Simplify $\frac{2x+3}{x^2+3x+2}-\frac{5}{x+1}$ as a single fraction.

$= \frac{2x+3}{(x+1)(x+2)}-\frac{5}{x+1}$ [M1]

$= \frac{2x+3-5(x+2)}{(x+1)(x+2)}$ [M1]

$= \frac{2x+3-5x-10}{(x+1)(x+2)}$

$= \frac{-3x-7}{(x+1)(x+2)}$ [A1]

Answer .... [3]

.

11 **(a)** Solve the inequality $x - 7 \leq \frac{3x-5}{2} < 8$.

Represent your solution on the number line below.

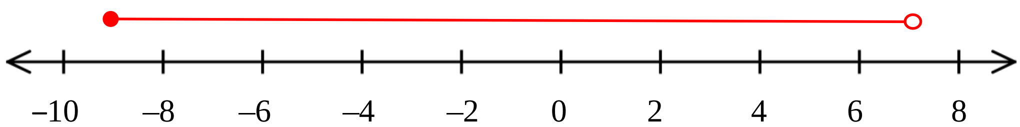
$x - 7 \leq \frac{3x-5}{2}$ $\qquad$ $\frac{3x-5}{2} < 8$

$2x - 14 \leq 3x - 5$ $\qquad$ $3x - 5 < 16$

$-9 \leq x$ [M1] $\qquad$ $x < 7$ [M1]

Hence, $-9 \leq x < 7$

Answer [A1]



[3]

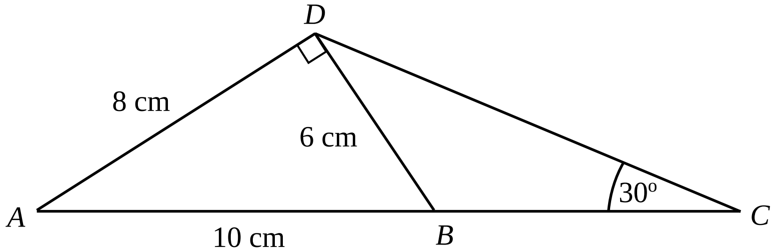
(b) Hence, write down the largest prime number that satisfy the inequality

$x - 7 \leq \frac{3x-5}{2} < 8.$

Largest prime number x = 5 [A1]

Answer x = ... [1]

12 In the diagram below, ADB is a right-angled triangle with angle $ADB = 90°$ and ACD is a triangle with AD = 8 cm and angle $DCA = 30°$.
B is a point on AC such that AB = 10 cm and DB = 6 cm.



(a) Write down, as a fraction in its simplest form, the value of $sin\angle DBC$.

$sin\angle DBC = sin\angle DBA = \frac{8}{10} = \frac{4}{5}$ [B1]

Answer .... [1]

(b) **Hence**, calculate the length of DC.

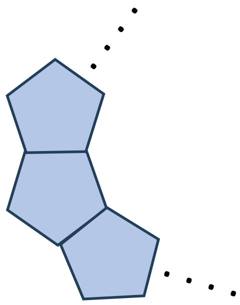
$\frac{DC}{sin\angle DBC} = \frac{6}{\sin 30°}$ [M1]

$\therefore DC = \frac{6}{sin30°} \times \frac{4}{5} = 9.6\ cm$ [A1]

Answer .... cm [2]

13 Dorothy wants to make a bracelet using gemstones. Every gemstone is a regular pentagon of the same size.

How many gemstones does she need to make a bracelet assuming that there is no gap between each gemstone?



Interior angle of each pentagon = $\frac{(5-2)}{5} \times 180° = 108°$ [M1]

So interior angle of polygon inside = $360° - 108° - 108° = 144°$ [M1]

$\therefore \frac{n-2}{n} \times 180° = 144°$ or $\frac{360°}{n} = 36°$

$\therefore n = 10$ [A1]

Answer .... [3]

14 Each term in the following sequence is found by multiplying the same constant to the previous term.

p, 12, q, 48, r, …

(a) Write down two possible values of q.

