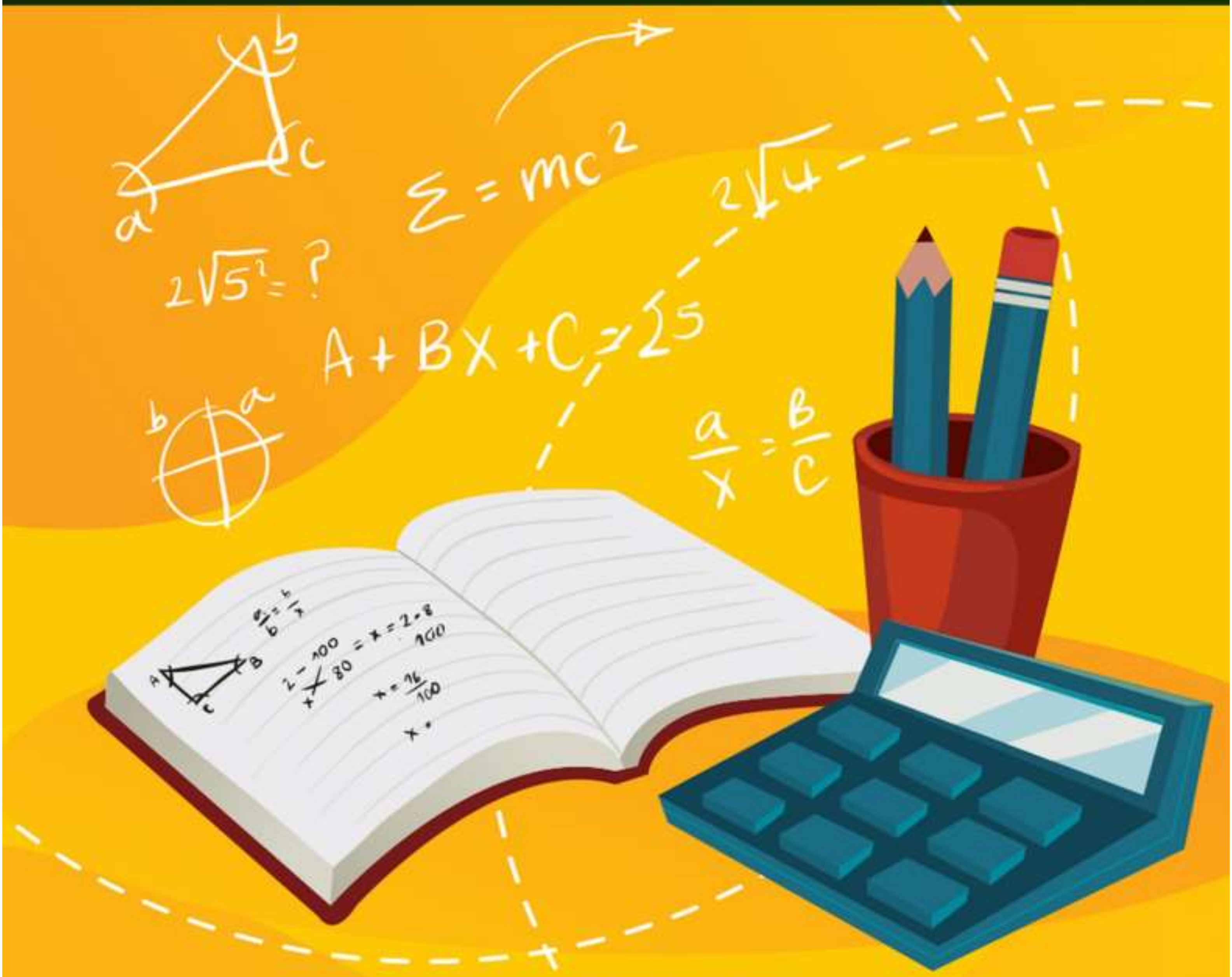
MATH REVISION NOTES FOR

SECONDARY SCHOOL STUDENTS

LEARN ALL THE CONCEPTS OF SECONDARY MATH



JIMMY LING

Published by Jimmy Maths Pte Ltd
Website: www.jimmymaths.com

While every care has been taken to compile this book, errors may still arise in the course of compilation and production. If you notice any error, kindly email to jimmyling@jimmymaths.com so that we can review it.

About the Author

Jimmy Ling graduated from NUS with a degree in Applied Mathematics and has been teaching Mathematics after graduation.

He is the co-founder and director of Grade Solution Learning Centre, a leading tuition centre in Singapore with more than 1000+ students enrolled monthly.

He is also the founder and director of Jimmy Maths, which has more than 10,000 students learning Math online through our online courses.

He is the author of the following books:

- Your Complete Guide to Math Concepts
- 80 Useful Tricks to Solve Math Problems Easily
- PSLE Maths Specimen Papers (You can buy this in Popular bookstores)
- Math Revision Notes for Secondary Students
- Real-World Applications of Math Concepts

He was featured on Love 972 to give tips in doing well for Secondary Math. You can watch the replay here >> https://www.youtube.com/watch?v=qILm1FvzyOM&t=808s

His Youtube channel which shares Math videos regularly has more than 1k subscribers. You can watch his videos here >> https://www.youtube.com/channel/UC5RoF52CtQLYH_W8iX1FrwQ

Preface

Hi there, thank you for downloading this book.

You have received a quick and concise summary of all the key concepts of every chapters in Secondary School Math.

Most students dread revising using the textbooks because they are too thick and lengthy.

That is why we compile this book to help your child extract and understand all the key concepts of each chapter.

This Revision Notes is broken down into different chapters, according to the latest MOE syllabus.

Each chapter is further broken down into different concepts and each concept is explained with examples.

Read and understand the concepts. Understand the examples behind the concepts to make sure that you know how to apply them next time.

Practice is essential. After understanding the concepts, your child needs to put in sufficient practice to master them. Use this book as a homework companion when practicing Math questions.

I hope that through this book, your child will enjoy learning Math as much as I do.

If your child needs more help, feel free to check out our online courses and tuition classes below.

We look forward to teaching your child one day.

Jimmy Ling
Jimmy Maths and Grade Solution Team

Online Courses

How to Learn from our online courses:

1) Watch the videos to learn the concepts
2) Do the assignments given after the videos
3) Check your answers with the solutions provided
4) Ask questions through Whatsapp and seek homework help
5) Submit evaluation tests which we will mark and provide feedback personally

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***Our Spaces are very limited. If you find a class which suits your child, sign up fast!**

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For Sec 1 Students

Chapter 1: Factors and Multiples

Prime Numbers

Prime number is a natural number greater than 1 that has no positive divisors other than 1 and itself.

Examples: 2, 3, 5, 7, 11, 13, 17, 19, 23….

Prime Factorisation

Process of breaking up a composite number into its prime factors.

Example

$$\begin{array}{r|l} 2 & 1176 \\ \hline 2 & 588 \\ \hline 2 & 294 \\ \hline 3 & 147 \\ \hline 7 & 49 \\ \hline 7 & 7 \\ \hline & 1 \end{array}$$

$$\begin{aligned} 1176 &= 2\times2\times2\times3\times7\times7 \\ &= 2^3\times3\times7^2 \text{ (Index Notation)} \end{aligned}$$

Highest Common Factor (H.C.F)

The Highest Common Factor (H.C.F) of two (or more) numbers is the **largest number** that **divides evenly** into all the numbers.

To find the H.C.F in index form, compare the factors and take the **Lower Power**

Example

Find the H.C.F of $2\times3^2\times5$ and $2^2\times3^3\times7\times13$.

$$2\times3^2\times5$$

$$2^2\times3^3 \quad \times7\times13$$

$$HCF=2\times3^2$$

Lowest Common Multiple (L.C.M)

The Lowest Common Multiple (LCM) of two (or more) numbers is the **smallest number** that is **divisible by all the numbers**.

To find the L.C.M in index form, compare the factors and take the **Higher Power**

Example

Compare and take the **Higher Power**

$2\times3^2\times5^2\times11$

$2^3\times3\times5^3$

$LCM=2^3\times3^2\times5^3\times11$

Perfect Square

$4=2^2$ $\sqrt{4}=2$

$9=3^2$ $\sqrt{9}=3$

$16=2^4$ $\sqrt{16}=2^2$

$25=5^2$ $\sqrt{25}=5$

Perfect Squares → 4, 9, 16, 25...
→ All the powers of the factors are divisible by 2

Perfect Cube

$8=2^3$ $\sqrt[3]{8}=2$

$27=3^3$ $\sqrt[3]{27}=3$

$64=2^6$ $\sqrt[3]{64}=2^2$

Perfect Cube → 8, 27, 64...
→ All the powers of the factors are divisible by 3

Chapter 2: Real Numbers

Type of Numbers

Integers → Numbers without Decimals

Positive Integers → 1, 2, 3 , 4, 5…..
Negative Integers → -1, -2, -3, -4, -5…..

Rational Numbers → Numbers which can be expressed as Fractions

All integer are rational	All fractions are rational
Rational → 1, 2, 3, -1, -2, -3…	$\frac{1}{2}, \frac{2}{3}, -\frac{3}{4}, \frac{22}{7}$

Irrational → $\pi, \sqrt{2}, \sqrt[3]{5}, \frac{10}{\sqrt{3}}$

Decimals

Terminating → 0.2, 0.25

Recurring → $0.\dot{3} = 0.3333333...$

$0.21\dot{3} = 0.213333333...$

$0.\dot{2}1\dot{3} = 0.213213213...$

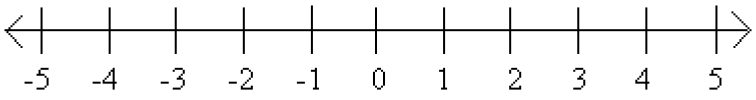
All Terminating and Recurring Decimals are Rational

$0.2 = \frac{1}{5}$

$0.\dot{3} = 0.3333333... = \frac{1}{3}$

$0.1\dot{6} = 0.16666... = \frac{1}{6}$

Number Line



More Than, Less Than

"a > b" means a is more than b.

Example: $-2>-3$

"a < b" means a is less than b.

Example: $-3<-2$

Times and Divide for Negative Number

The 4 Rules of Multiplication

1. Plus × Plus = Plus

   $2 \times 1 = 2$

2. Plus × Minus = Minus

   $2 \times -1 = -2$

3. Minus × Plus = Minus

   $-2 \times 1 = -2$

4. Minus × Minus = Plus

   $-2 \times -1 = 2$

Order of Operations

(BODMAS rule)

1. **B**rackets
2. **O**rder (Power **O**f)
3. **D**ivision and **M**ultiplication, left to right
4. **A**ddition and **S**ubtraction, left to right

Example

$$\begin{aligned}-2+[3-(-4)^2]\times(-5) &= -2+[3-16]\times(-5)\\ &= -2+(-13)\times(-5)\\ &= -2+65\\ &= 63\end{aligned}$$

Chapter 3: Estimation

Rounding Off

Decimal Place

$24.16 \approx 24.2$ (1 d.p)

Nearest 10, 100 etc

$7894 \approx 7890$ (Nearest Tens)

≈ 7900 (Nearest Hundreds)

Nearest unit

$789g = 0.789kg$

$\approx 1\ kg$ (Nearest Kg)

Significant Figures

Significant figures show the accuracy in measurements.

The higher the significant figures, the accurate the answer is.

Rules of Significant Figures

1.23 has 3 significant figures (All the digits give us information on how accurate the measurement is).

0.123 has 3 significant figures (The first zero only tell us the size, and not the accuracy of the measurement).

0.1230 has 4 significant figures (The last zero tells us that this number has been rounded off, so it is significant).

0.10023 has 5 significant figures (All the zeros between the significant figures are significant).

10.0 has 3 significant figures (The zero between the significant figures is significant too).

10 has either 2 significant figures or 1 significant figure (Depending on the question)

For example, 10.2 ≈ 10 (2 sf)

Or 10.2 ≈ 10 (1 sf)

Chapter 4: Algebraic Simplification

Remember The 4 Rules of Multiplication!

1. Plus × Plus = Plus
2. Plus × Minus = Minus
3. Minus × Plus = Minus
4. Minus × Minus = Plus

Substitution

Example

Given that $x = 5$ and $y = -2$, evaluate $\dfrac{x}{2x+y} - \dfrac{y}{x}$.

$$\frac{x}{2x+y} - \frac{y}{x}$$

$$= \frac{5}{2(5)+(-2)} - \frac{-2}{5}$$

$$= \frac{5}{10-2} + \frac{2}{5}$$

$$= \frac{5}{8} + \frac{2}{5}$$

$$= \frac{41}{40}$$

Algebra Simplification for Times and Divide

Examples

1. $3a \times 5a \;= 15a^2$

2. $27a \div 9b \times 3c$

$$= 27a \times \frac{1}{9b} \times 3c$$

$$= \frac{9ac}{b}$$

Algebraic Expansion

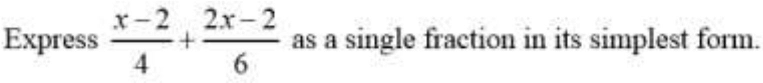
Example

Expand and Simplify $2(a+b)-3(2a-b)$

$$\begin{aligned}&2(a+b)-3(2a-b)\\&=2a+2b-6a+3b\\&=2a-6a+2b+3b\\&=-4a+5b\end{aligned}$$

Algebraic Fraction

Example



Express $\frac{x-2}{4}+\frac{2x-2}{6}$ as a single fraction in its simplest form.

$$\frac{x-2}{4}+\frac{2x-2}{6}$$

$$=\frac{3(x-2)}{12}+\frac{2(2x-2)}{12}$$

$$=\frac{3x-6+4x-4}{12}$$

$$=\frac{7x-10}{12}$$

Algebraic Factorisation

Identify the H.C.F of the terms and put outside the bracket.

Examples

1. $12xy-36yz$

$=12y(x-3z)$

2. $4x-8x^2$

$=4x(1-2x)$

Chapter 5: Algebraic Equations

Solving Linear Equations

Example

Solve for *x* in the following equation:

$$3(4x-1)=7(2x-5)$$
$$3(4x-1)=7(2x-5)$$
$$12x-3=14x-35$$
$$12x-14x=-35+3$$
$$-2x=-32$$
$$x=\frac{-32}{-2}$$
$$x=16$$

Solving Equations Involving Algebraic Fractions

Solve for *x* in the following equation:

$$\frac{2p+1}{6}-\frac{6-5p}{5}=\frac{12p-15}{10}$$

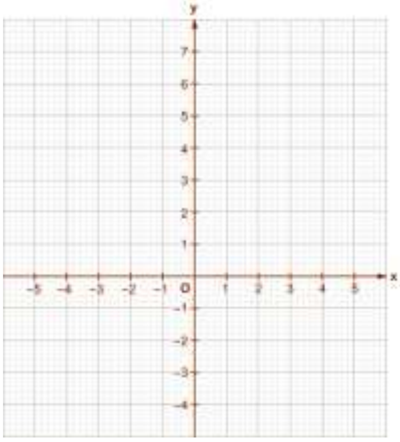
Multiply each term by 30 (The LCM of 6, 5 and 10)

$$\left(\frac{2p+1}{6}\right)\times 30-\left(\frac{6-5p}{5}\right)\times 30=\left(\frac{12p-15}{10}\right)\times 30$$
$$5(2p+1)-6(6-5p)=3(12p-15)$$
$$10p+5-36+30p=36p-45$$
$$10p+30p-36p=-45-5+36$$
$$4p=-14$$
$$p=\frac{7}{2}$$

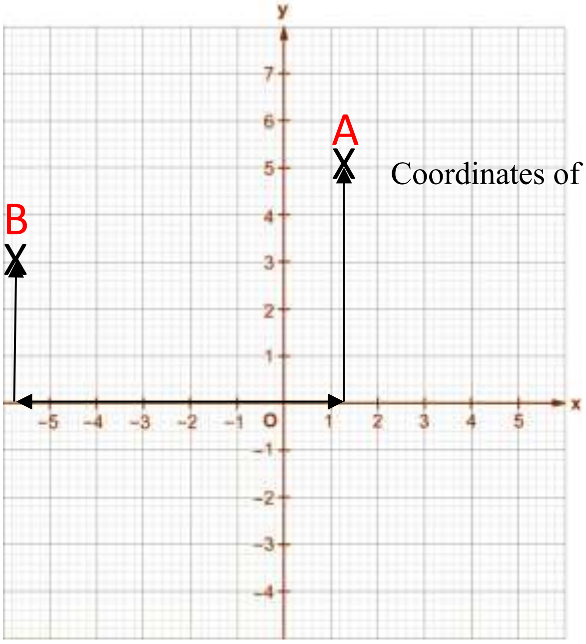
Chapter 6: Coordinates and Linear Functions

Coordinates

Cartesian Plane



- The position of a point on the Cartesian plane
- Expressed in the form of an ordered pair (x, y)
- *x is the x-coordinate* and y, is the y-coordinate.



