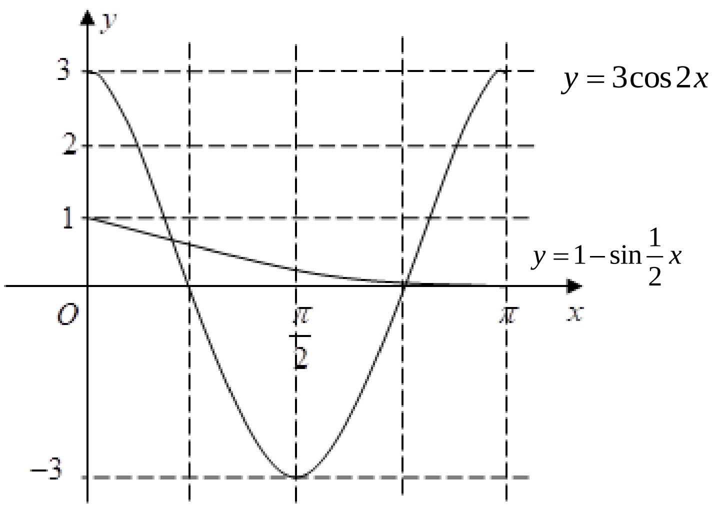
Secondary 4E Prelims 2021 Add Math Paper 1 (Solution)

Q.	Solution	Remarks
1(i)	$\sqrt{(p-8)^2+(12-11)^2}=\sqrt{(8-0)^2+(11-2)^2}$ M1 $(p-8)^2+1=145$ $(p-8)^2=144$ M1 $p-8=12$ or -12 $p=20$ or -4 (reject) A1 (must reject –4) OR $p^2-16p+64+1-145=0$ $p^2-16p-80=0$ M1 $(p-20)(p+4)=0$ $p=20$ or -4 (reject) A1	
(ii)	Gradient of EF = $\frac{12-2}{20-0}=\frac{1}{2}$ (allow ecf) Gradient of perp. bisector = –2 M1 $y=-2x+c$ Subst (8, 11) into equation, $11=-2(8)+c$ $c=27$ Equation of bisector is $y=-2x+27$ A1 OR Midpoint = $\left(\frac{0+20}{2},\frac{2+12}{2}\right)=(10,7)$ Gradient of perp. bisector = $\frac{11-7}{8-10}=-2$ M1 (ecf) $y=-2x+c$ Subst (10, 7) [or (8, 11)] into equation, $7=-2(10)+c$ $c=27$ Equation of bisector is $y=-2x+27$ A1	

2(i)	 For each graph, B2 (–1 mark for incorrect interval (*x*-axis) –1 mark for incorrect min/max (*y*-axis) –1 mark for incorrect graph shape)	
(ii)	$3\cos 2x + \sin\frac{1}{2}x = 1$ for $-2\pi \le x \le 2\pi$ $3\cos 2x = 1 - \sin\frac{1}{2}x$ Number of solution is 2×4 = 8 B1	
3(i)	$(1-ax)^4 = (1)^4 + \binom{4}{1}(1)^3(-ax) + + \binom{4}{2}(1)^2(-ax)^2 + \ldots$ M1 $= 1 - 4ax + 6a^2x^2 + \ldots$ Comparing coeff. of *x*, $-4a = -12$ M1 $a = 3$ (shown) A1 OR $T_{r+1} = \binom{4}{r}(1)^{4-r}(-ax)^r = \binom{4}{r}(1)^{4-r}(-a)^r x^r$ Let $x^r = x$ $r = 1$ M1 $\binom{4}{1}(1)^{4-1}(-a)^1 = -12$ M1 $-4a = -12$ $a = 3$ (shown) A1	

(ii)	$(1-3x)^4 = 1-4(3)x+6(3)^2x^2+...$ $= 1-12x+54x^2+...$ M1 $(2x+1)(1-ax)^4 = (2x+1)(1-12x+54x^2+...)$ Coefficient of x^2 = (2)(–12) + (1)(54) M1 = 30 A1	
4(i) **(ii)**	$\mathrm{f}(x) = \dfrac{x^2-3}{x^2+5}$ $\mathrm{f}'(x) = \dfrac{(x^2+5)(2x)-(x^2-3)(2x)}{(x^2+5)^2}$ M1 $= \dfrac{(2x)(x^2+5-x^2+3)}{(x^2+5)^2}$ $= \dfrac{16x}{(x^2+5)^2}$ M1 Given that $x>0$ $16x>0$ M1 $\dfrac{16x}{(x^2+5)^2}>0$ Since, $\mathrm{f}'(x)>0$, f is an increasing function. A1 $\dfrac{dy}{dx} = \dfrac{16x}{(x^2+5)^2}$ At x = 1, $\dfrac{dy}{dx} = \dfrac{16(1)}{(1^2+5)^2} = \dfrac{4}{9}$ $\dfrac{dx}{dt} = \dfrac{dx}{dy} \times \dfrac{dy}{dt}$ $\dfrac{dx}{dt} = \dfrac{9}{4} \times 0.2 = 0.45$ units/s M1, A1	
5(i)	$P = 3.75e^{kt}$ $5.79 = 3.75\ e^{k(20)}$ $e^{20k} = \dfrac{5.79}{3.75}$ M1 $k = \ln\left(\dfrac{5.79}{3.75}\right) \div 20$ = 0.0217188 M1	

