N2: Ratio and Proportion (Common Questions w/ Real-World Applications)

- Ratios involving Rational Numbers
- Map scales
- Direct and Inverse Proportion

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1. The surface area of a sphere (S) is directly proportional to the square of its radius. If the radius increases by 15%, find the percentage increase in the surface area.	[2]
2. Arthur, Clement, John are to share a bag of sweets amongst themselves in the ratio $2 : 3 : 4$. John obtained 6 sweets more than Arthur, find	
(a) the total number of sweets received by Arthur	[2]
(b) the number of sweets received by Clement	[1]
3. y is inversely proportional to the square root of x and the difference in the values of y when $x = 9$ and when $x = 16$ is 3. Find	
(a) an equation connecting x and y.	[3]
(b) the exact value of x when $y = 5$.	[1]
4. On a map, a straight road measuring 600 m is represented by a line segment of length 7.5 cm.	
(a) Express the scale of the map in the ratio $1 : r$.	[1]
(b) The length of a canal on the map is 25 cm. Find the actual length in km.	[1]
(c) The actual area of a school is $22\,400\ m^2$. Find the area of the school on the map.	[2]
5. A map of Singapore has a scale of 1 : 80 000.	
(a) The actual length of the Singapore River is 3.2 km. Calculate the length, in centimetres, of the river on the map.	[2]
(b) The actual area of the Bishan-Ang Mo Kio Park is $620\,000\ m^2$. Calculate the area, in square centimetres, of the park on the map.	[2]

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6. A map is drawn to a scale of 1 : 40 000.	
(a) This scale can be expressed as 1 cm represents n km. Find n.	[1]
(b) The distance between a seaport and an airport on the map is 60 cm. Find the actual distance, in kilometres, between the seaport and the airport.	[1]
(c) A bus depot has an actual area of 8 km^2. Find the area, in square centimetres, of the bus depot on the map.	[2]
7. If $a = \frac{4}{3}b$ and $b = \frac{5}{6}c$, write down the ratio of a : b : c.	[1]
8. It is given that y is inversely proportional to the cube of x. If x is doubled, find the percentage decrease in y.	[3]
9. A packet of sweets is divided among Ahmad, Benny, Carl and Dexter. The number of sweets that Ahmad, Benny and Carl have are in the ratio 5 : 9 : 7. If Carl gets 12 sweets more than Ahmad and Benny gets twice as many sweets as Dexter, find	
(a) the total number of sweets	[2]
(b) the number of sweets that Dexter gets	[1]
10. (a) Express the ratio of 1 minute and 30 seconds to 1 hour in its simplest form.	[1]
10. (b) If $5a = 4b$ and $8b = 3c$, find a : b : c.	[1]
11. A map is drawn to a scale of 1 : 20 000.	
(a) The distance from one end of a grassland to the other end is represented by a line of length 8 cm on the map. Calculate the actual distance, giving your answer in kilometres.	[1]
(b) The actual area of the grassland is 2.5 hectares. Calculate the area on the map, giving your answer in square centimetres. $\left[1\ hectare = 10\,000\,m^2\right]$.	[2]

12. The map of The Singapore Zoo has a scale of 1 : 3000.	
(a) If the length from the entrance to Reptile Garden is 4.5 cm on the map, calculate the actual distance, in kilometres.	[1]
(b) The actual area of the Baboon Enclosure is $0.0225\ km^2$. Calculate the area on the map that represents the enclosure, giving your answer in square centimetres.	[1]
13. A map is drawn to a scale of 1 : 50 000.	
(a) Two towns are 24 km apart. Calculate, in centimetres, their distance apart on the map.	[1]
(b) On the map, a farm has an area of $20\ cm^2$. Calculate, in square kilometres, the actual area of the farm.	[1]
14. The map of a national park is drawn to a scale of $1 : n$. A lake, which has an actual area of $7.5\ km^2$, is represented by an area of $4.8\ cm^2$ on the map.	
(a) Find the value of n.	[2]
(b) Calculate the actual perimeter of the lake, in km, if its perimeter on the map is 9 cm.	[1]

N2: Ratio and Proportion (Marking Scheme)

1. The surface area of a sphere (S) is directly proportional to the square of its radius. If the radius increases by 15%, find the percentage increase in the surface area. $S = kr^2$ Let $S = 10, r = 2$ $10 = k(2)^2$ $k = \frac{10}{4}$ $\frac{2}{100} \times 115\% = 2.3$ $S = \frac{10}{4}(2.3)^2$ $S = 13.225$ Percentage increase $= \frac{new\ value - original\ value}{original\ value} \times 100\%$ Percentage increase $= \frac{13.225-10}{10} \times 100\%$ Percentage increase $= 32.25$ $\therefore 32.25\%$	[2]
2. Arthur, Clement, John are to share a bag of sweets amongst themselves in the ratio $2 : 3 : 4$. John obtained 6 sweets more than Arthur, find	
(a) the total number of sweets received by Arthur $A : C : J$ $2 : 3 : 4$ $4u - 2u \rightarrow 6\ sweets$ $\therefore$ 6 sweets	[2]
(b) the number of sweets received by Clement $1u \rightarrow 3\ sweets$ $3u \rightarrow 9\ sweets$ $\therefore$ 9 sweets	[1]