Let the multiplying constant be x.

$12x^2 = 48$ [M1]

$x = \pm 2$

Hence $q = 12 \times 2 = 24$ or $q = 12 \times (-2) = -24$ [A1]

Answer q = ... or .... [2]

(b) Write down the value of $\frac{p}{r}$.

$\frac{p}{r} = \frac{6}{96} = \frac{1}{16}$ [B1]

Answer …………............................... [1]

(c) Write down the nth term of the sequence.

$T_n = 6 \times 2^{n-1}$ or 3×2^n [B1]

Answer …………............................... [1]

(d) Explain why 400 is not a term in this sequence.

(1) …, 48, 96, 192, 384, 768, …

....

OR (2) $\frac{400}{6} = 66.67$ is not a multiple of 2

....

Hence, 400 is not a term in this sequence. [B1]

.... [1]

15 A bag initially contains 6 blue balls, x green balls and y red balls. The probability of drawing a green ball is $\frac{1}{4}$. If 4 blue balls are added into the bag and 1 red ball is removed from the bag, the possibility of drawing a green ball from the bag is $\frac{2}{9}$.

Find the value of x.

$$\frac{x}{6+x+y} = \frac{1}{4}$$

$$4x = 6 + x + y$$

$3x - 6 = y$ …(1) [M1]

$$\frac{x}{10+x+y-1} = \frac{2}{9}$$

$$9x = 18 + 2x + 2y$$

$7x - 18 = 2y$ …(2) [M1]

Solving (1) & (2) simultaneously,

$7x - 18 = 2(3x - 6)$ [M1]

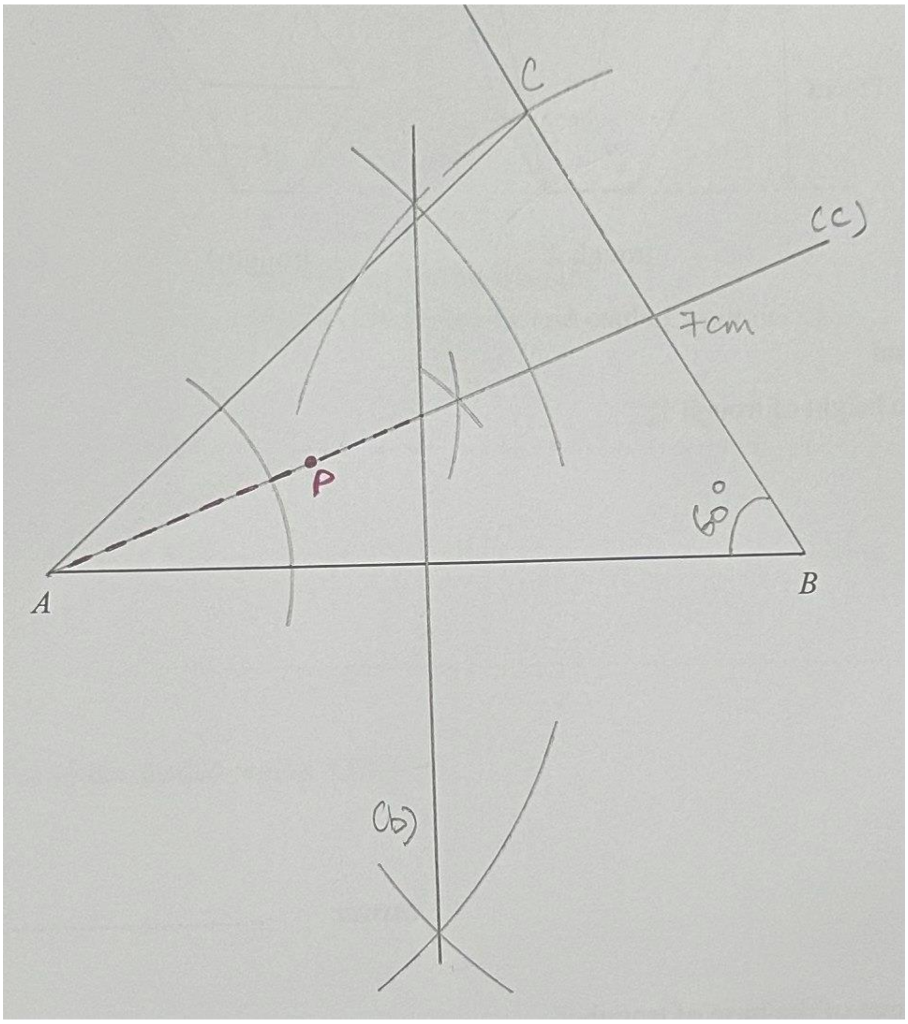
$\therefore x = 6$ [A1]