	When $t = 30$		
	$P = 3.75\, e^{0.0217188\,(30)}$	M1	
	$= 7.19$		
	population = 7.19 million	A1	
(ii)	$10 = 3.75\, e^{0.0217188\, t}$		
	$e^{0.0217188\, t} = \dfrac{10}{3.75}$		
	$t = \ln\left(\dfrac{10}{3.75}\right) \div 0.0217188$	M1	
	$= 45.16$ years		
	the year = 2045 (Note: no need to round up)	A1	
6(i)	$\dfrac{d^2y}{dx^2} = \dfrac{18}{(x-2)^3}$		
	$\dfrac{dy}{dx} = \int 18(x-2)^{-3}\, dx$		
	$= \dfrac{18(x-2)^{-2}}{-2(1)} + c \quad = -9(x-2)^{-2} + c$	M1	
	Given $\dfrac{dy}{dx} = 2$ and $x = 5$,		
	$2 = -9(5-2)^{-2} + c \rightarrow c = 3$	M1	
	$\dfrac{dy}{dx} = -9(x-2)^{-2} + 3$		
	$y = \int [-9(x-2)^{-2} + 3]\, dx$		
	$= \dfrac{-9(x-2)^{-1}}{-1(1)} + 3x + c \quad = 9(x-2)^{-1} + 3x + c$	M1	
	Given $x = 5$ and $y = 20$,		
	$20 = 9(5-2)^{-1} + 3(5) + c \rightarrow c = 2$		
	Equation of the curve is $y = 9(x-2)^{-1} + 3x + 2$	A1	
(ii)	$\dfrac{dy}{dx} = -9(5-2)^{-2} + 3 = 2$		
	Gradient of normal = $-\dfrac{1}{2}$	M1	

$y = -\frac{1}{2}x + c$

$20 = -\frac{1}{2}(5) + c$

$c = 22\frac{1}{2}$

Equation of the normal is $y = -\frac{1}{2}x + 22\frac{1}{2}$ A1

7(i)

$3x^2 - 6x + 5 = 3\left(x^2 - 2x + \frac{5}{3}\right)$ or $3\left(x^2 - 2x\right) + 5$

$= 3\left[\left(x-1\right)^2 - (1)^2 + \frac{5}{3}\right]$ or $3\left[\left(x-1\right)^2 - (1)^2\right] + 5$ M1

$= 3\left(x-1\right)^2 + 2$ A1

$-x^2 - 4x - 3 = -\left(x^2 + 4x + 3\right)$ or $-\left(x^2 + 4x\right) - 3$

$= -\left[\left(x+2\right)^2 - (2)^2 + 3\right]$ or $-\left[\left(x+2\right)^2 - (2)^2\right] - 3$ M1

$= -\left(x+2\right)^2 + 1$ A1

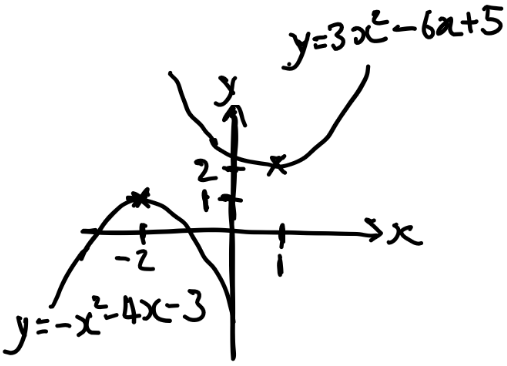
(ii)

$3x^2 - 6x + 5 = 3\left(x-1\right)^2 + 2$

Graph is curving upwards,
with the turning point at (1, 2) M1

$-x^2 - 4x - 3 = -\left(x+2\right)^2 + 1$

Graph is curving downwards,
with the turning point at (–2, 1) M1



Hence, as seen from the diagram, they will not intersect. A1

OR

The y-coordinate of the minimum value of $3x^2 - 6x + 5$ is larger than the maximum value of $-x^2 - 4x - 3$.