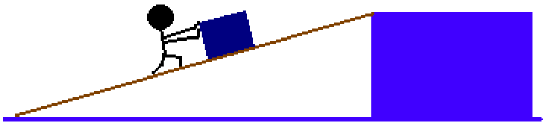
Coordinates of A: (2, 5)

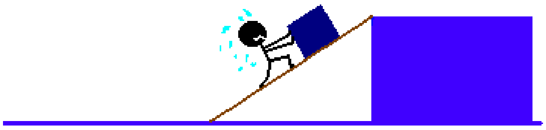
Coordinates of B: (-5, 3)

Gradient

Determines the slope of a line

This line has a lower gradient.

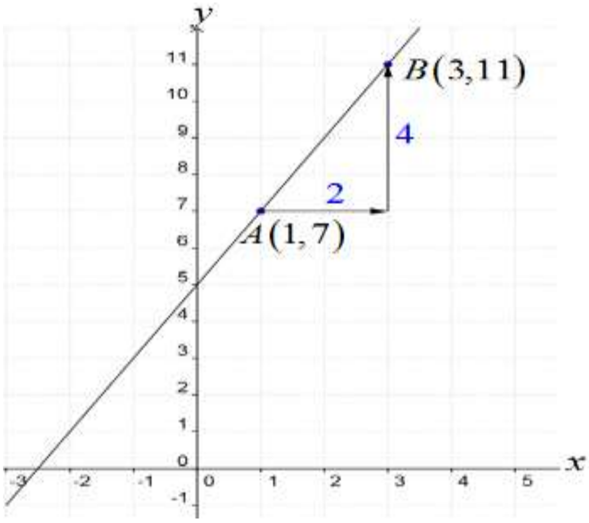




This line has a higher gradient.

Calculation of Gradient

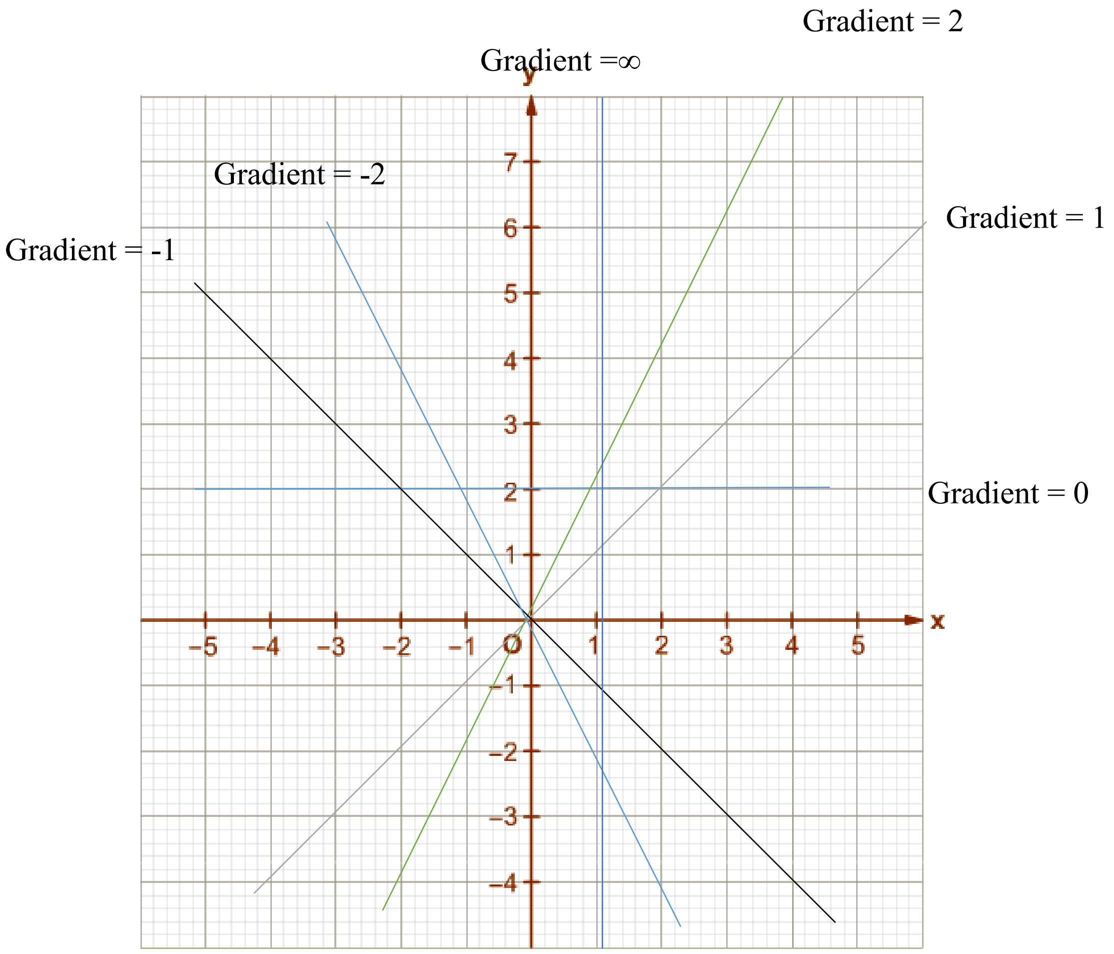
$$\text{Gradient} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{Change in y}}{\text{Change in x}}$$



Change in $y = 11 - 7 = 4$

Change in $x = 3 - 1 = 2$
Gradient = $4 \div 2 = 2$

Different Gradients

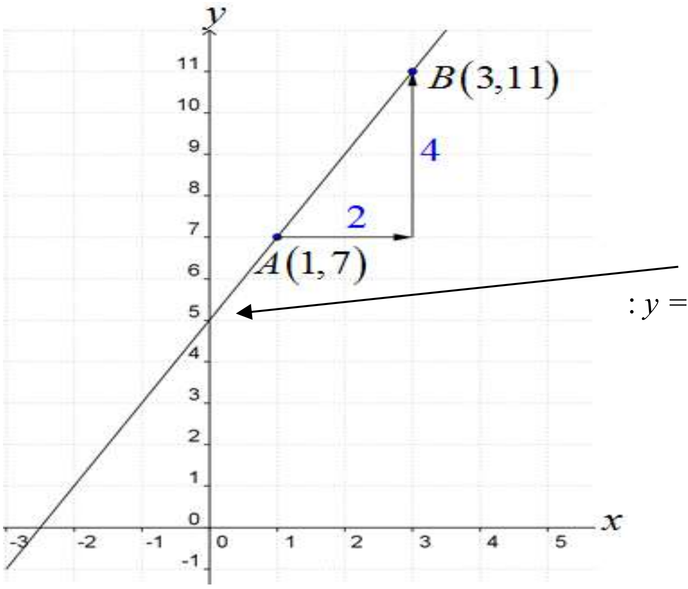


Equation of Straight Line

$y = mx + c$

m = gradient

c = y-intercept (The point when the graph cuts the y-axis)



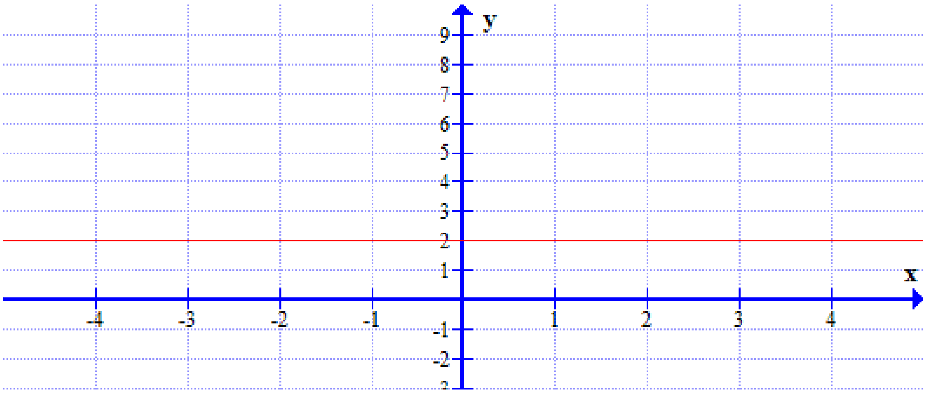
m = gradient = 4 ÷ 2 = 2

c = y-intercept = 5

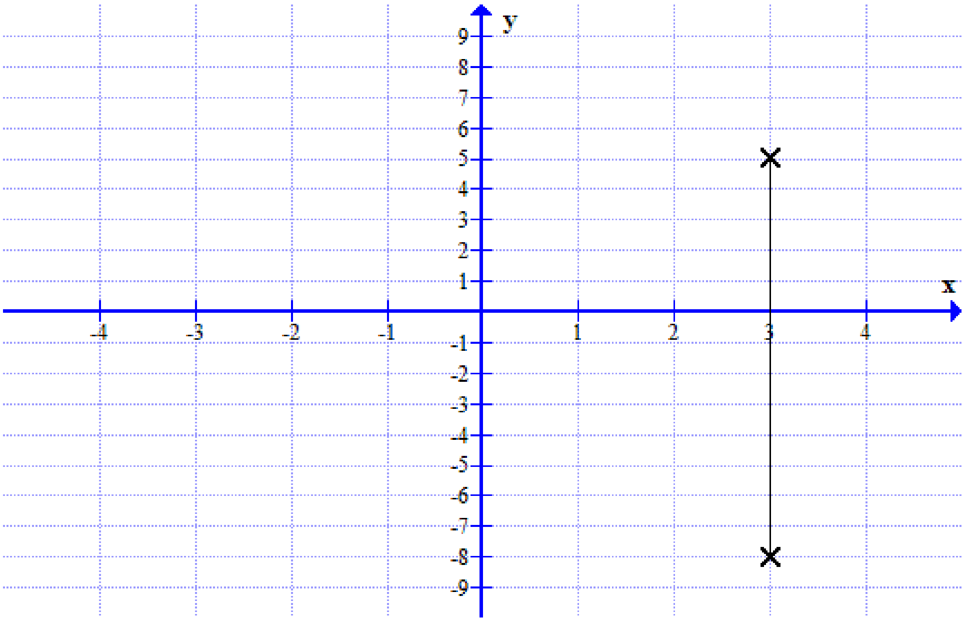
: $y = 2x + 5$

Graphs of Horizontal and Vertical Lines

Equation of a horizontal line is always: $y = a$



Equation: y = 2



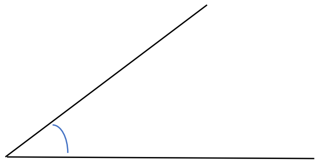
Equation: $x = 3$

Equation of a vertical line is always: $x = a$

Chapter 7: Geometry (Angles)

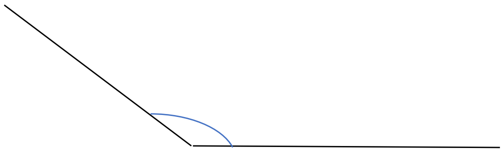
1) There are different types of angles.

   a) An **acute** angle is less than 90°.

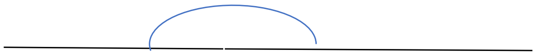


   b) A **right angle** is equal to 90°.

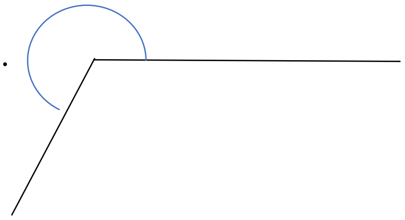
   c) An **obtuse** angle is between 90° and 180°.



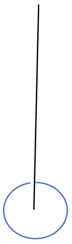
   d) A **straight line** is equal to 180°.



   e) A **reflex angle** is larger than 180° but less than 360°.



   f) A **complete turn** is 360°.

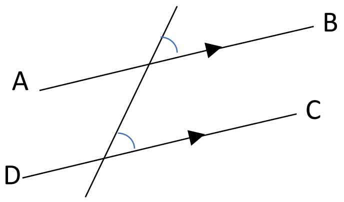


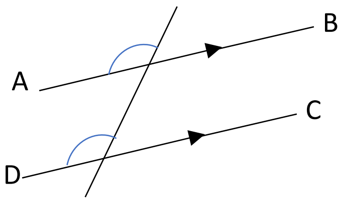
2) Two angles are said to be **complementary angles** if their sum is 90°.

3) Two angles are called **supplementary angles** if their sum is 180°.

4) **Vertically opposite angles** are those on the opposite sides of two intersecting lines.

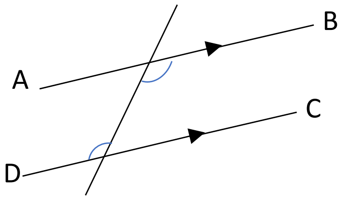
5) Two angles are said to be **corresponding angles** if they have the same relative positive at the each intersection where a line cuts across a pair of parallel lines.

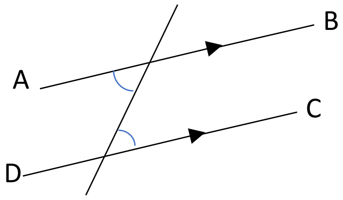




The corresponding angles are equal to each other.

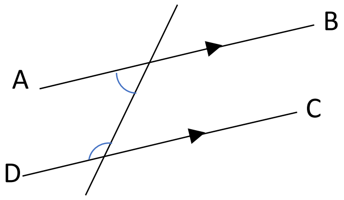
6) Two angles are called **alternate angles** if they lie on the different side of the line cutting across a pair of parallel lines.

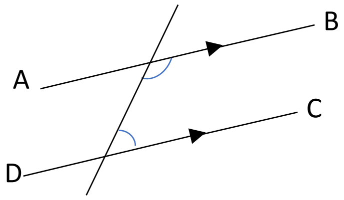




The alternate angles are equal to each other.

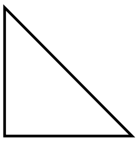
7) Two angles are known as **interior angles** if they lie on the same side of the line that lie on the different side of the line cutting across a pair of parallel lines.

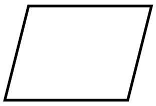


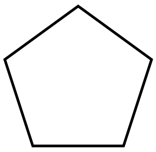


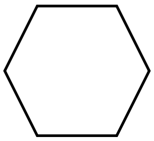
Chapter 8: Polygons

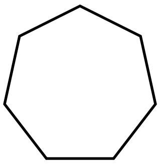
1) A **polygon** is a closed plane figure with three or more straight lines. Polygons are named according to the number of sides they have.

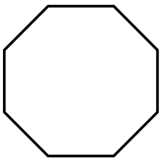

Triangle
(3-sided)

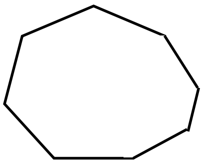

Quadrilateral
(4-sided)

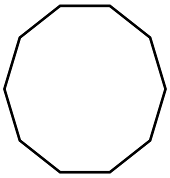

Pentagon
(5-sided)


Hexagon
(6-sided)


Heptagon
(7-sided)


Octagon
(8-sided)


Nonagon
(9-sided)

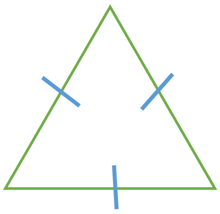

Decagon
(10-sided)

Note: Polygon with n sides is called an *n*-gon.