Answer x = ... [4]

16 **(a)** Construct triangle *ABC* where *AB* = 10 cm, *BC* = 7 cm and angle *ABC* = 60°.
AB has already been drawn. [1]

Answer ***(a)*, *(b)*, *(c)*** *and* ***(d)***

[B1× 4]

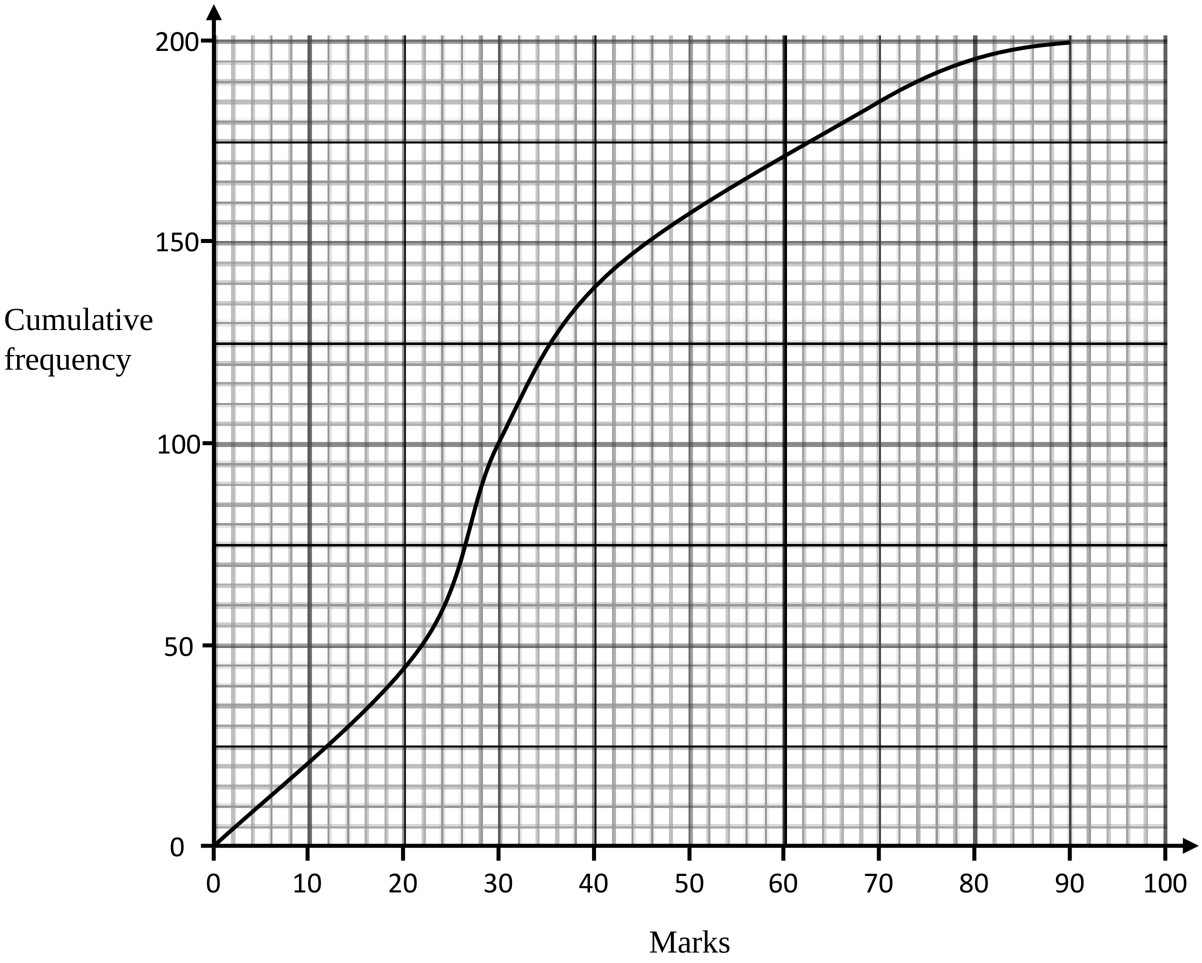


(b) Construct the perpendicular bisector of *AB*. [1]

(c) Construct the bisector of angle *BAC*. [1]

(d) Mark clearly a possible point which is inside the triangle, equidistant from the lines *AB* and *AC*, and is nearer to point *A* than point *B*.
Label this point P. [1]

17 The cumulative frequency graph shows the distribution of the marks of 200 students for a Biology test



(a) Use the curve to estimate

(i) the median mark for the test,

Median = 30 [B1]

Answer .... [1]

(ii) the interquartile range for the test,

IQR = 46 – 22 = 24 [B1]

Answer .... [1]

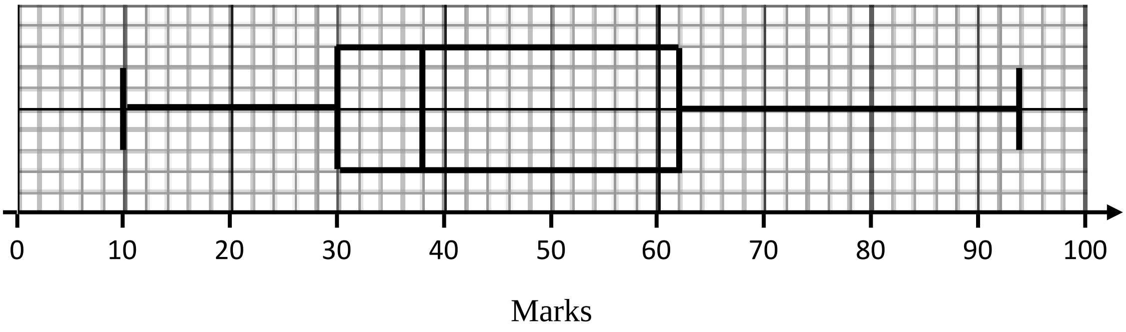