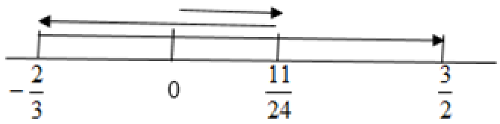
8(i)	$\sin 75° = \sin(30° + 45°)$		
	$= \sin 30° \cos 45° + \cos 30° \sin 45°$	M1	
	$= \frac{1}{2} \times \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{2}$		
	$= \frac{\sqrt{2} + \sqrt{6}}{4}$ (shown)	A1	
(ii)	$\text{cosec} 75° = \frac{1}{\sin 75°} = \frac{4}{\sqrt{2} + \sqrt{6}}$	M1 (for $\frac{1}{\sin 75°}$)	
	$= \frac{4}{\sqrt{2} + \sqrt{6}} \frac{\times(\sqrt{2} - \sqrt{6})}{\times(\sqrt{2} - \sqrt{6})}$	M1	
	$= \frac{4\sqrt{2} - 4\sqrt{6}}{2 - 6}$		
	$= -\sqrt{2} + \sqrt{6}$	A1	
	$\text{cosec}^2 75° = \left(-\sqrt{2} + \sqrt{6}\right)^2$		
	$= 2 - \sqrt{12} - \sqrt{12} + 6$	M1	
	$= 8 - 2\sqrt{4 \times 3}$		
	$= 8 - 4\sqrt{3}$	A1	
	OR		
	$\text{cosec} 75° = \frac{1}{\sin 75°} = \frac{4}{\sqrt{2} + \sqrt{6}}$	M1	
	$\text{cosec}^2 75° = \left(\frac{4}{\sqrt{2} + \sqrt{6}}\right)^2$		
	$= \frac{16}{2 + \sqrt{12} + \sqrt{12} + 6}$	M1	
	$= \frac{16}{8 + 2\sqrt{4 \times 3}}$		
	$= \frac{16}{8 + 4\sqrt{3}} \frac{\times(8 - 4\sqrt{3})}{\times(8 - 4\sqrt{3})}$	M1, M1	
	$= \frac{16(8 - 4\sqrt{3})}{64 - 16(3)}$		
	$= 8 - 4\sqrt{3}$	A1	

9(i)	$2(3x)+3y=36$	M1	
	$3y=36-6x$		
	$y=12-2x$	A1	
(ii)	$V=\frac{1}{2}(x)(x)\sin 60°\times y$	M1	
	$=\frac{1}{2}x^2\left(\frac{\sqrt{3}}{2}\right)(12-2x)$		
	$=\left(\frac{\sqrt{3}}{2}\right)(6x^2-x^3)$	A1	
(iii)	$V=3\sqrt{3}x^2-\frac{\sqrt{3}}{2}x^3$		
	$\frac{dV}{dx}=6\sqrt{3}x-\frac{3\sqrt{3}}{2}x^2$	M1	
	Let $\frac{dV}{dx}=0$		
	$\sqrt{3}x\left(6-\frac{3}{2}x\right)=0$	M1	
	$x=0$ (reject) or $x=4$		
	$V=3\sqrt{3}(4)^2-\frac{\sqrt{3}}{2}(4)^3=16\sqrt{3}$ or 27.7 cm^3	A1	
	$\frac{d^2V}{dx^2}=6\sqrt{3}-3\sqrt{3}x$		
	When $x=4$		
	$\frac{d^2V}{dx^2}=6\sqrt{3}-3\sqrt{3}(4)=-6\sqrt{3}$ (or –10.4)		
	Since $\frac{d^2V}{dx^2}<0$, V = 27.7 cm^3 is a maximum value.	A1	
10(i)	$f(x)=2x^3-10x^2+ax+b$		
	Let $x=-1$		
	Remainder = $2(-1)^3-10(-1)^2+a(-1)+b$		
	$3=-12-a+b$	M1	
	$-a+b=15$ --- (1)		

(ii)	Let $x = 2$ Remainder = $2(2)^3 - 10(2)^2 + a(2) + b$ $-3 = -24 + 2a + b$ M1 $2a + b = 21$ --- (2) Solving the simultaneous equations, $a = 2$ and $b = 17$ A1, A1 $f(x) - 3 = 0$ $2x^3 - 10x^2 + (2)x + (17) - 3 = 0$ $2x^3 - 10x^2 + 2x + 14 = 0$ Let $x = -1$ Remainder = $2(-1)^3 - 10(-1)^2 + 2(-1) + 14 = 0$ Hence, $(x+1)$ is a factor M1 Using long division or comparing coefficient, $2x^3 - 10x^2 + 2x + 14 = (x+1)(2x^2 - 12x + 14)$ M1 $2x^3 - 10x^2 + 2x + 14 = 0$ $(x+1)(2x^2 - 12x + 14) = 0$ $x = -1$ or $2x^2 - 12x + 14 = 0$ A1 $x = \dfrac{12 \pm \sqrt{(-12)^2 - 4(2)(14)}}{2(2)}$ $x = \dfrac{12 \pm \sqrt{32}}{4}$ $x = \dfrac{12 \pm 4\sqrt{2}}{4} = 3 \pm \sqrt{2}$ A1	
11(i) **(ii)**	Angle A is common $\angle ADB = \angle ACD$ (Alternate segment theorem) M1 Hence triangle ADB is similar to triangle ACD (AA) A1 Since triangle ADB is similar to triangle ACD, $\dfrac{AC}{AD} = \dfrac{AD}{AB}$ $AC \times AB = AD^2$ M1	