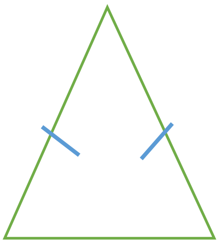
2) A **regular polygon** is a polygon with all its sides and angles equal.

3) Triangles can be classified according to the number of equal sides they have.

a) An **equilateral triangle** is a triangle with 3 equal sides. Each of its angles is 60°.



b) An **isosceles triangle** is a triangle with 2 equal sides. Its base angles are equal.



4) The **sum of the interior angles** of a triangle is always 180°.

5) A **quadrilateral** is a 4-sided polygon. Any quadrilateral can be divided into 2 triangles along its diagonal. Since the angles of a triangle add up to 180°, the angle sum of a quadrilateral is $180° \times 2 = 360°$.

6) A **regular polygon** is a polygon with equal angles and equal sides. Its n interior angles add up to $(n-2) \times 180°$.

7) The sum of the exterior angles of an n-sided polygon is always 360°.

Example 11

Find the exterior angle of a 12-sided regular polygon.

Solution

Let x the size of each exterior angle.

$12 \times x = 360°$

$x° = 30°$

Example 12

The ratio of an interior angle to an exterior angle of a regular n-sided polygon is 8 ∶ 1. How many sides does the polygon have?

Let 8a and a be the interior angle and the exterior angle at the same vertex respectively.

exterior angle + interior angle = 180° (**adjacent ∠s on a straight line)**

$$a + 8a = 180°$$

$$a = 20°$$

$$\text{Sum of exterior angles} = 360°$$

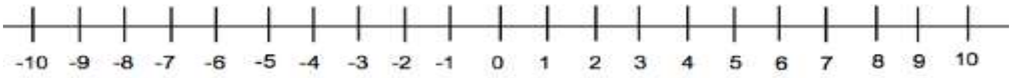
$$20°(n) = 360°$$

$$n = 18$$

∴ *The polygon has 18 sides.*

Chapter 9: Inequalities

Number Line



1) *a* is more than *b*

$a > b$

2) *a* is lesser than *b*

$a < b$

3) *a* is more than or equal to *b*

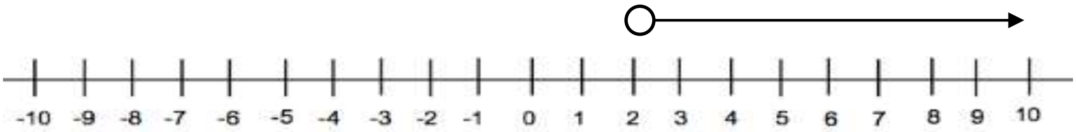
$a \geq b$

4) *a* is lesser than or equal to *b*

$a \leq b$

Examples

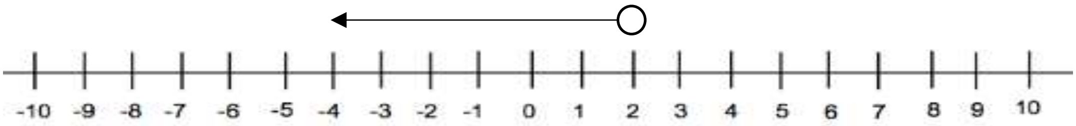
1) *x* is more than 2



$x > 2$

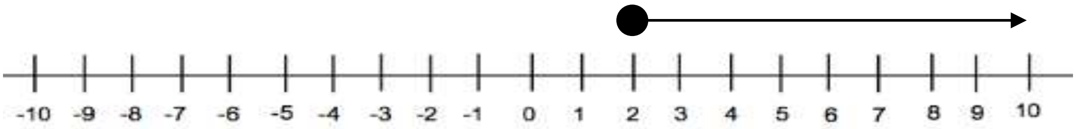
2) *x* is lesser than 2

$x < 2$



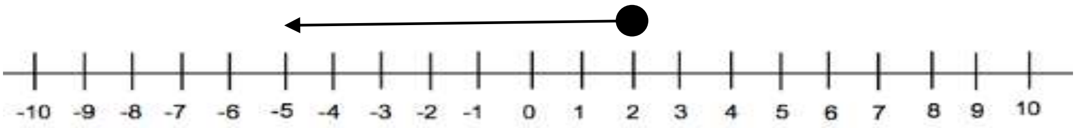
3) *x* is more than or equal to 2

$x \geq 2$



4) x is lesser than or equal to 2

$x \leq 2$



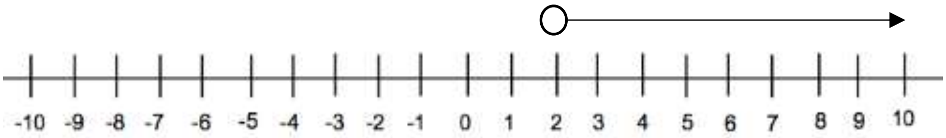
Maximum and Minimum

If x is an integer and $x > 2$,

What is the max value of x?

There is no max!

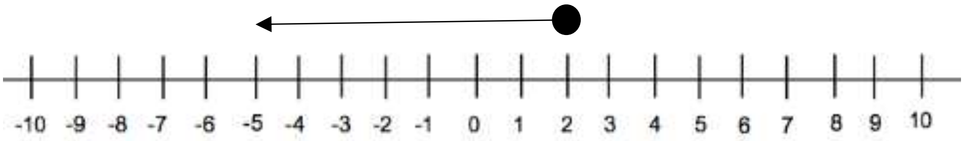
What is the min value of x?
Ans: 3



If x is an integer and $x \leq 2$,

What is the max value of x?
Ans: 2

What is the min value of x?
There is no min!



Chapter 10: Patterns

Patterns with Same Difference

Important Formula: Nth Term = a + (n – 1) d

a → 1^{st} term, d → common difference

Example: 1, 3, 5, 7, 9

n	
1	1
2	3 = 1 + 1 × 2
3	5 = 1 + 2 × 2
4	7 = 1 + 3 × 2
5	9 = 1 + 4 × 2
n	Nth term = a + (n – 1) d

a → first term

d → difference

$$\begin{aligned}\text{nth term} &= 1+(n-1)2\\ &= 1+2n-2\\ &= 2n-1\end{aligned}$$

$$\begin{aligned}\text{50th term} &= 2(50)-1\\ &= 99\end{aligned}$$

Patterns with Square Numbers

Example: 1, 4, 9, 16, 25…

n	
1	$1 = 1^2$
2	$4 = 2^2$
3	$9 = 3^2$
n	Nth term = n^2

Sum of Consecutive Numbers

Example: 1, 3, 6, 10, 15

n	
1	1
2	3 = 1 + 2
3	6 = 1 + 2 + 3
4	10 = 1 + 2 + 3 + 4
5	15 = 1 + 2 + 3 + 4 + 5
n	Nth term = 1 + 2 + 3 + … + n

$$\begin{aligned}\text{nth term} &= 1+2+3+...+(n-1)+n \\ &= (n+1)\times\frac{n}{2} \\ &= \frac{n(n+1)}{2}\end{aligned}$$

Chapter 11: Ratio and Proportion

Ratio Involving Rational Number

What is a ratio?

- A ratio is a comparison of 2 quantities in the **same units**

Example:

3 m : 20 cm $15:1=\frac{15}{1}$

= 300 cm : 20 cm

= 15 : 1

Equivalent Ratios

Example 1

$$\frac{1}{2}:3=\frac{2}{3}x:5$$
$$\frac{1}{2}\div 3\times 5=\frac{2}{3}x$$
$$\frac{5}{6}=\frac{2}{3}x$$
$$x=\frac{5}{6}\div\frac{2}{3}=\frac{5}{4}$$

Combining Ratios (Repeated Identity Concept)

Example 1

If a : b = 2 : 3 and b : c = 4 : 5 , what is a : b : c?

a : b	b : c
=2 : 3	=4 : 5
=8 : 12	=12 : 15

a : b : c

=8 :12 : 15

Chapter 12: Percentage

Discount

A discount is a reduction in the price.

Discount = Marked Price – Selling Price

$$\text{Percentage Discount} = \frac{\text{Discount}}{\text{Marked Price}} \times 100\%$$

GST

GST (Goods and Services Tax) is an increase on the price. In Singapore, the GST is 7%.

$$\text{Price after GST} = (100\% + GST) \times \text{Selling Price}$$

Conversion

To convert a fraction or decimal to percentage, multiply it by 100%.

$$\begin{aligned} \frac{1}{4} &= \frac{1}{4} \times 100\% \\ &= 25\% \end{aligned}$$

$$\begin{aligned} 1\frac{1}{2} &= \frac{3}{2} \times 100\% \\ &= 150\% \end{aligned}$$

$$\begin{aligned} 0.8 &= 0.8 \times 100\% \\ &= 80\% \end{aligned}$$

$$\begin{aligned} 1.2 &= 1.2 \times 100\% \\ &= 120\% \end{aligned}$$

Finding Percentage of a Number

To find percentage of a number,

1) Write the percentage as a fraction
2) Multiply the fraction by the number

$$\begin{aligned} 25\% \text{ of } 80 &= \frac{25}{100} \times 80 \\ &= \frac{1}{4} \times 80 \\ &= 20 \end{aligned}$$

$$\begin{aligned} 125\% \text{ of } 80 &= \frac{125}{100} \times 80 \\ &= \frac{5}{4} \times 80 \\ &= 100 \end{aligned}$$

Express one Quantity as a Percentage of Another

To express one quantity as a percentage of another,

1) Express as a fraction
2) Multiply the fraction by 100%

Express 15 cm as a percentage of 0.8 m

0.8 m = 80 cm

$$\frac{15}{80} \times 100\% = 18.75\%$$

Reverse Percentage

25% of x is equal to 30, what is x?

1) Express 25% as a fraction.
2) Divide 30 by the fraction.

$$\frac{25}{100} x = 30$$

$$\begin{aligned} x &= 30 \div \frac{25}{100} \\ &= 30 \div \frac{1}{4} \\ &= 30 \times 4 \\ &= 120 \end{aligned}$$

Percentage Increase and Decrease

$$\text{Percentage Increase} = \frac{Increase}{Original} \times 100\%$$

$$\text{Percentage Decrease} = \frac{Decrease}{Original} \times 100\%$$

The price of a bag increased from \$28 to \$32. Find the percentage increase?

$$32 - 28 = 4$$

$$\frac{4}{28} \times 100\% = 14\frac{2}{7}\%$$

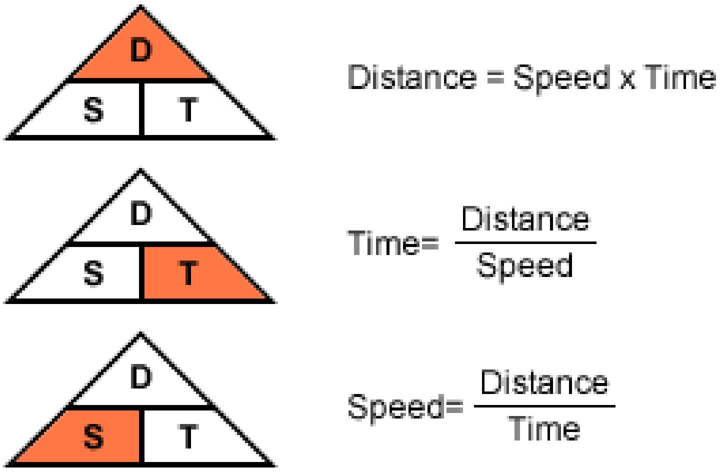
Chapter 13: Rate and Speed

Rate

Definition of Rate

- Measures how fast something changes
- Rate of Change of Distance = Speed (m/s)
- Rate of Change of Volume = Water Rate (litres per min)
- Printing Rate (Example: 100 pages/min)
- Working Rate (Example: Clean 1 house in 3 days)

Speed



Distance = Speed x Time

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

- Average Speed is Total Distance ÷ Total Time

Conversion

Express 120 km/h in m/s.

Recall: 1 km = 1000 m

1 hour = 3600 sec

120 km/h = 120 000 m/h

= 120 000 ÷ 3600 m/s

= 13 m/s (Ans)

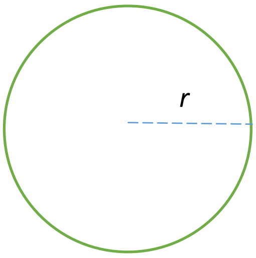
Chapter 14: Measurement

Perimeter

1. We use **millimetres** (mm), **centimetres** (cm), **metres** (m) and **kilometres** (km) as units for measuring lengths or distances.

   The units are related as follows:
   1km = 1000m; 1m = 100cm; 1cm = 10mm

2. Perimeter of a **circle** is referred to as the circumference. The **circumference**, *C*, of a circle of radius, r, can be obtained by using this formula, circumference = $2 \times \pi \times r$ where $\pi = 3.14$ or $\frac{22}{7}$.



Circumference of circle = $2\pi r$

Area

3. Area of a closed figure is the amount of space it covers. It is measured in square units.

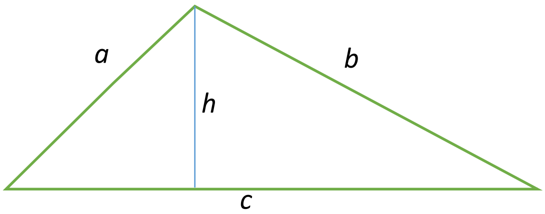
4. Note:
   a) $1\text{ cm}^2 = 1\text{ cm} \times 1\text{ cm}$
   $= 10\text{ mm} \times 10\text{ mm}$
   $= 100\text{ mm}^2$

   b) $1\text{ m}^2 = 1\text{ m} \times 1\text{ m}$
   $= 100\text{ cm} \times 100\text{ cm}$
   $= 10\ 000\text{ cm}^2$

   c) $1\text{ km}^2 = 1\text{ km} \times 1\text{ km}$
   $= 1\ 000\text{ m} \times 1\ 000\text{ m}$
   $= 1\ 000\ 000\text{ m}^2$

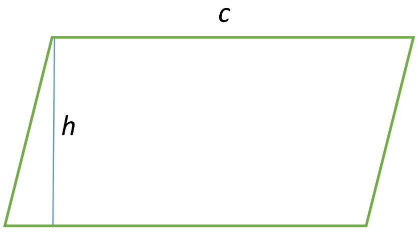
   d) Hectare (ha) is a unit for measuring large land areas such as farms.
   $1\text{ ha} = 10\ 000\text{ m}^2$

5. Area of a **triangle** = $\frac{1}{2}$ × base × height



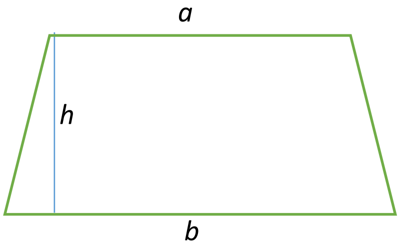
Area of triangle = $\frac{1}{2} \times c \times h$

6. Area of a **parallelogram** = base × height



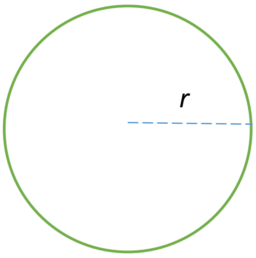
Area of parallelogram = $c \times h$

7. Area of a **trapezium** = $\frac{1}{2}$ × (sum of parallel sides) × height



Area of trapezium = $\frac{1}{2} \times (a + b) \times h$

8. Area of a **circle** = $\pi \times r^2$, where $\pi = 3.14$ or $\frac{22}{7}$.



Area of circle = $\pi \times r^2$

Volume

Volume of an object is the amount of space it occupies. We use cubic metre (m^3), cubic centimetre (cm^3) and millimetre (mm^3) as units for measuring volumes.