(iii) the percentage of the students scored more than 70 marks for the test.

$\frac{200-185}{200} \times 100\%$ [M1]

= 7.5% [A1]

Answer .... % [2]

(b) The box-and-whisker plot represents the distribution of marks of the same 200 students for a Chemistry test.



Make two comparisons between the marks for the two tests.

Use figures to support your answers.

1. Students did better in Chemistry test on average since the median of Chemistry (38) is greater than the median of Biology (30). [A1]

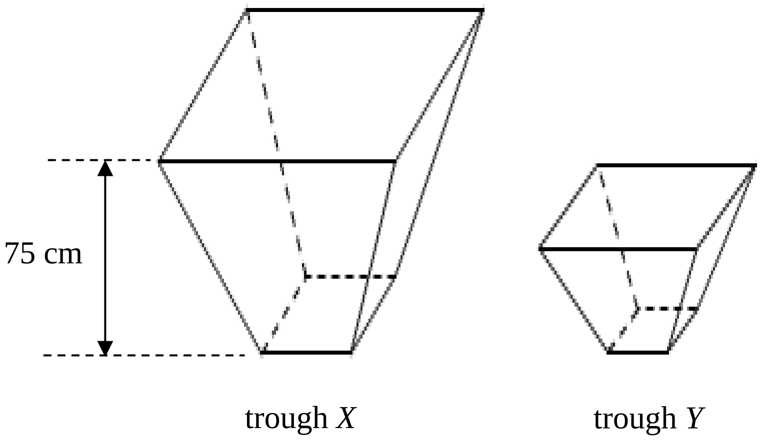
.... [1]

2. Students have more consistent marks in Biology test since the IQR of Biology (24) is smaller than the IQR of Chemistry (32). [A1]

.... [1]

18 The volumes of two geometrically similar troughs X and Y are 4050 cm^3 and 1200 cm^3 respectively.

Trough X has a height of 75 cm and base area of 162 cm^2.