3. y is inversely proportional to the square root of x and the difference in the values of y when $x = 9$ and when $x = 16$ is 3. Find	
(a) an equation connecting x and y. $y = \frac{k}{\sqrt{x}}$ $y = \frac{k}{3}$ $y = \frac{k}{4}$ $\frac{k}{3} - \frac{k}{4} = 3$ $k = 36$ $y = \frac{36}{\sqrt{x}}$ $\therefore y = \frac{36}{\sqrt{x}}$	[3]
(b) the exact value of x when $y = 5$. $5 = \frac{36}{\sqrt{x}}$ $5\sqrt{x} = 36$ $\sqrt{x} = 7.2$ $x = \frac{1296}{25}$ $x = 51\frac{21}{25}$ $\therefore x = 51\frac{21}{25}$	[1]

4. On a map, a straight road measuring 600 m is represented by a line segment of length 7.5 cm.	
(a) Express the scale of the map in the ratio $1 : r$. map : actual 7.5 cm : 600 m 7. 5 cm : 60 000 cm ★ [1 m = 100 cm] 1 cm : 8000 cm $\therefore 1 : 8000$	[1]
(b) The length of a canal on the map is 25 cm. Find the actual length in km. 1 cm : 80 m 25 cm : 2000 m 25 cm : 2 km ★ [1 km = 1000 m] $\therefore$ 2 km	[1]
(c) The actual area of a school is 22 400 m^2. Find the area of the school on the map. 1 cm : 80 m $1\ cm^2 : 6400\ m^2$ $\frac{22400}{6400} = 3.5$ $3.\ 5\ cm^2 : 22\ 400\ m^2$ $\therefore 3.\ 5\ cm^2$	[2]

5. A map of Singapore has a scale of 1 : 80 000.	
(a) The actual length of the Singapore River is 3.2 km. Calculate the length, in centimetres, of the river on the map. 1 cm : 80 000 cm 1 cm : $\frac{80\,000}{10^5}$ km ★ [1 km = 10^5 cm] 1 cm : 0.8 km 4 cm : 3.2 km ∴ 4 cm	[2]
(b) The actual area of the Bishan-Ang Mo Kio Park is 620 000 m^2. Calculate the area, in square centimetres, of the park on the map. 1 cm : $\frac{80\,000}{100}$ m 1 cm : 800 m 1 cm^2 : 640 000 m^2 $\frac{620\,000}{640\,000} = 0.96875$ 0.96875 cm^2 : 620 000 m^2 ∴ 0.96875 cm^2	[2]

6. A map is drawn to a scale of 1 : 40 000.

(a) This scale can be expressed as 1 cm represents n km. Find n. [1]

1 cm : 40 000 cm

1 cm : $\frac{40\,000}{10^5}$ km

1 cm : 0.4 km

n = 0.4

$\therefore n = 0.4$

(b) The distance between a seaport and an airport on the map is 60 cm. [1]
Find the actual distance, in kilometres, between the seaport and the airport.

1 cm : 0.4 km

60 cm : 24 km

∴ 24 km

(c) A bus depot has an actual area of 8 km^2. [2]
Find the area, in square centimetres, of the bus depot on the map.

1 cm : 0.4 km

1 cm^2 : 0.16 km^2

$\frac{8}{0.16} = 50$

50 cm^2 : 8 km^2

∴ 50 cm^2

7. If $a = \frac{4}{3}b$ and $b = \frac{5}{6}c$, write down the ratio of a : b : c. [1]

Method 1:

$\frac{a}{b} = \frac{4}{3} = \frac{20}{15}$

$\frac{b}{c} = \frac{5}{6} = \frac{15}{18}$

a : b : c

20 : 15 : 18

Method 2:

$a = \frac{4}{3}b$ and $b = \frac{5}{6}c$

substitute $b = \frac{5}{6}c$ to $a = \frac{4}{3}b$

$a = \frac{4}{3}\left(\frac{5}{6}c\right)$

$a = \frac{20}{18}c,\ b = \frac{5}{6}c,\ c = c$

a : b : c

20 : 15 : 18

∴ 20 : 15 : 18

8. It is given that y is inversely proportional to the cube of x. If x is doubled, find the percentage decrease in y. $y = \frac{k}{x^3}$ Let $y = 2, x = 3$ $2 = \frac{k}{27}$ $k = 54$ ★ [k is a CONSTANT] When x is doubled $= 3 \times 2$ $y = \frac{54}{6^3}$ $y = \frac{1}{4}$ Percentage decrease $= \frac{\frac{1}{4} - 2}{2} \times 100\%$ Percentage decrease $= 87.5\%$ $\therefore 87.5\%$	[3]
9. A packet of sweets is divided among Ahmad, Benny, Carl and Dexter. The number of sweets that Ahmad, Benny and Carl have are in the ratio 5 : 9 : 7. If Carl gets 12 sweets more than Ahmad and Benny gets twice as many sweets as Dexter, find	
(a) the total number of sweets A : B : C 5 : 9 : 7 $7u - 5u = 2u$ $2u \rightarrow 12\ sweets$ $1u \rightarrow 6\ sweets$ Dexter $4.5u$ Total number of sweets $= (5 + 9 + 7 + 4.5) \times 6$ Total number of sweets $= 153$ $\therefore$ 153 sweets	[2]
(b) the number of sweets that Dexter gets $4.5 \times 6 = 27$ $\therefore$ 27 sweets	[1]

10. (a) Express the ratio of 1 minute and 30 seconds to 1 hour in its simplest form. 1 minute and 30 seconds → 90 seconds 1 hour → 3600 seconds 90 : 3600 ★ [Make them the same unit] 1 : 40 ∴ 1 : 40	[1]
10. (b) If $5a = 4b$ and $8b = 3c$, find a : b : c. $a = \frac{4}{5}b,\ c = \frac{8}{3}b$ a : b : c $\frac{4}{5}b : b : \frac{8}{3}b$ $\frac{4}{5} : 1 : \frac{8}{3}$ 12 : 15 : 40 ∴ 12 : 15 : 40	[1]
11. A map is drawn to a scale of 1 : 20 000.	
(a) The distance from one end of a grassland to the other end is represented by a line of length 8 cm on the map. Calculate the actual distance, giving your answer in kilometres. 1 cm : 20 000 cm 1 cm : $\frac{20\,000}{10^5}$ km 1 cm : 0.2 km 8 cm : 1.6 km ∴ 1.6 km	[1]
(b) The actual area of the grassland is 2.5 hectares. Calculate the area on the map, giving your answer in square centimetres. $\left[1\ hectare = 10\,000\ m^2\right]$. 1 cm : 20 000 cm 1 cm : 200 m $1\ cm^2 : 40\,000\ m^2$ $2.5\ hectares = 2.5 \times 10\,000$ $2.5\ hectares = 25\,000$ $\frac{25\,000}{40\,000} = 0.625$ $\therefore 0.625\ cm^2$	[2]

12. The map of The Singapore Zoo has a scale of 1 : 3000.	
(a) If the length from the entrance to Reptile Garden is 4.5 cm on the map, calculate the actual distance, in kilometres. 1 cm : 3000 cm 1 cm : $\frac{3000}{10^5}$ km 1 cm : 0.03 km 4.5 cm : 0.135 km $\therefore$ 0.135 km	[1]
(b) The actual area of the Baboon Enclosure is 0.0225 km^2. Calculate the area on the map that represents the enclosure, giving your answer in square centimetres. 1 cm : 0.03 km 1 cm^2 : 0.0009 km^2 $\frac{0.0225}{0.0009} = 25$ 25 cm^2 : 0.0009 km^2 $\therefore$ 25 cm^2	[1]
13. A map is drawn to a scale of 1 : 50 000.	
(a) Two towns are 24 km apart. Calculate, in centimetres, their distance apart on the map. 1 cm : 50 000 cm 1 cm : $\frac{50\,000}{10^5}$ km 1 cm : 0.5 km $\frac{24}{0.5} = 48$ 48 cm : 24 km $\therefore$ 48 cm	[1]
(b) On the map, a farm has an area of 20 cm^2. Calculate, in square kilometres, the actual area of the farm. 1 cm : 0.5 km 1 cm^2 : 0.25 km^2 20 cm^2 : 5 km^2 $\therefore$ 5 km^2	[1]

14. The map of a national park is drawn to a scale of $1 : n$. A lake, which has an actual area of $7.5\ km^2$, is represented by an area of $4.8\ cm^2$ on the map.	
(a) Find the value of n. $1 : n$ $4.8\ cm^2 : 7.5\ km^2$ $1\ cm^2 : \frac{25}{16}\ km^2$ 1 cm : 1.25 km $1 \text{ cm} : 1.25 \times 10^5 \text{ cm}$ 1 cm : 125 000 cm 1 : 125 000 $n = 125\,000$ $\therefore n = 125\,000$	[2]
(b) Calculate the actual perimeter of the lake, in km, if its perimeter on the map is 9 cm. 1 cm : 1.25 km 9 cm : 11.25 km $\therefore 11.25\ km$	[1]