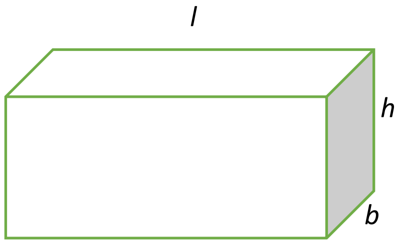
9. Note:

a) $1\ cm^3 = 1\ cm \times 1\ cm \times 1\ cm$
$= 10\ mm \times 10\ mm \times 10\ mm$
$= 1000\ mm^3$

b) $1\ m^3 = 1\ m \times 1\ m \times 1\ m$
$= 100\ cm \times 100\ cm \times 100\ cm$
$= 1\ 000\ 000\ cm^3$

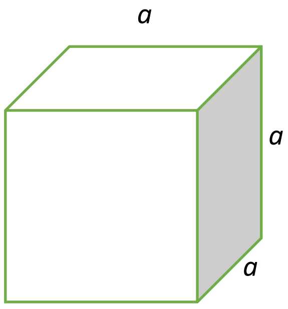
c) $1\ km^3 = 1\ km \times 1\ km \times 1\ km$
$= 1\ 000\ m \times 1\ 000\ m \times 1\ 000\ m$
$= 1\ 000\ 000\ 000\ m^3$

10. Volume of a **cuboid** = length × breadth × height



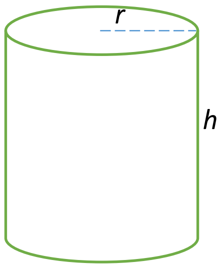
Volume of cuboid = $l \times b \times h$

11. Volume of a **cube** = length × length × length



Volume of cube = a^3

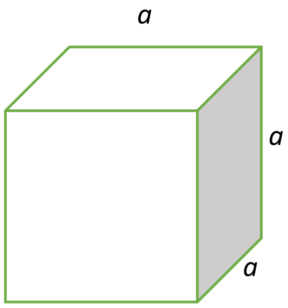
12. A cylinder is a special prism with a circular cross-section. Hence, its volume can be found by multiplying the area of the circular base by its height.



Volume of cylinder = $\pi r^2 h$

Surface Area

13. A cube has 6 equal faces. Therefore, its surface area is obtained by multiplying the area of one face by 6.

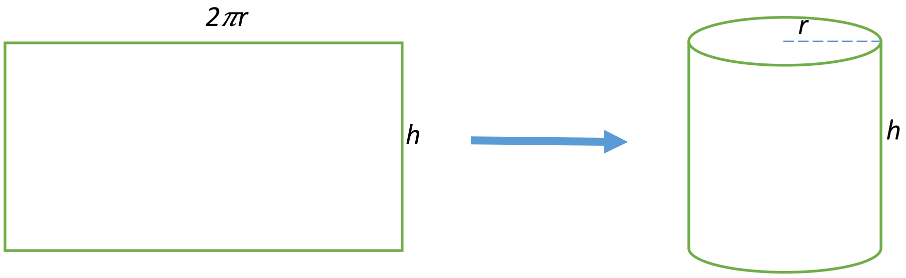


Surface area of cube

$= 6 \times a \times a$

$= 6a^2$

14. The lateral surface of a cylinder is also called the curved surface. If we rolled up a piece of rectangular sheet, the rectangular sheet will become the curved surface of the cylinder.



Area of rectangle = curved surface area of the cylinder

$= 2\pi rh$

Hence, total surface area of a solid cylinder = area of curved surface + 2 × area of base

$= 2\pi rh + 2\pi r^2$

Mass and Density

15. The density of a substance is the mass of one unit volume of the substance. It is calculated using the formula:

$$Density = \frac{Mass}{Volume}$$

Chapter 15: Data Handling

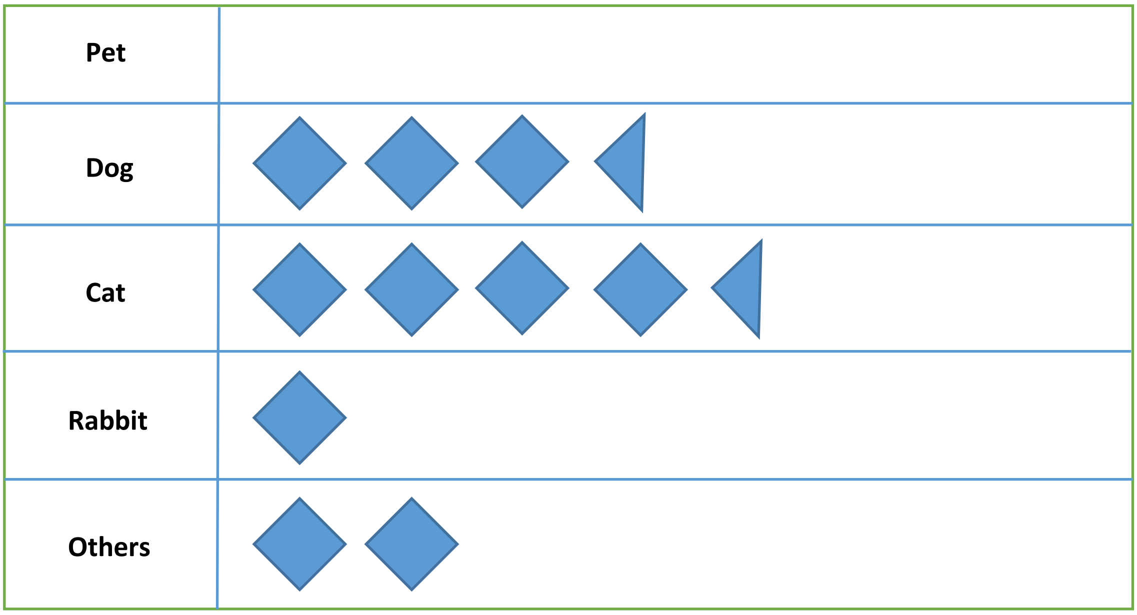
1. Statistics is a science of collecting, organising, interpreting and analysing data in order to assist the user to make decisions.

2. A survey on the pets owned by a group of 44 children was conducted and its result were tallied as shown below.

Pet Name	Tally	Frequency
Dog	~~////~~ ~~////~~ ////	14
Cat	~~////~~ ~~////~~ ~~////~~ ///	18
Rabbit	////	4
Others	~~////~~ ///	8

In the table, the number of times each pet appears is called as **frequency**.

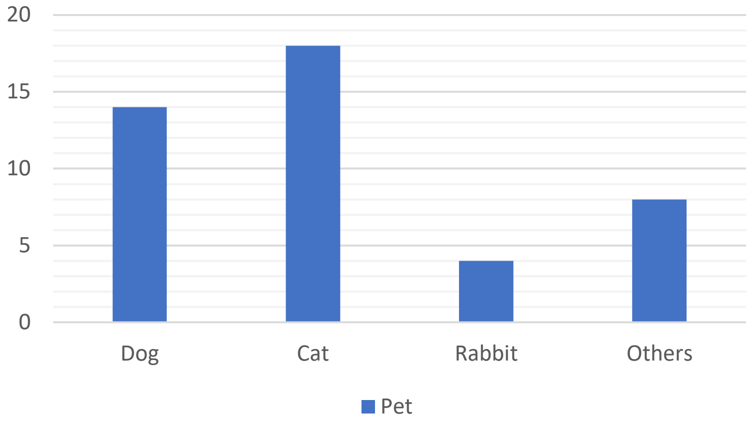
We can present the table of data using a **pictogram** shown below.



Key: Each ◆ represents 4 animals.

3. A **pictogram** is statistical diagram that uses pictures to represent data. It gives a quick comparison of different categories. However, it is not suitable for large quantities of data.
Note: When drawing a pictogram, you must remember to include a key to explain what the individual symbol represents.

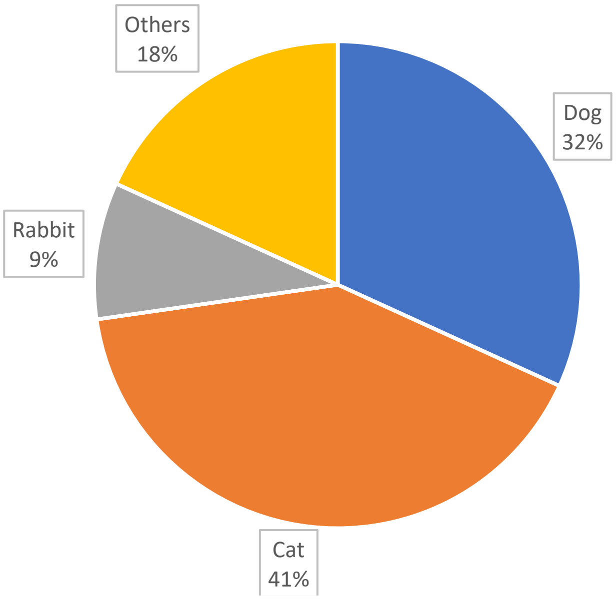
4. The information about the pets owned by 48 children can be represented by a horizontal or a vertical **bar graph**.



Vertical bar graph

5. A **pie chart** uses sectors of a circle to represent relative quantities. It compares the proportions of a whole instead of the actual numerical values. However, it is not suitable to display too many categories. Also, we need to calculate the angle size of each sector before drawing a pie chart.

6. The following pie chart shows the number of pets owned by 44 children.

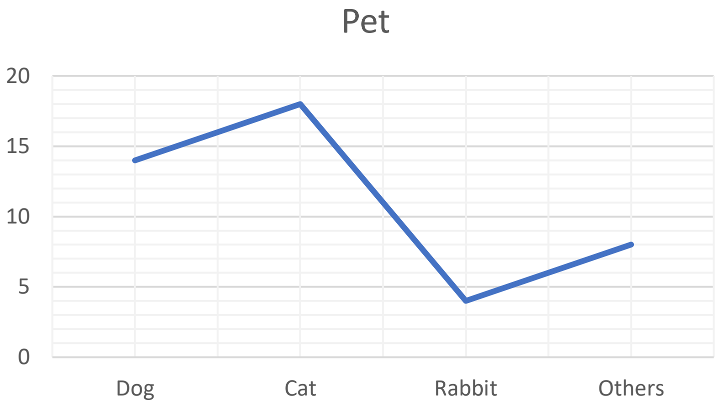


Observe that

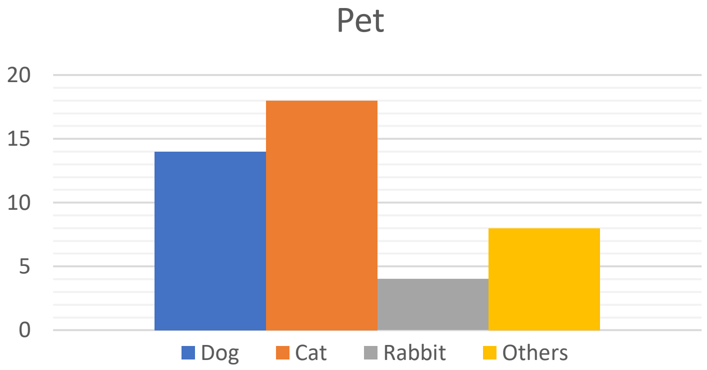
a) Each sector corresponds to the percentage of pets for that category.

b) The angle of each sector is proportional to the number of pets for that category.

7. A **line graph** is drawn by plotting the frequencies at various points and then joining the points with line segments. This is particularly useful when we wish to show trends over time.



8. A **histogram** is a vertical bar graph with no space in between the bars. The area of each bar is proportional to the frequency it represents.



Vertical Histogram

In Summary

A **bar graph** is used to **compare quantities.**

A **pie chart** is used to **compare proportions.**

A **line chart** is used to **show trends.**

A **histogram** is similar to bar graph except it has **no gaps between the bars** and **the area is proportional to the frequency.**

For Sec 2 Students

Chapter 1: Map Scales

Scale Factor

a. Transformation resulted in a figure that has the **same shape as original, but a different size.**

b. Transformation can be divided into two: enlargement and reduction.

c. Scale factor: The ratio of a pair of corresponding sides or the enlarged or reduced figure to the original figure.

$$\boxed{\textit{Scale factor} = \frac{\textit{Length of a side of the enlarged or reduced figure}}{\textit{Length of the corresponding side of the original figure}}}$$

d. Scale factor = 1, the transformed figure and its original is the same.

Scale factor > 1, the transformed figure is bigger than its original size.

Scale factor < 1, the transformed figure is smaller than its original size.

Length Scale

The scale of a drawing can be expressed as a ratio of $1 : n$

Example: 1:500 means 1 cm on the drawing represent 500 cm on the actual object.

Area Scale

If the map scale is 1 : n, the area scale will be $1 \times 1 : n \times n$.

Example

A football field map was drawn with a scale of 1 : 1500.

Scale = 1cm : 15 m

$$\text{Area Scale} = (1\times1)\text{ cm}^2 : (15 \times 15)\text{ m}^2$$

$$= 1\text{ cm}^2 : 225\text{ m}^2$$

Chapter 2: Direct and Inverse Proportion

Direct Proportion

2 quantities x and y are said to be **directly proportional** to each other if $x = ky$, where k is a constant.

Example

A is directly proportional to B, given that $A = 125$ when $B = 5$, find an equation connecting A and B

$$A = kB$$
$$125 = k \times 5$$
$$k = 25$$
$$A = 25B$$

Inverse Proportion

2 quantities X and Y are said to be **inversely proportional** to each other if $x = \frac{k}{y}$, where k is a constant.

Example

(i) A is inversely proportional to B, given that $A = 125$ when $B = 5$, find an equation connecting A and B

$$A = \frac{k}{B}$$
$$125 = \frac{k}{5}$$
$$k = 625$$
$$A = \frac{625}{B}$$

Chapter 3: Algebraic Expansion and Factorisation

Algebraic Expansion

Type 1: Expand and Simplify (Rainbow Method)

a) $a(b+c)=ab+ac$

Example

$$x(5-y)$$

$$=x(5)+x(-y)$$

$$=5x-xy$$

Take note that when we remove the bracket, the signs are included in each multiplication.

Type 2: Expand with formula (Algebraic Identity)

a) $(a+b)^2=a^2+2ab+b^2$

b) $(a-b)^2=a^2-2ab+b^2$

c) $(a+b)(a-b)=a^2-b^2$

Example 1

$(x+2)^2=x^2+4x+4$

Example 2

$(x-2)^2=x^2-4x+4$

Example 3

$(x+2)(x-2)=x^2-4$

Algebraic Factorisation

Type 1: Taking out Highest Common Factor (HCF)

To factorise the algebraic expression given, we need to identify the common factor.

Example 1

$$5x-xy$$

$$=x(5-y)$$

Example 2

$$a^2b+2ab-ab^2$$

$$=ab(a+2-b)$$

Type 2: Group Factorisation

We need to group the terms to find the common factors.

In some situations, we may need to change the operator in order to find the common factor.

Example 1

$$2x^2-8x-4+x$$

$$=2x(x-4)-1(4-x)$$

$$=2x(x-4)+1(x-4)$$

$$=(2x+1)(x-4)$$

Example 2

Factorise $2x+2y+ax+ay$ completely.

$$2x+2y+ax+ay$$

$$=2(x+y)+a(x+y)$$

$$=(2+a)(x+y)$$

Type 3: Factorisation by formula (Algebraic Identity)

$$a^2 + 2ab + b^2 = (a+b)^2$$

$$a^2 - 2ab + b^2 = (a-b)^2$$

$$a^2 - b^2 = (a+b)(a-b)$$

Example 1

Factorise $x^2 + 18x + 81$ completely.

$$x^2 + 18x + 81$$

$$= x^2 + 2(x)(9) + (9)^2$$

$$= (x+9)^2$$

Example 2

Factorise $x^2 - 8x + 16$ completely.

$$x^2 - 8x + 16$$

$$= x^2 - 2(x)(4) + (4)^2$$

$$= (x-4)^2$$

Example 3

Factorise $x^4 - 81$ completely.

$$x^4 - 81$$

$$= (x^2)^2 - (9)^2$$

$$= (x^2+9)(x^2-9)$$

$$= (x^2+9)(x^2-3^2)$$

$= (x^2+9)(x+3)(x-3)$ (This is a complete factorisation.)