Find

(a) the height of trough Y,

$\frac{h_y}{75} = \sqrt[3]{\frac{1200}{4050}}$ [M1]

$\therefore h_y = \frac{2}{3} \times 75 = 50$ cm [A1]

Answer .... cm [2]

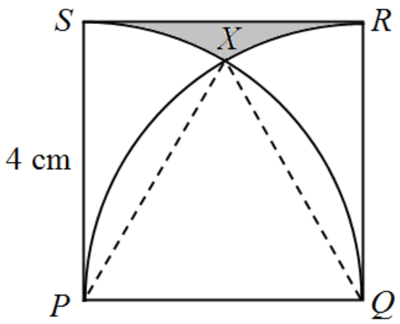
(b) the area of the base of trough Y.

$\frac{A_y}{162} = \left(\frac{2}{3}\right)^2$ [M1]

$\therefore A_y = \frac{4}{9} \times 162 = 72$ cm^2 [A1]

Answer .... cm^2 [2]

19 The diagram shows a square *PQRS* of side 4 cm. Two quadrants are drawn with *P* and *Q* as their centres respectively. The two quadrants intersect at point *X*.



(a) Catherine said the triangle *PXQ* is an equilateral triangle.
State whether you agree or disagree with Catherine and explain your decision.

agree PX = QX = PQ = radius of the two same size quadrants. [B1]

I ...., because ....

....

.... [1]

(b) Find the area of the shaded region *SXR*.

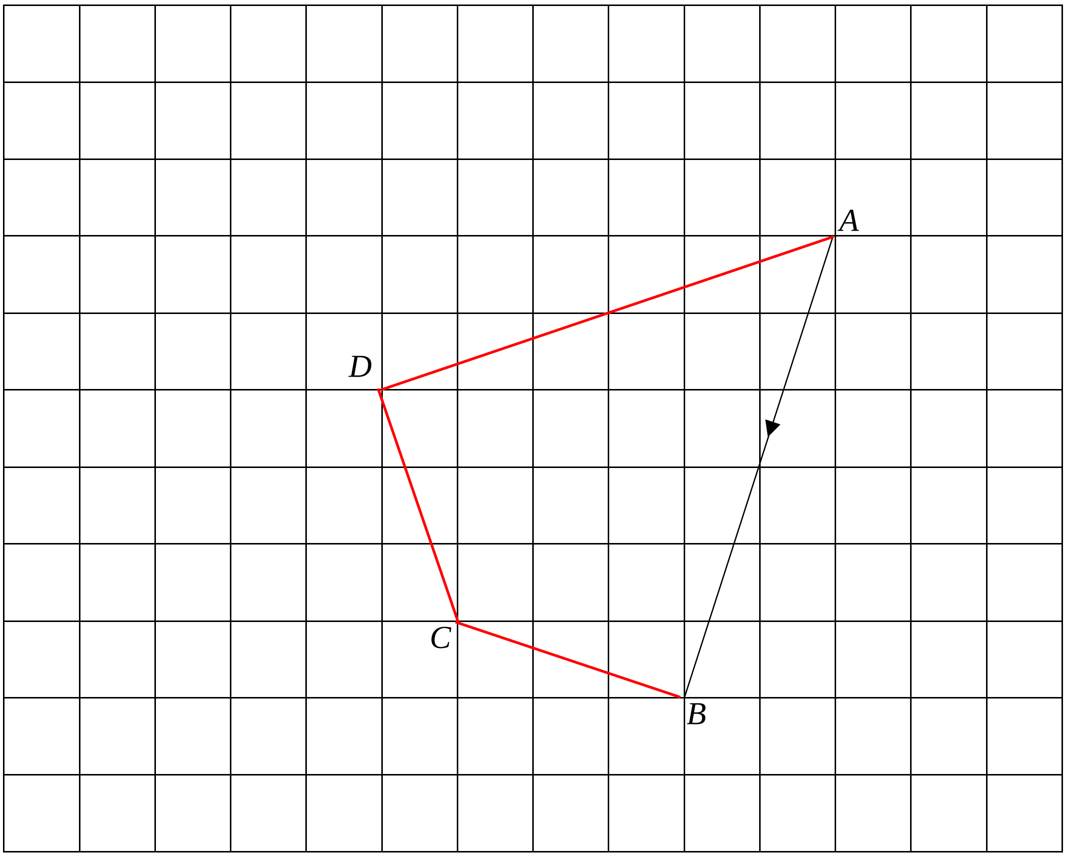
Area of square – 2(Area of sector SPX) – Area of triangle PXQ