	Given that $AD = 2AB$		
	$AC \times AB = (2AB)^2$	M1	
	$AC = \dfrac{4AB^2}{AB} = 4AB$ (shown)	A1	
(iii)	$DF = FC$ means F the midpoint of DC.		
	DE is the diameter means O is the midpoint of DE.		
	Hence, OF is parallel to EC (midpoint theorem)	M1	
	$\angle DEC = \angle DOF$ (corresponding angles)		
	$\angle DEC = \angle CDG$ (alternate segment theorem)	M1	
	Hence, angle $CDG =$ angle DOF (shown)	A1	
12(i)	$LHS = \sec x - \dfrac{\cos x}{1+\sin x}$		
	$= \dfrac{1}{\cos x} - \dfrac{\cos x}{1+\sin x}$		
	$= \dfrac{1(1+\sin x)}{\cos x(1+\sin x)} - \dfrac{\cos x(\cos x)}{1+\sin x(\cos x)}$	M1	
	$= \dfrac{\sin x + (1-\cos^2 x)}{\cos x(1+\sin x)}$		
	$= \dfrac{\sin x + \sin^2 x}{\cos x(1+\sin x)}$	M1	
	$= \dfrac{\sin x(1+\sin x)}{\cos x(1+\sin x)}$	M1	
	$= \tan x = RHS$ (proved)	A1	
(ii)	$\sec x - \dfrac{\cos x}{1+\sin x} = \dfrac{\cot x}{3}$		
	$\tan x = \dfrac{1}{3\tan x}$		
	$\tan^2 x = \dfrac{1}{3}$		
	$\tan x = \pm\sqrt{\dfrac{1}{3}}$ (Q1, Q2, Q3, Q4)	M1	
	Acute angle of $x = \tan^{-1}\dfrac{1}{\sqrt{3}} = \dfrac{\pi}{6}$	A1	

(iii)	$y = \sec x - \dfrac{\cos x}{1+\sin x}$ --- (1) $y = -\dfrac{\cot x}{3}$ --- (2) Subst (1) into (2), $\sec x - \dfrac{\cos x}{1+\sin x} = -\dfrac{\cot x}{3}$ $\tan x = \dfrac{-1}{3\tan x}$ $\tan^2 x = \dfrac{-1}{3}$ $\tan x = \pm\sqrt{\dfrac{-1}{3}}$ (no solution) Hence, there is no intersection between the 2 curves.	 M1 A1	
13(i) **(ii)** **(iii)**	Let $v = 0$ $2t^2 - 5t + 2 = 0$ $(2t-1)(t-2) = 0$ $t = \dfrac{1}{2}$ or $t = 2$ $a = \dfrac{dv}{dt} = 4t - 5$ Let $a < 0$ (decelerating) $4t - 5 < 0$ $t < \dfrac{5}{4}$ seconds Hence, $0 \le t < \dfrac{5}{4}$ $s = \int v\ dt = \int (2t^2 - 5t + 2)dt$ $= \dfrac{2t^3}{3} - \dfrac{5t^2}{2} + 2t + c$ When t =0, s =0 , $0 = \dfrac{2(0)^3}{3} - \dfrac{5(0)^2}{2} + 2(0) + c$ → $c = 0$ Hence, $s = \dfrac{2t^3}{3} - \dfrac{5t^2}{2} + 2t$	 M1 A1 M1 M1 A1 M1 A1	

(iv)	When $t=0$, $s=0$	
	When, $t=\frac{1}{2}$. $s=\frac{2}{3}(\frac{1}{2})^3-\frac{5}{2}(\frac{1}{2})^2+2(\frac{1}{2})=\frac{11}{24}$ M1	
	When $t=2$, $s=\frac{2}{3}(2)^3-\frac{5}{2}(2)^2+2(2)=-\frac{2}{3}$	
	When $t=3$, $s=\frac{2}{3}(3)^3-\frac{5}{2}(3)^2+2(3)=\frac{3}{2}$	
		
	Total distance travelled $d=\frac{11}{24}+(\frac{11}{24}+\frac{2}{3})+(\frac{3}{2}+\frac{2}{3})$ M1	
	$=3\frac{3}{4}$ m A1	