Type 4: Quadratic Factorisation

Example 1

Factorise a^2+5a+6

Observe that

a) the quadratic term, a^2, is the product of a and a;
b) the coefficient of x, 5, is the sum of 2 and 3; and
c) the constant term, 6, is the product of 2 and 3.

Using cross-multiplication method to factorise a^2+5a+6, follow the following steps.

Step 1: Factorise the quadratic term, a^2, into two linear factors.

$$a^2=a\times a$$

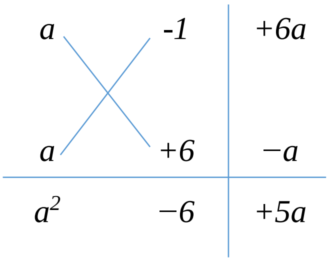
Step 2: Factorise the constant term, 6, into product of two numbers.

$$6=1\times 6$$

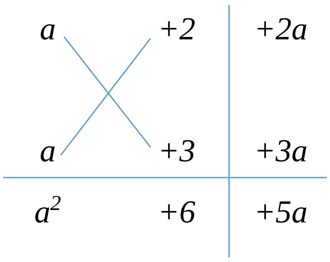
$$6=2\times 3$$

Step 3: Place the factors of the quadratic term and the constant term in two columns.

Cross multiply the factors and write the products in the third column. Add up the products in the third column to see which pair of factors gives +5a as the middle term.



a	-1	$+6a$
a	$+6$	$-a$
a^2	-6	$+5a$



a	$+2$	$+2a$
a	$+3$	$+3a$
a^2	$+6$	$+5a$

When we consider $6=1\times 6$, the sign for 6 will be negative and is different from the algebraic expression required, therefore $(a-1)(a+6)$ cannot be the factorisation of a^2+5a+6.

$$\therefore a^2+5a+6=(a+2)(a+3)$$

Chapter 4: Algebraic Simplification

Type 1: Algebraic Simplification by Cancellation

Example 1

$$\frac{\cancel{x}y}{x^{\cancel{2}}} = \frac{y}{x}$$

$$\searrow x \times x$$

Type 2: Algebraic Simplification by Cancellation – Factorisation

Example 1

$$\frac{ac+bc+ad+bd}{a^2-b^2} = \frac{c(a+b)+d(a+b)}{(a+b)(a-b)}$$

$$= \frac{\cancel{(a+b)}(c+d)}{\cancel{(a+b)}(a-b)}$$

$$= \frac{(c+d)}{(a-b)}$$

Example 2

$$\frac{2x^2-3x-5}{(2x-5)^2} = \frac{\cancel{(2x-5)}(x+1)}{(2x-5)^{\cancel{2}}}$$

$$= \frac{(x+1)}{(2x-5)}$$

$2x$	-5	$+2x$
x	$+1$	$-5x$
$2x^2$	-5	$-3x$

Chapter 5: Algebraic Fractions and Making Subject

Type 1: Algebraic Fractions

Example 1

$$\frac{3}{(a+b)}-\frac{a}{(a+b)^2}=\frac{3(a+b)}{(a+b)^2}-\frac{a}{(a+b)^2}$$
$$=\frac{3a+3b-a}{(a+b)^2}$$
$$=\frac{2a+3b}{(a+b)^2}$$

Example 2

$$\frac{3}{a-b}+\frac{1}{b-a}=\frac{3}{a-b}-\frac{1}{a-b}$$ (Change sign to make denominator same)
$$=\frac{2}{a-b}$$

Type 2: Algebraic Fractions with Factorisation

Example 1

$$\frac{5}{x^2+3x}-\frac{2}{x+3}=\frac{5}{x(x+3)}-\frac{2}{x+3}$$
$$=\frac{5}{x(x+3)}-\frac{2x}{x(x+3)}$$
$$=\frac{5-2x}{x(x+3)}$$

Example 2

$$\frac{5}{x^2+3x}+\frac{1}{x^2-9}=\frac{5}{x(x+3)}+\frac{1}{(x+3)(x-3)}$$
$$=\frac{5(x-3)}{x(x+3)(x-3)}+\frac{x}{x(x+3)(x-3)}$$
$$=\frac{5x-15+x}{x(x+3)(x-3)}$$
$$=\frac{6x-15}{x(x+3)(x-3)}$$
$$=\frac{3(2x-5)}{x(x+3)(x-3)}$$

Type 3: Making Subject

Example 1

Make x the subject,

$$2ax+b=3c$$

$$2ax=3c-b$$

$$x=\frac{3c-b}{2a}$$

Example 2

Make b the subject,

$$x=\frac{\sqrt{b^2-4ac}}{2a}$$

$$2ax=\sqrt{b^2-4ac}$$

$$(2ax)^2=b^2-4ac$$

$$4a^2x^2=b^2-4ac$$

$$4a^2x^2+4ac=b^2$$

$$b^2=4a^2x^2+4ac$$

$$b=\pm\sqrt{4a^2x^2+4ac}$$

Chapter 6: Quadratic Equations

Type 1: Basic Concept of Quadratic Equations

$\text{Any Number} \times 0 = 0$

$0 \times \text{Any Number} = 0$

If $A \times B = 0$, then either $A = 0$ or $B = 0$

$x(x+1)=0$	$(x-2)(x+1)=0$
$x=0$ or $x+1=0$	$x-2=0$ or $x+1=0$
$x=0$ or $x=-1$	$x=2$ or $x=-1$

Type 2: Fraction Form

Cross Multiplication Method

$$\frac{a}{b} = \frac{c}{d}$$

$$ad = bc$$

Example 1

$$\frac{x+7}{6} + \frac{4}{x-3} = 0$$

$$\frac{x+7}{6} = -\frac{4}{x-3}$$

$$(x+7)(x-3) = -24$$

$$x^2 + 4x - 21 = -24$$

$$x^2 + 4x + 3 = 0$$

$$(x+3)(x+1) = 0$$

$$x+3=0 \; or \; x+1=0$$

$$x=-3 \qquad x=-1$$

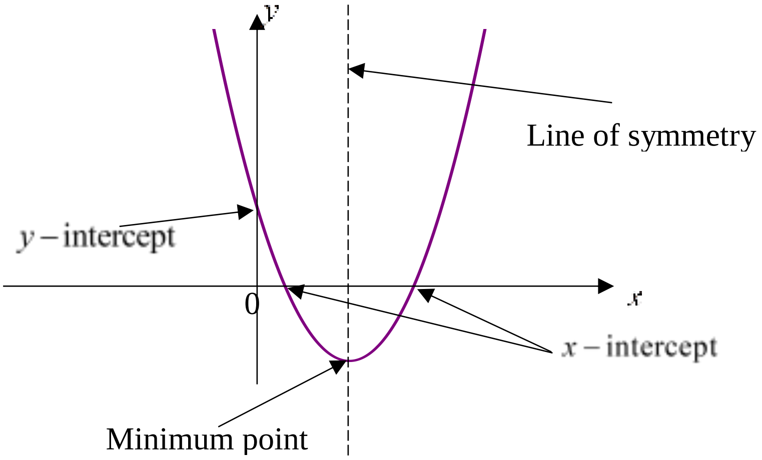
Chapter 7: Quadratic Graphs

1. The general form for the equation of a quadratic function is $y = ax^2 + bx + c$, where a , b and c are constants and a is not equal to 0.

2. The graph of a quadratic equation is called a parabola.

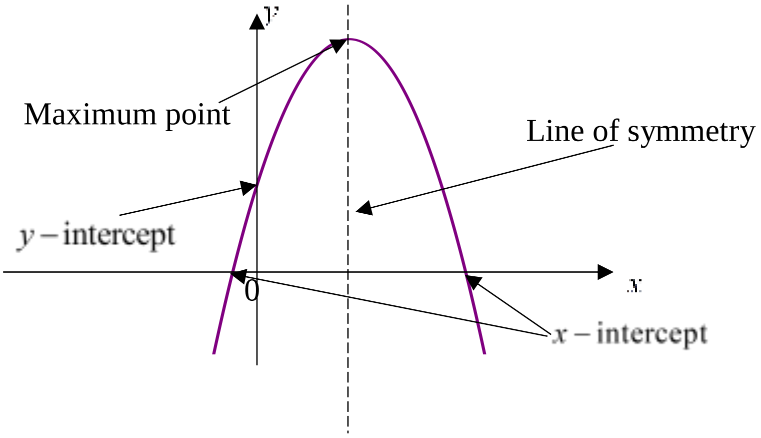
3. The graph of $y = ax^2 + bx + c$ passes through the point $(0, c)$ where c is the y-intercept.

4. a) When a is positive $(a > 0)$,



- The quadratic curve is an **upward** ‘U’ shape.
- The graph opens upwards indefinitely and has a **minimum point**.

b) When a is positive $(a < 0)$,



- The quadratic curve is an **downward** ‘U’ shape.
- The graph opens downwards indefinitely and has a **maximum point**.

5. The line of symmetry of the graph is the vertical line that passes through its minimum / maximum point.

Example 1

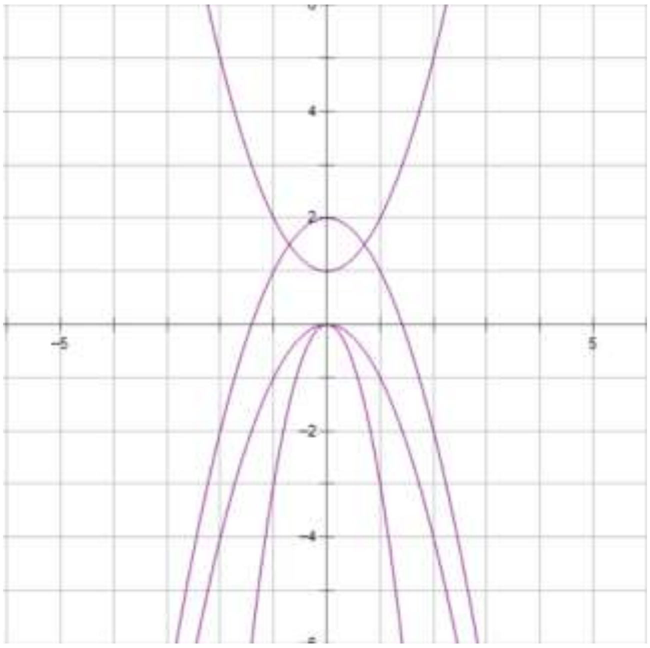
Match the possible equation to each graph by writing down the letter corresponding to the equation. Use each letter once only.

a) $y = -x^2$

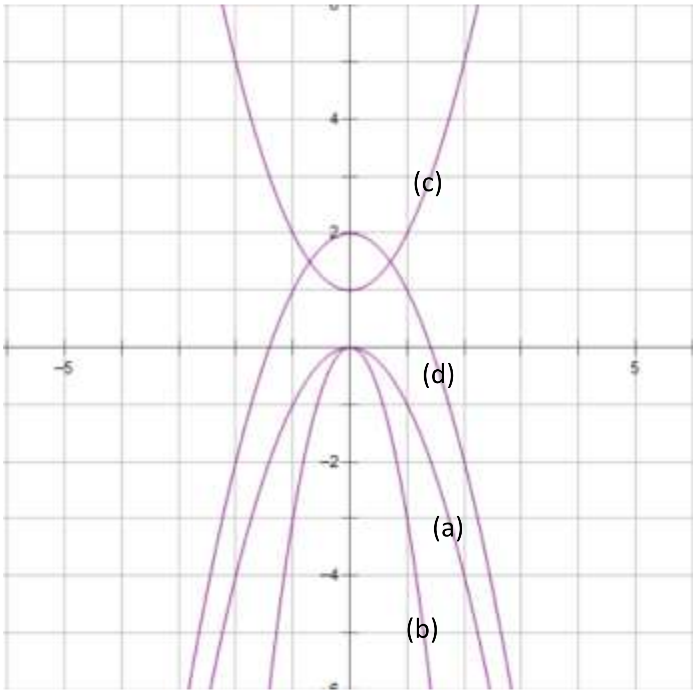
b) $y = -3x^2$

c) $y = x^2 + 1$

d) $y = -x^2 + 2$



Solution



Chapter 8: Simultaneous Equations

Elimination

Solve the following pairs of Simultaneous Equations by Elimination.

$2x+3y=-1$

$3x-4y=7$

Solutions:

$2x+3y=-1$ ------ (1)

$3x-4y=7$ ----- (2)

(1) × 3

$6x+9y=-3$ ----- (3)

(2) × 2

$6x-8y=14$ ----- (4)

(3) − (4) Be Careful

$9y-(-8y)=-3-14$

$17y=-17$

$y=-1$

Sub $y=-1$ into (1)

$2x+3(-1)=-1$

$2x-3=-1$

$2x=2$

$x=1$

$x=1,\ y=-1$ (Ans)

Substitution

$$2x+3y=-1$$
$$3x-4y=7$$

Solutions:

$2x+3y=-1$ ------ (1)

$3x-4y=7$ ----- (2)

From (1)

$$2x=-1-3y$$

$x=\frac{-1-3y}{2}$ ----- (3)

Sub (3) into (2)

$$3\left(\frac{-1-3y}{2}\right)-4y=7$$

$$3(-1-3y)-8y=14$$

$$-3-9y-8y=14$$

$$-17y=17$$

$$y=-1$$

Sub $y=-1$ into (3)

$$x=\frac{-1-3(-1)}{2}$$

$$=\frac{-1+3}{2}$$
$$=\frac{2}{2}$$
$$=1$$

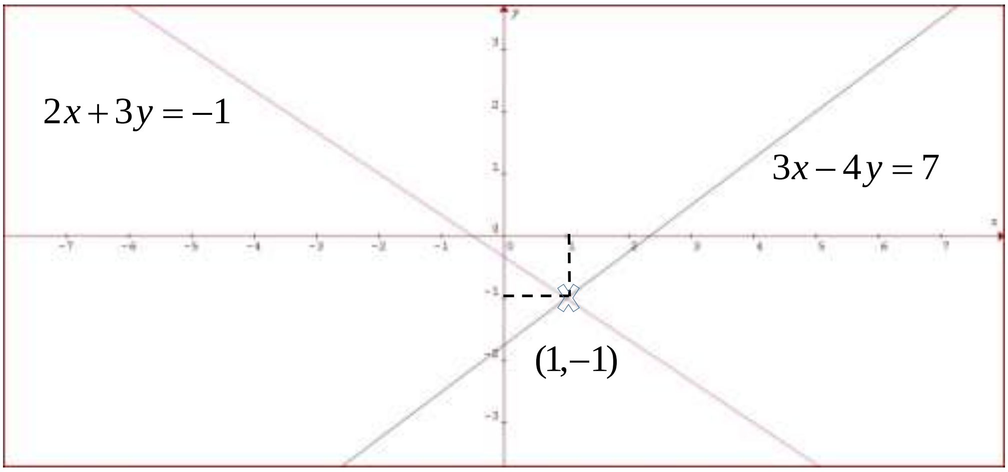
$x=1,\ y=-1$ (Ans)

Graphical Method

Solve the following pairs of Simultaneous Equations graphically.

$2x+3y=-1$

$3x-4y=7$



Ans: $x=1,\ y=-1$

Chapter 9: Financial Mathematics

Simple Interest

For a sum of money (**Principal** sum), **P**, deposited in a bank at **R**% interest per annum for **T** years, the **simple interest** (**I**) is given by:

$$\boxed{I = \frac{PRT}{100}}$$

Compound Interest

Compound interest can also be computed half-yearly, quarterly or monthly. In this situation, we need to find out the interest rate per half year, per quarter or per month.

$$\boxed{Amount = P\left(1+\frac{R}{100}\right)^{n}}$$

where for a sum of money (**Principal** sum), **P**, deposited in a bank at **R**% interest per annum for **n**, the number of times compounded according to the time span given in a question.