$= 4 \times 4 - 2\left(\frac{30°}{360°} \times \pi \times 4^2\right) - \frac{1}{2} \times 4^2 \times sin60°$ [M2]

= 0.694 cm^2 [A1]

Answer .... cm^2 [3]

20 The diagram shows the points A and B where $\overrightarrow{AB} = \begin{pmatrix} -2 \\ -6 \end{pmatrix}$.



(a) Given that $\overrightarrow{BC} = \begin{pmatrix} -3 \\ 1 \end{pmatrix}$, mark and label the point C on the diagram. [1]

Point C in grid above [B1]

(b) D is the point such that $|\overrightarrow{BC}| = |\overrightarrow{CD}|$ and $ABCD$ is a kite. Express $\overrightarrow{CD}$ as a vector.

$\overrightarrow{CD} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$ [B1]

Answer .... [1]

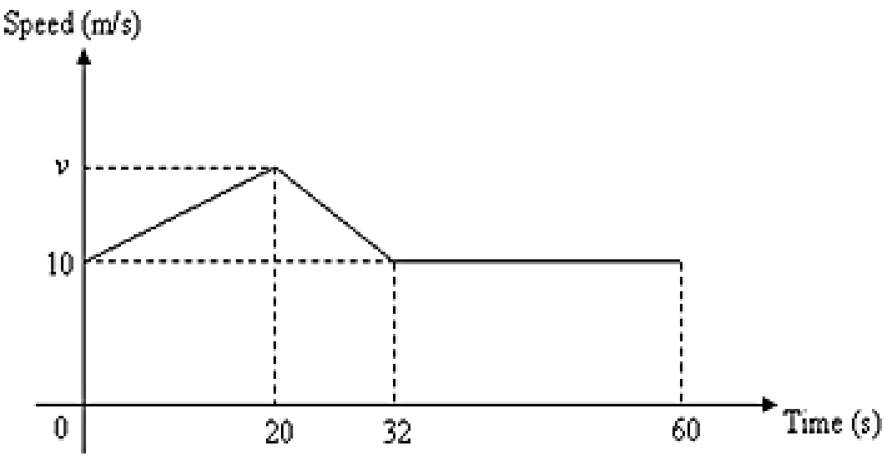
(c) Find $|\overrightarrow{AC}|$.

$|\overrightarrow{AC}| = \sqrt{(-5)^2 + (-5)^2}$ [M1]

$= 7.07$ [A1]

Answer .... units [2]

21 The graph shows the speed-time graph of a car during a period of 60 seconds.
The distance travelled in the first 20 seconds is 250 m.



(a) Show that the maximum speed v is 15 m/s.

Answer

$\frac{1}{2}(10+v)\times 20 = 250$ [M1]

Hence, $v = 15\ m/s$

[1]

(b) Calculate the deceleration during the motion.