Money Exchange

Exchange rate = Home currency ÷ New currency

Example

The exchange rate between the US dollars (US$) and the Singapore dollars (S$) on a certain day was US $1 = S $1.3542

Convert US $200 to Singapore dollars:

Solution: US $200 = S $1.3542 × 200 = S $270.84

Convert S $500 to US dollars:

Solution: S $500 ÷ 1.3542 = US $369.22

Income Tax:

Chargeable Income = Assessment Income – Personal Relief

Assessment Income = Annual Income – Donations

Example

Alan’s chargeable income was $75,000. Using the table for Year of Assessment 2017 (below), calculate Alan’s income tax payable.

Solution: (The computation does not include reliefs and donations)

Chargeable Income	Income Tax Rate (%)	Gross Tax Payable ($)
First $40,000	-	550
Next $40,000	7	2,800

$$\text{Income tax payable} = 550 + \frac{7}{100} \times (75{,}000 - 40{,}000)$$

$$= 550 + 2{,}450$$

$$= \$3{,}000 \quad \text{(Ans)}$$

Hire Purchase

We can buy goods by paying for it immediately (cash payment) or by spreading the payment over a period of time.

To buy on hire-purchase, often a deposit or initial down payment is required. The loan plus interest is then calculated and paid through regular instalments.

Total Interest = Loan × Flat Rate x Loan Period (in years)

Repayment Amount = Loan + Total Interest

Monthly Repayment (Instalment) = Repayment Amount ÷ Loan Period (in months)

Chapter 10: Speed-Time Graph

- Definitions

Initial = at the beginning	$t = 0$
Instantaneously at rest	$v = 0$
Stationary	$v = 0$,
Constant velocity	$a = 0$

1. Velocity is the rate of change of displacement or distance in a particular direction with respect to time. (Can be Negative → Going in Negative Direction)

Formula :

$$\boxed{\text{Velocity} = \frac{\text{distance moved in a given direction}}{\text{time taken}}}$$

Positive / negative velocity = the object is moving forward / backward respectively

2. Speed is the rate of change of distance. (Cannot be Negative)

Formula :

$$\boxed{\text{Speed} = \frac{\text{distance travelled}}{\text{time taken}}}$$

3. Average speed

Formula :

$$\boxed{\text{Average Speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}}}$$

4. Acceleration is the rate of change of velocity with respect to time.

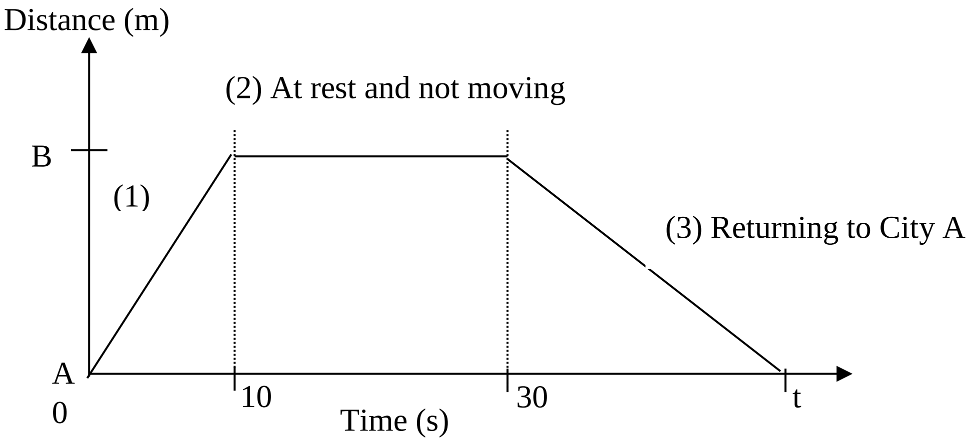
Formula :

$$\text{Acceleration} = \frac{\text{change in velocity}}{\text{time taken}}$$

$$\text{Acceleration} = \frac{\text{final velocity - initial velocity}}{\text{time taken}}$$

Distance-time

The diagram shows a distance-time graph of a car travelling from City A to City B.

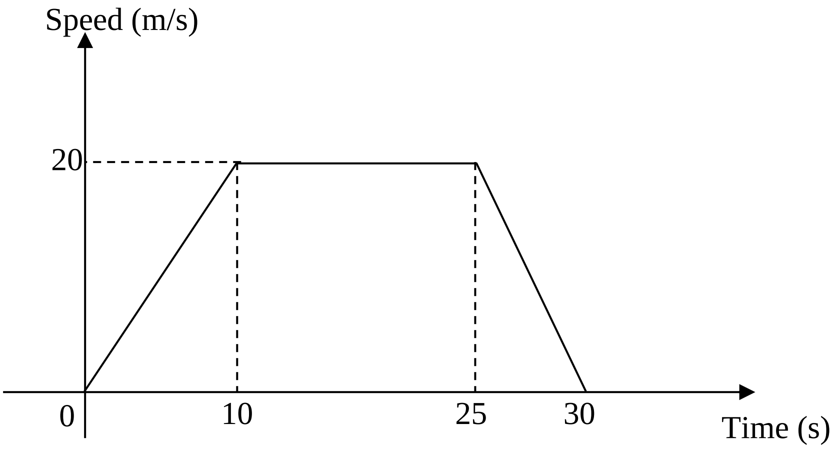


The gradient of a distance–time graph gives the **speed**.

Sign: (1) Positive (+) gradient indicates car travelling away from City A to City B.
(2) Zero gradient indicate car is at rest and not moving.
(3) Negative (–) gradient indicates car is returning towards City A.

Speed-time

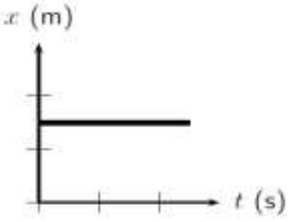
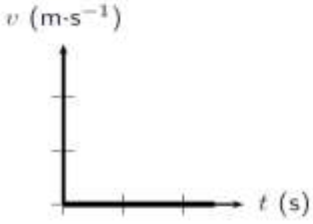
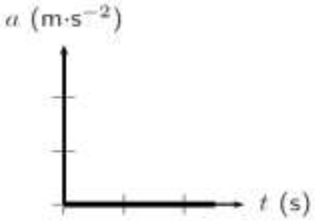
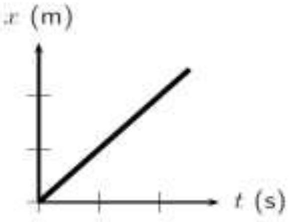
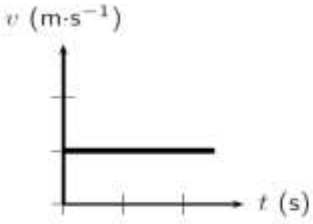
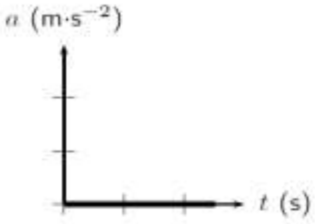
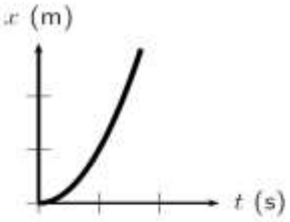
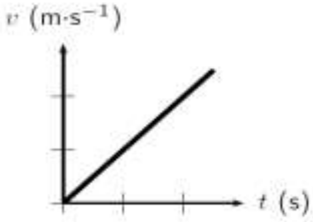
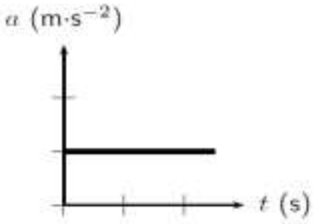
The diagram shows a speed-time graph of a car



The gradient of a speed–time graph gives the **acceleration**.

The area under a speed–time graph gives the **distance travelled**.

$$\text{Average Speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

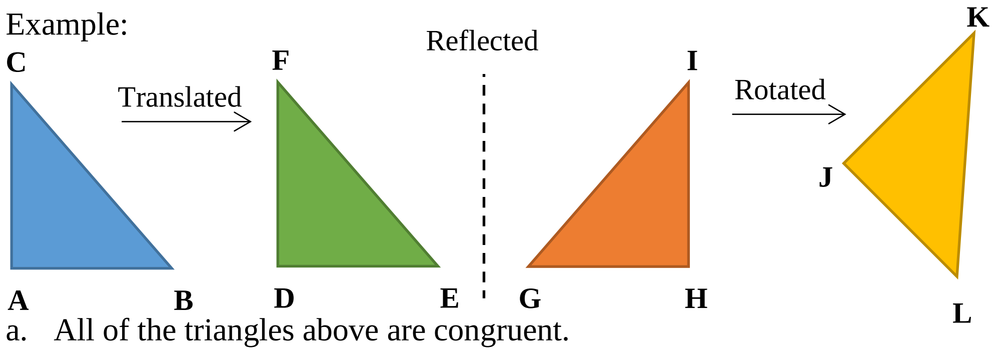
	Distance–time graph	**Speed–time graph**	**Acceleration–time graph**
Stationary object			
Uniform motion			
Motion with constant acceleration			

Chapter 11: Congruence And Similarity

Congruence

Two figures are said to be **congruent** if they are **identical in shape and size.**

Example:



a. All of the triangles above are congruent.

b. Congruency statement: $\Delta ABC \equiv \Delta\ DEF \equiv \Delta\ HGI \equiv \Delta\ JLK$ ('≡' denotes 'is congruent to').

When we write congruency statement, the letters representing the vertices of the shape have to be written in marching pairs.

Example:

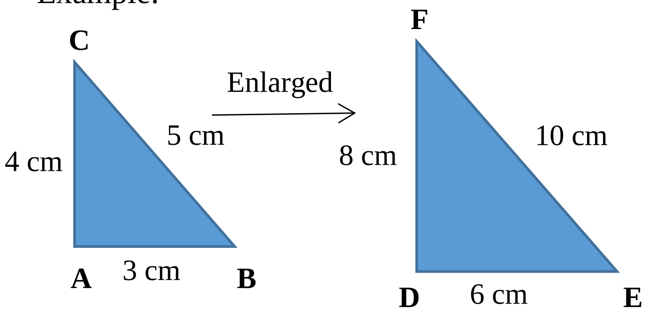
$\Delta ABC \equiv \Delta\ JLK$

This means that A and J, B and L, C and K are corresponding vertices.

Similarity

Two figures are said to be **similar** if they have the **same shape but different sizes.**

Example:



a. The two triangles above are similar but not congruent due to the different sizes.

b. Similarity statement: ΔABC is similar to $\Delta\ DEF$. When we write similarity statement, the letters representing the vertices of the shape have to be written in marching pairs.

For example: ΔABC is similar to $\Delta\ DEF$

c. The corresponding angles of similar figures are equal.

Example: ΔABC is similar to $\Delta\ DEF$

$\angle BAC = \angle EDF,\ \angle ABC = \angle DEF, \angle ACB = \angle DFE$

d. The ratios of corresponding sides of similar figures are equal.

Example: ΔABC is similar to $\Delta\ DEF$.

Therefore:

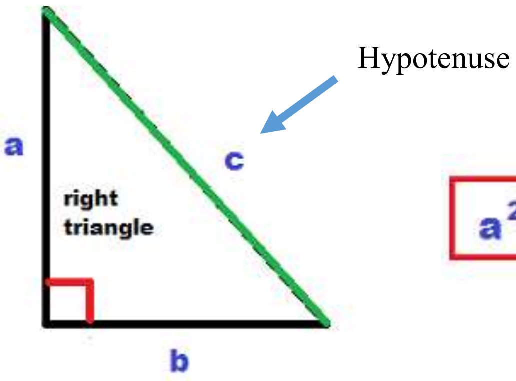
$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$$

$$\frac{3\text{ cm}}{6\text{ cm}} = \frac{4\text{ cm}}{8\text{ cm}} = \frac{5\text{ cm}}{10\text{ cm}} = \frac{1}{2}$$

e. The above mentioned rules are applicable to all polygons.

Chapter 12: Pythagoras Theorem and Trigonometry

Pythagoras Theorem



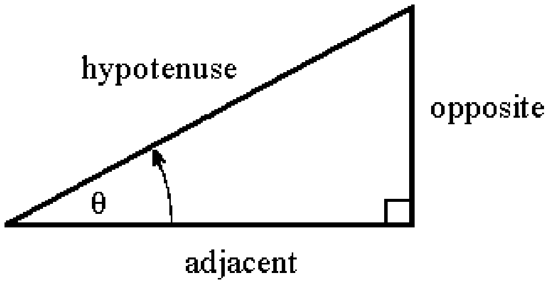
$$a^2 + b^2 = c^2$$

Trigonometry

$$\sin\theta = \frac{opposite}{hypotenuse} \quad \text{SOH}$$

$$\cos\theta = \frac{adjacent}{hypotenuse} \quad \text{CAH}$$

$$\tan\theta = \frac{opposite}{adjacent} \quad \text{TOA}$$

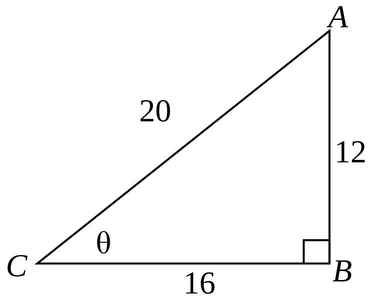


Example

In the right-angled triangle ΔABC , AB = 12 cm, BC = 16 cm and AC = 20 cm.

Find the values of

(a) $\tan\theta$,
(b) $\cos\theta$,
(c) $\sin\theta$,
(d) $\angle\theta$.



$$(a)\ \tan\theta = \frac{12}{16} = \frac{3}{4}$$

$$(b)\ \cos\theta = \frac{16}{20} = \frac{4}{5}$$

$$(c)\ \sin\theta = \frac{12}{20} = \frac{3}{5}$$

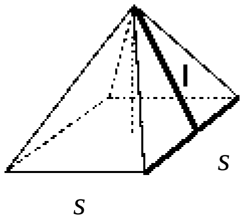
(d) $\angle\theta = \tan^{-1}\left(\frac{3}{4}\right) = 36.9°$

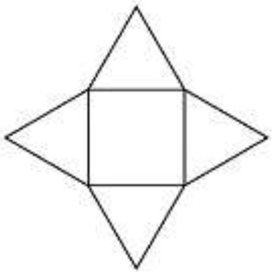
Chapter 13: Measurement

Pyramid

A pyramid is a solid that has a base with a perpendicular vertex and slant lateral faces.

The base can be a triangle, a square or a rectangle.





Nets of pyramid

Volume = $\frac{1}{3} \times base\ area \times h$

$Base\ area$ = s^2

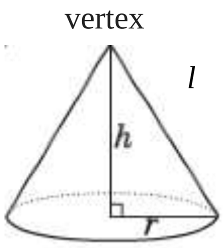
$$Surface\ area = \frac{1}{2} \times s \times l \times 4\ sides$$
$$= 2sl$$

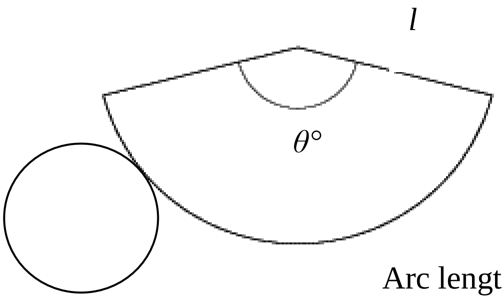
Surface area of pyramid = base area + total area of slant Δ faces

$= s^2 + 2sl$

Cone

A cone is a solid with a circular base and a vertex.