$Gradient = \dfrac{15-10}{20-32} = -\dfrac{5}{12}$

$\therefore\ deceleration = \frac{5}{12}\ or\ 0.417\ m/s^2$ [B1]

Answer .... m/s^2 [1]

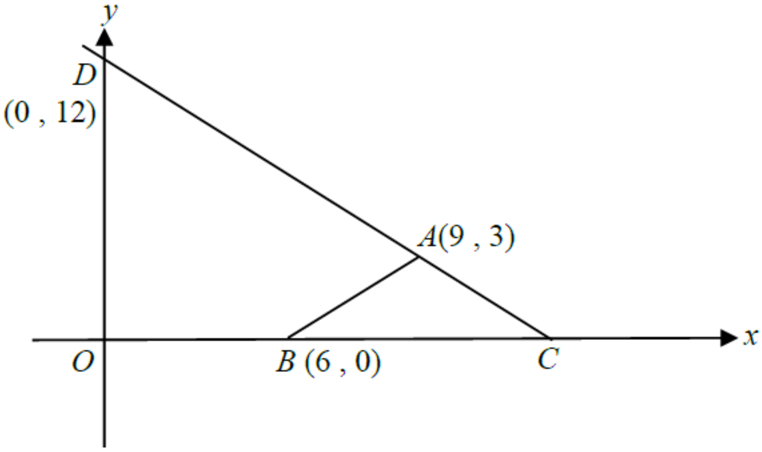
(c) Calculate the average speed during the 60 seconds.

Total distance = $250 + \frac{1}{2}(15+10)\times 12 + 28\times 10 = 680\ m$ [M1]

$\therefore\ average\ speed = \frac{680}{60} = 11\frac{1}{3}\ m/s$ [A1]

Answer .... m/s [2]

22 In the figure below, O is the origin, A is the point (9, 3), B is the point (6, 0) and D is the point (0, 12).



(a) C is another point on the x-axis such that $AB = AC$.

Find the coordinates of C.

$C = (12\,,0)$ [B1]

Answer C (.............. , ...) [1]

(b) Find the equation of line AB.

$Gradient = \frac{3-0}{9-6} = 1$ [M1]

$\therefore\ equation\ of\ AB\ is\quad y = x - 6.$ [A1]

Answer .... [2]

(c) Find the shortest distance from O to the straight line CD.

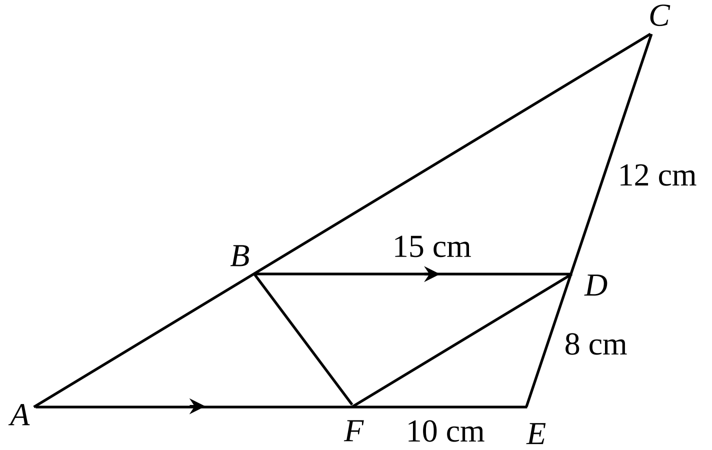
$CD = \sqrt{12^2 + 12^2} = 16.97$ [M1]

$\frac{1}{2} \times 16.97 \times h = \frac{1}{2} \times 12 \times 12$

$\therefore h = 8.49\ (correct\ to\ 3\ s.f.)$ [A1]

Answer .... units [2]

23 In the diagram, *ABC*, *CDE* and *AFE* are straight lines. *BD* is parallel to *AE*.
BD = 15 cm, *CD* = 12 cm, *DE* = 8 cm and *EF* = 10 cm.



(a) Show that triangles *CDB* and *DEF* are similar.
Give a reason for each statement you make.