Arc length = circumference of circular base

Curved surface area = πrl

Volume = $\frac{1}{3}\pi r^2 h$

Surface area of a cone = Area of circle + Area of curved surface

$= \pi r^2 + \pi rl$

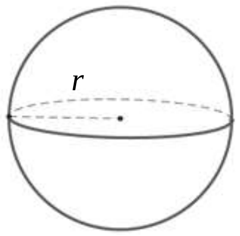
r = radius

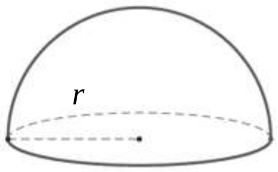
l = length of slant height = $\sqrt{r^2 + h^2}$

Sphere and Hemisphere (half-sphere)

Every point on the surface of a sphere is equidistance from the centre.

A hemisphere is half a sphere.





Volume = $\frac{4}{3}\pi r^3$

Surface area of a sphere = $4\pi r^2$

Volume = $\frac{1}{2}\times$ volume of sphere $=\frac{2}{3}\pi r^3$

Surface area = $\frac{1}{2}\times$spherical surface area + area of circle

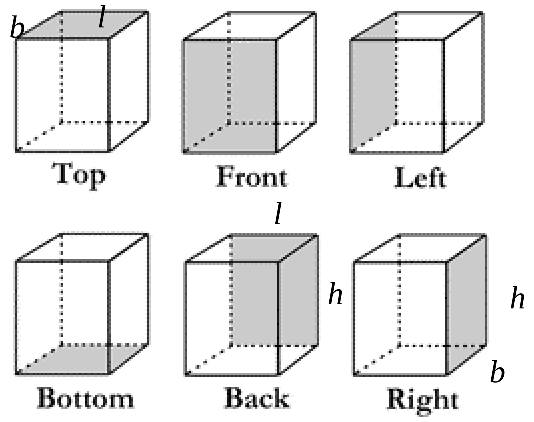
$$=2\pi r^2+\pi r^2=3\pi r^2$$

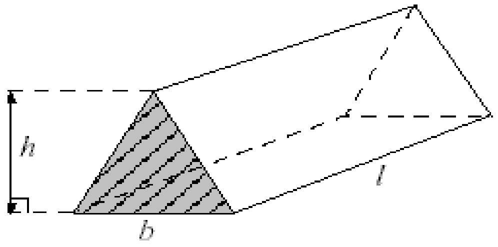
Prism

Volume = base area × height

Or area of cross-section × length

Surface area of a prism = Area of all the faces



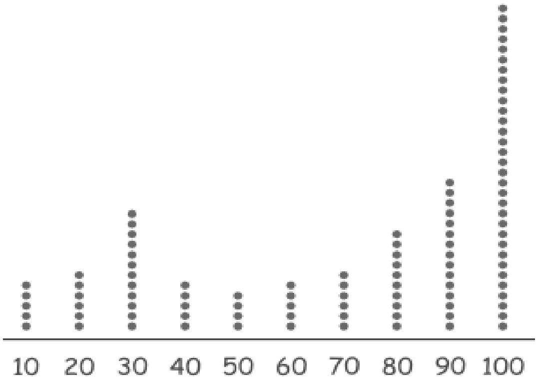


Volume = $=\frac{1}{2}\times b\times h\times l$

Chapter 14: Data Analysis

Dot Diagram

A dot diagram consists of a horizontal number line and dots placed above the corresponding number line value. The number of dots above each value indicates how many times each value occurred.



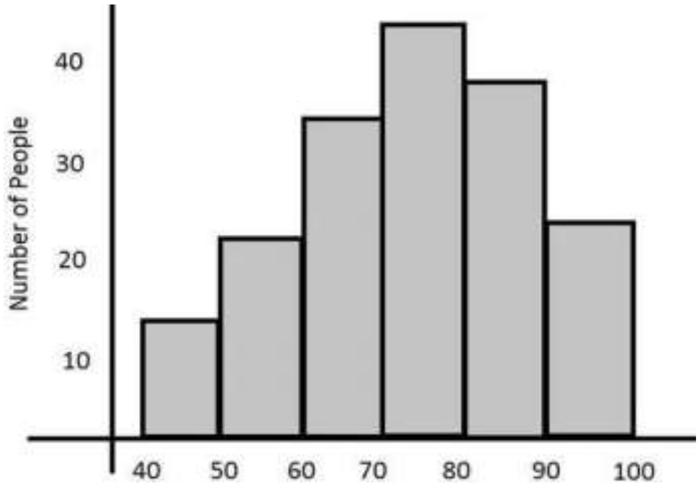
A sample of dot diagram   Scores of students in a common test

Histogram

We use histogram to represent grouped data with class interval.

A histogram consists of rectangles whose area are proportional to the frequency.

Grades on the Math Test



Grade on the test

The data is grouped into different class interval

There is no spacing between the different rectangles.

The area of the rectangles are proportional to the frequencies.

Stem and Leaf

Stem-and-leaf diagram is used to organize a large amount of data.

In a stem and leaf diagram, each value is split into two parts, the stem and the leaf.

The leaf of a number is usually the last digit in a number.

The stem is the remaining digit or digits to the left.

Title: Masses of marbles

Stem	Leaf
5	0 6 8
6	1 3 5 7
7	2 4 6

Key: 5 | 8 means 58 g

Advantages and disadvantages of different forms of statistical representation:

The choice of statistical representation depends on the type of data collected and the purpose of collecting the data.

Dot diagram

An advantage is that it is an easy way to display small sets of data that do not contain many distinct values.

A disadvantage is that if a set of data is too large, the diagram may appeared packed.

Histogram

An advantage is that the data sets with the highest and the lowest frequencies can be easily determined.

Stem-and-leaf diagram

An advantage is that individual data values are retained.

The shape of the data distribution can be easily observed if appropriate units are used.

A disadvantage is that it is not useful if there are extreme values in a set of data as many stems with no leaves will be present.

Measures of central tendency

A measure of central tendency refers to a value that indicates the approximate centre of a set of data. The 3 most common measures of central tendency are the mean, median, mode.

MEAN

1. The mean of a data set is the sum of all the data values divided by the total number of data.

$$Mean = \frac{Sum\ of\ all\ data}{Total\ number\ of\ data}$$

MEDIAN

1. The middle value of a set of ordered numbers (ascending or descending) is the median.

2. For a set of *n* data arranged in ascending or descending order
   If the total number of data is odd, the median is the middle value.
   If the total number of data is even, the median is the mean of the 2 middle values.
   The middle position of a set of data $= \frac{n+1}{2}$

MODE

1. Mode of a data set is the value which occurs the most frequently (i.e. the data that has the highest frequency) or

2. Modal class is the class interval with the highest frequency.

Example

Consider the data: 1, 2, 3, 4, 5, 5,

Mean = $\frac{1+2+3+4+5+5}{6}$

$= \frac{10}{3}$

Median = $\frac{3+4}{2}$

= 3.5

Mode = 5

Chapter 15: Probability

Introduction of Probability

Probability is a measure of chance.

The probability of an event, A is:

$$\boxed{P(A) = \frac{k}{m}}$$

Where k is the number of outcomes of A while m is the total number of possible outcomes.

When the probability is 0, it is impossible to happen.

When the probability is 1, it will certainly happen.

$0 \le \text{Probability} \le 1$

Mutually Exclusive Events and Addition Law

Two events are called mutually exclusive events if they **cannot occur at the same time.**

Eg. In tossing a coin, the event A of getting 1 head and the event of event B of getting 1 tail are exclusive events. They cannot happen at the same time.

Addition Law

If A and B are exclusive events, then the probability that either A or B occurring is given by

$$P(A \text{ or } B) = P(A) + P(B)$$

Example

A bag contains 4 red balls, 5 green balls and 6 yellow balls. If a ball is drawn at random from the bag, find the probability that the ball is either green or yellow.

Total of balls in the bags

$= 4 + 5 + 6$

$= 15$

P(ball is either green or yellow)

= P(ball is green) + P(ball is yellow)

$= \frac{5}{15} + \frac{6}{15}$

$= \frac{11}{15}$

Independent Events and Multiplication Law

Two events are called independent events if the occurrence of one event does not affect the probability of occurrence of the other event.

Eg. In tossing a coin and a die, the event A that the coin is a head and the event B that the number on the die is even are independent events.

Multiplication Law

If A and B are independent events, then the probability that both A and B occurring is given by

$$P(A \text{ and } B) = P(A) \times P(B)$$

Example

A bag contains 4 red balls and 6 green balls. Two balls are drawn from the bag one by one with replacement. Find the probability that

i) both balls are red,

ii) one ball is red and one ball is green.

i)

P(both balls are red)

= P(1st ball is red) × P(2nd ball is red)

$$= \frac{4}{10} \times \frac{4}{10}$$

$$= \frac{16}{100}$$

$$= \frac{4}{25}$$

ii) P(one ball is red and one ball is green)

= P[(1st ball is red and 2nd ball is green) or (1st ball is green and 2nd ball is red)]

$$= \left(\frac{4}{10} \times \frac{6}{10}\right) + \left(\frac{6}{10} \times \frac{4}{10}\right)$$

$$= \frac{24}{100} + \frac{24}{100}$$

$$= \frac{48}{100}$$

$$= \frac{12}{25}$$

For Sec 3 Students

Chapter 1: Indices

a^n means the n^{th} power of a, where $a > 0$ and n is a positive integer

a is known as base and n is the index or power

$$a^m = \underbrace{a \times a \times a \ldots \times a}_{\text{m factors}}$$

Example: 10 000 = $10 \times 10 \times 10 \times 10 = 10^4$

$2^6 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$

Indices Law

(a, b, m and n are positive integers)

Same base

1. $a^m \times a^n = a^{m+n}$
2. $a^m \div a^n = a^{m-n}$

Same index but different base

3. $a^m \times b^m = (a \times b)^m$
4. $a^m \div b^m = \left(\dfrac{a}{b}\right)^m$
5. $\left(a^m\right)^n = a^{mn}$
6. $a^0 = 1$ *any number to the power of 0 is 1*
7. $a^1 = a$ *no need to show when index is 1*
8. $a^{-m} = \dfrac{1}{a^m}$ *negative index* → *division*
9. $\left(\dfrac{a}{b}\right)^{-m} = \left(\dfrac{b}{a}\right)^m$
10. $a^{\frac{1}{m}} = \sqrt[m]{a}$ *when index is a fraction* → *root form*
11. $a^{\frac{m}{n}} = \left(a^{\frac{1}{n}}\right)^m = \sqrt[n]{a^m}$

Example

1.	$2^3 \times 2^6 = 2^9$
2.	$2^7 \div 2^2 = 2^5$
3.	$2^3 \times 5^3 = 10^3$
4.	$2^3 \div 5^3 = \left(\dfrac{2}{5}\right)^3$
5.	$(2^3)^4 = 2^{12}$
6.	$2^0 = 1$
7.	$2^1 = 2$
8.	$2^{-3} = \dfrac{1}{2^3}$
9.	$\left(\dfrac{2}{5}\right)^{-3} = \left(\dfrac{5}{2}\right)^3$
10.	$2^{\frac{1}{3}} = \sqrt[3]{2}$
11.	$2^{\frac{3}{5}} = 2^{3\times\frac{1}{5}} = \left(\sqrt[5]{2^3}\right)$

Indices Simplification

Example 1 Simplify $2^4 \times (3^2)^2 \div \left(5^{\frac{2}{3}}\right)^6$

$$= 2^4 \times 3^4 \div 5^4 \qquad\qquad a^m \times b^m = (a \times b)^m$$

Solving Indices Equations

Solving equations involving same base or same index

$$a^m = a^n \quad \Rightarrow \quad m = n$$
$$a^m = b^m \quad \Rightarrow \quad a = b$$

Example

Solve $125^{x+1} = 0.2$

$$\left(5^3\right)^{x+1} = \frac{1}{5}$$

$$5^{3x+3} = 5^{-1}$$

Comparing the power

$$3x + 3 = -1$$

$$x = -\frac{4}{3} \text{ (Ans)}$$

Example – Substitution

Solve $2^{2x+3} + 2^{x+2} + 3(2^x) = 1.$

Let U = 2^x , $2^{2x}2^3 + 2^x2^2 + 3\left(2^x\right) = 1$

$$8U^2 + 4U + 3U = 1$$
$$8U^2 + 7U - 1 = 0$$

$$(8U - 1)(U + 1) = 0$$

$$U = \frac{1}{8} \quad \text{or} \quad U = -1$$

$$2^x = 2^{-3} \qquad 2^x = -2^0 \qquad \text{(N.A)}$$

$$x = -3 \ \ (ans)$$

Chapter 2: Quadratic Equations

A quadratic equation is in the form of $ax^2 + bx + c = 0$ where a, b and c are constants, and $a \neq 0$.

It can be solved by :

- (i) Factorisation
- (ii) Completing the square
- (iii) Using quadratic formula

(i) Factorisation

Example: $2x^2 - 3x - 9 = 0$

By trial and error, cross multiply

$$\begin{array}{ccc|c} 2x & & 3 & 3x \\ & \times & & \\ x & & -3 & -6x \\ \hline 2x^2 & & -9 & -3x \end{array}$$

$$(2x+3)(x-3) = 0$$

$$2x+3=0 \quad or \quad x-3=0$$

$$x = -\frac{3}{2} \quad or \quad x = 3 \quad (ans)$$

(ii) Completing the square

To solve the quadratic equation $ax^2 + bx + c = 0$, where $a \neq 0$

Step 1 Change coefficient of x^2 equal to 1 $$a(x^2 + \frac{b}{a}x + \frac{c}{a}) = 0$$

Step 2 Leave x^2 and x terms on LHS $$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

Step 3 *Coefficient of* $x \div 2$, *square it, add to both sides* $$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$$

Step 4 Factorise the equation $$\left(x + \frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$$

Step 5 Simplify

Example: $2x^2-3x-9=0$

$$x^2-\frac{3}{2}x-\frac{9}{2}=0$$

$$x^2-\frac{3}{2}x=\frac{9}{2}$$

$$x^2-\frac{3}{2}x+\left(-\frac{3}{4}\right)^2=\frac{9}{2}+\left(-\frac{3}{4}\right)^2$$

$$\left(x-\frac{3}{4}\right)^2=\frac{9}{2}+\frac{9}{16}$$

$$=\frac{81}{16}$$

$$\left(x-\frac{3}{4}\right)=\pm\frac{9}{4}$$

$$x=\frac{3}{4}+\frac{9}{4} \quad or \quad x=\frac{3}{4}-\frac{9}{4}$$

$$=3 \qquad or \qquad =-\frac{3}{2} \quad (ans)$$

(iii) Quadratic Formula

To solve the quadratic equation $ax^2+bx+c=0$, where $a\neq 0$ using the formula

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Example: $2x^2-4x+1=0$

$$a=2,\ \ b=-4,\ \ c=1$$

$$x=\frac{-(-4)\pm\sqrt{(-4)^2-4(2)(1)}}{2(2)}$$

$$x=\frac{4\pm\sqrt{8}}{4}$$

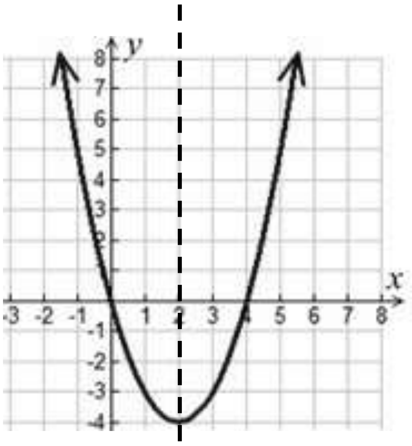
$$x=1.71 \text{ or } 0.293 \ (to\ 3\ s.f.)\ (ans)$$

Note:

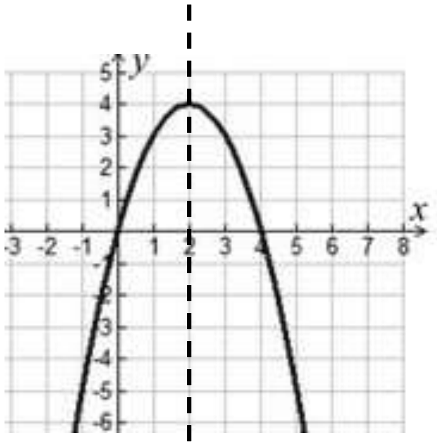
In the case where $b^2-4ac<0$, $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$ *has no solution (no real roots)*

2. Graphical Method

To solve the quadratic equation $ax^2 + bx + c = 0$, where $a \neq 0$ and the highest power of x is 2, the solution can be obtained from the intersection of the graph $y = ax^2 + bx + c$ and the x-axis. The graph cuts the x-axis when $y = 0$.