$\angle BDC = \angle FED$ $(corresponding\ \angle s, BD \parallel AE)$, $\frac{BD}{FE} = \frac{15}{10} = \frac{3}{2}$ and $\frac{CD}{DE} = \frac{12}{8} = \frac{3}{2}$

[B1]

....

$\therefore \Delta CDB\ and\ \Delta DEF\ are\ similar.$ $(Ratio\ of\ corr\ sides\ and\ included\ angle\ are\ equal\ /$ $SAS\ similarity\ test)$ [B1]

.... [2]

(b) Find the ratio of area of triangle *CDB* : area of triangle *DEF*.

$(12:8)^2 = 9 : 4$ [B1]

Answer .... : .... [1]

(c) Show that *ABDF* is a parallelogram.

$\angle CBD = \angle DFE$ $(from\ part\ (a)\ \Delta CDB\ and\ \Delta DEF\ are\ similar)$ [B1]

....

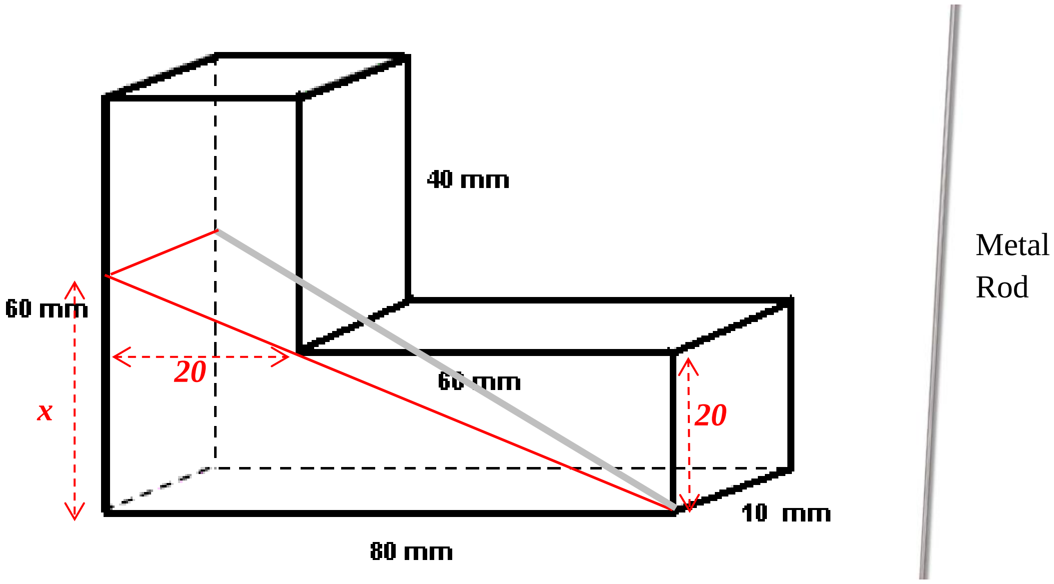
$\angle BAF = \angle CBD$ $(corresponding\ \angle s, BD \parallel AE)$ and so $\angle BAF = \angle DFE$ [B1]

....

$\therefore$ $AB \parallel FD$ and hence *ABDF* is a parallelogram.

.... [2]

24 The diagram shows a closed L-shaped rectangular structure with dimensions stated.



(a) Calculate the volume of the L-shaped rectangular structure.

$= 80 \times 20 \times 10 + 40 \times 20 \times 10$ [M1]

$= 24\,000$ mm^3 [A1]

Answer .... mm^3 [2]

(b) Edison wants to put a thin and long metal rod inside the structure.
What is the longest length of the metal rod that Edison can put in?

$$\frac{x}{20} = \frac{80}{60} \ (similar\ triangles)$$

$x = 26\frac{2}{3}$ [M1]

$l = \sqrt{\left(26\frac{2}{3}\right)^2 + 80^2 + 10^2}$ $(pythagoras\ theorem)$ [M1]

$\therefore$ *longest length of metal rod*, $l = 84.9\ mm$ [A1]

Answer .... mm [3]

End of Paper