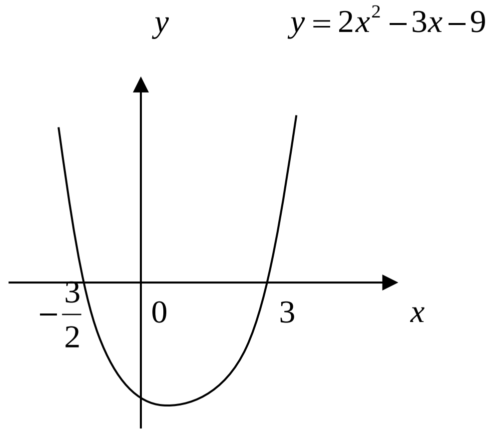


(a) If $a > 0$, the graph open upwards



(b) If $a < 0$, the graph open downwards

Example: $y = 2x^2 - 3x - 9$



$$\text{when } y = 0, \quad 2x^2 - 3x - 9 = 0$$
$$(2x+3)(x-3) = 0$$
$$x = -\frac{3}{2} \quad or \quad x = 3 \quad (ans)$$

Chapter 3: Linear Inequalities

1. Solving linear inequalities in one variable

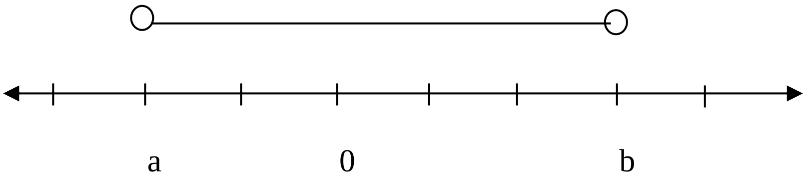
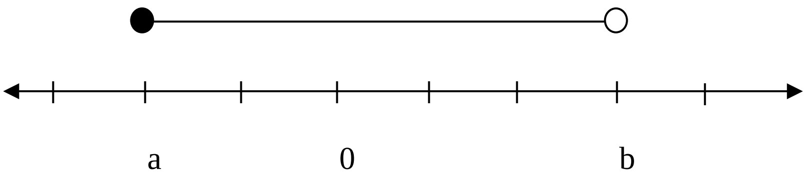
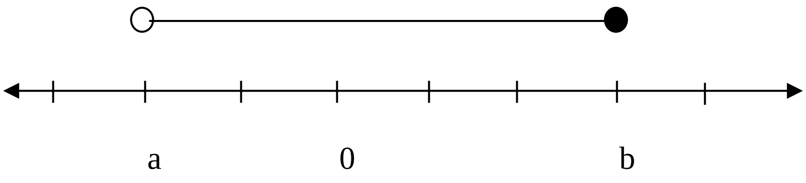
Linear inequalities are linear equations with either of the following signs

<	" is less than "
>	" is greater than "
≤	" is less than or equal to "
≥	" is greater than or equal to "

2. Representing the solution on the number line

The solution set to a linear inequality can be represented on a number line with the below symbols

○	open dot for < and > (means the endpoint value is not included)
●	solid dot for ≤ and ≥ (means the critical endpoint value is included)
⟷	arrows

Inequality notation	Number line
$a<x<b$	
$a\le x<b$	
$a<x\le b$	

Special rules for linear inequalities:

- When an inequality is multiplied or divided by a negative number, the inequality sign must be reversed.

Example (1) $-2x > 10$

Multiply by –1, $(-2x)(-1) < (10)(-1)$ * sign reversed to " < "

$2x < -10$

$\therefore x < -5$

- Reverse the inequality sign whenever you take the reciprocal of both sides

Example $3 > \frac{1}{x}$

$\frac{1}{3} < x$ * sign reversed to " < "

$\therefore x > \frac{1}{3}$

Combined inequality

Example (1) Solve $-7 \le 3x - 5 \le 2$

separate it into 2 inequalities, $-7 \le 3x - 5, \quad 3x - 5 \le 2$

move the variable to one side, $-7 + 5 \le 3x, \quad 3x \le 2 + 5$

simplify, $-2 \le 3x, \quad 3x \le 7$

$-\frac{2}{3} \le x, \quad x \le \frac{7}{3}$

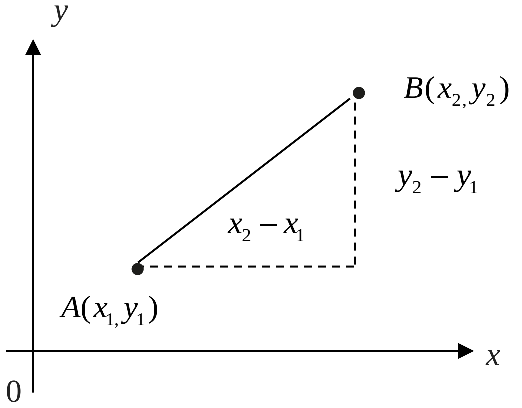
$\therefore -\frac{2}{3} \le x \le \frac{7}{3} \quad (ans)$

Chapter 4: Coordinate Geometry

Finding Length

The length of the line segment joining two points $A(x_{1,}y_1)$ *and* $B(x_{2,}y_2)$ is given by

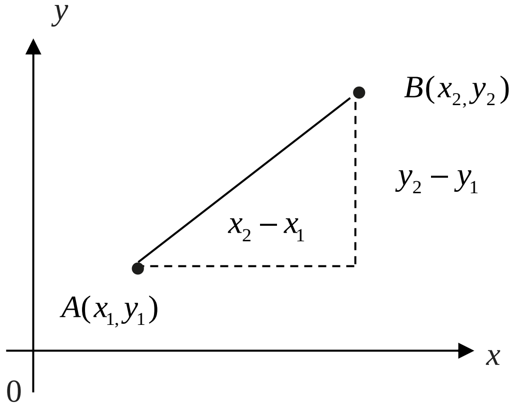
$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



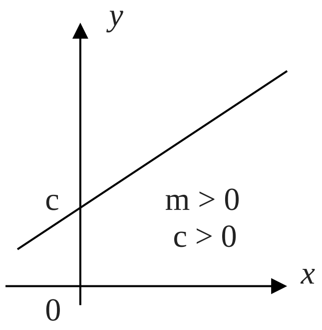
Finding gradient and parallel lines

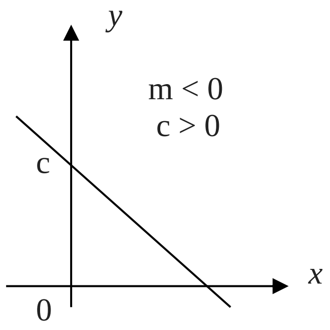
The gradient of the line joining two points $A(x_{1,}y_1)$ *and* $B(x_{2,}y_2)$ is given by

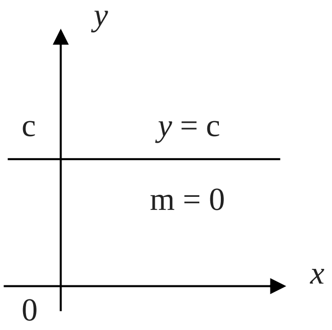
$$gradient = m = \frac{y_2 - y_1}{x_2 - x_1} \quad where\ x_1 \neq x_2$$



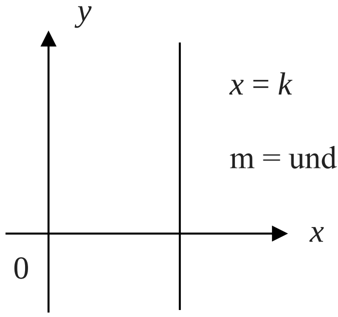
The gradient of a line can be positive, negative or zero.







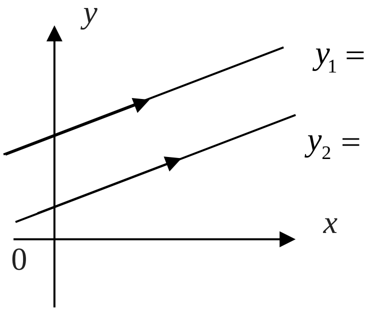
Gradient of special line



$x = k$

m = undefined (∞)

Parallel lines have the same gradient (m)

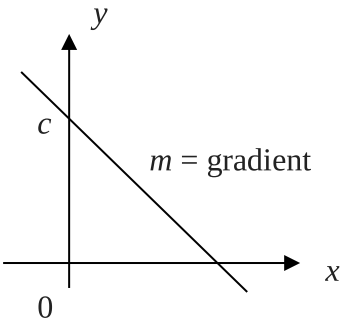


$y_1 = mx_1 + c_1$

$y_2 = mx_2 + c_2$

Finding Equation of Straight Line

To find the equation of the straight line given the gradient (m), and the y–intercept (c).



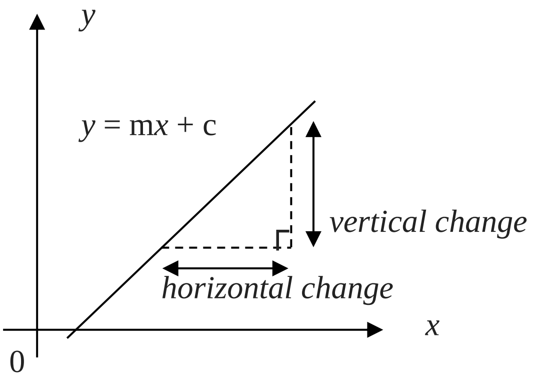
m = gradient

The equation of a straight line is $y = mx + c$ where m = gradient and

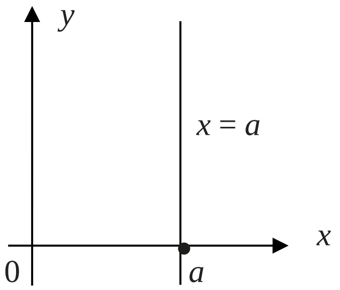
c = y–intercept

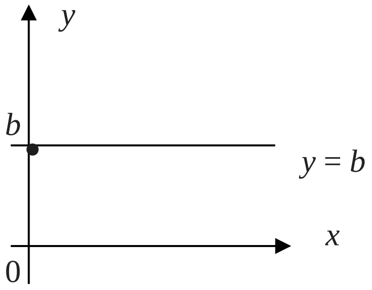
Chapter 5: Graphs

Straight Line Graphs (y = mx + c, $x = a$, $y = b$)



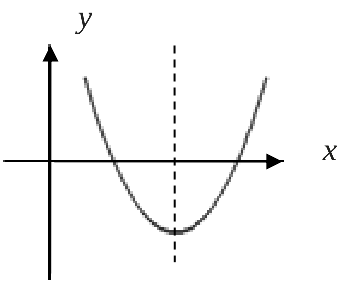
$$\text{gradient} = \frac{vertical\ change}{horizontal\ change}$$



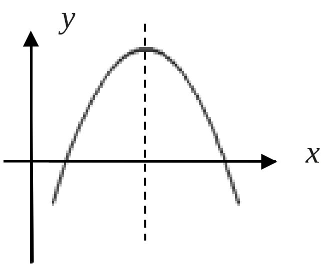


Quadratic Graphs

(a) $y = ax^2 + bx + c$ where a, b and c are constants and $a \neq 0$



$a > 0$



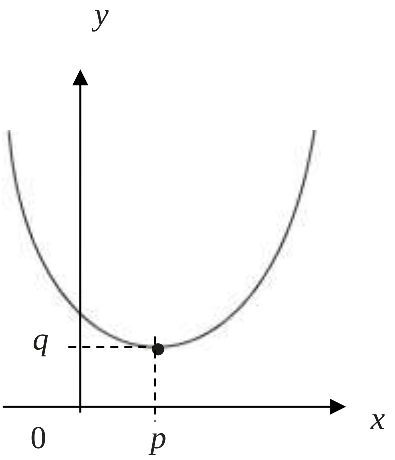
$a < 0$

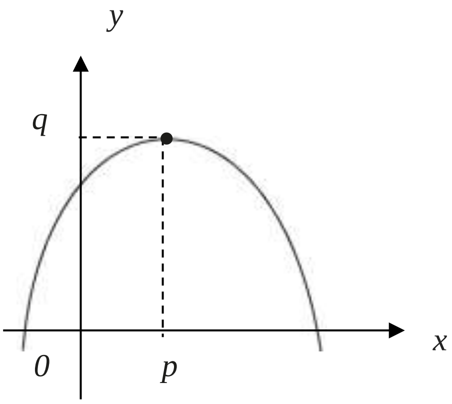
If $a > 0$, the graph has a minimum point

If $a < 0$, the graph has a maximum point

The graph of $y = ax^2 + bx + c$ has an axis of symmetry which passes through the minimum or maximum point.

(b) $y = (x-p)^2 + q$ $y = -(x-p)^2 + q$





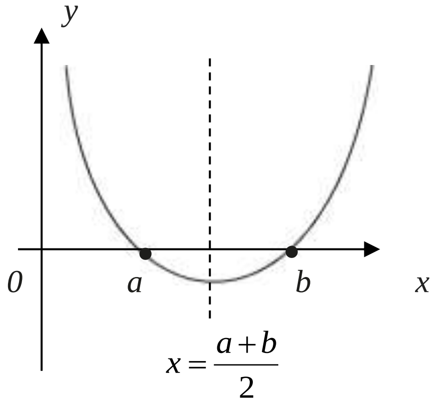
minimum point at (*p*, *q*) maximum point at (*p*, *q*)

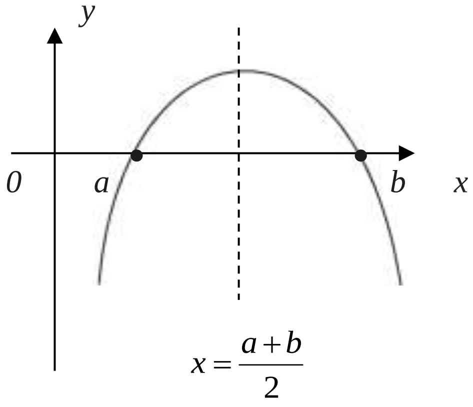
To find the x – intercepts, let $y = 0$

To find the y – intercepts, let $x = 0$

The line of symmetry of the graph in the form $y = \pm(x-p)^2 + q$ is $x = p$.

(c) $y = (x-a)(x-b)$ $y = -(x-a)(x-b)$



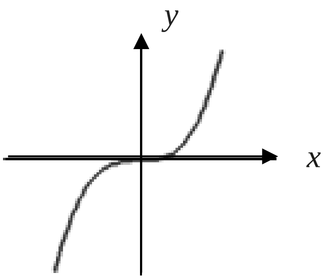


x–intercept are x = a and x = b

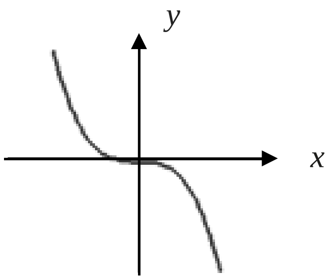
The line of symmetry of the graph in the form $y = \pm(x-a)(x-b)$ is $x = \frac{a+b}{2}$.

Cubic Graphs

$y = ax^n,\ when\ n = 3,\ a \neq 0$



a > 0

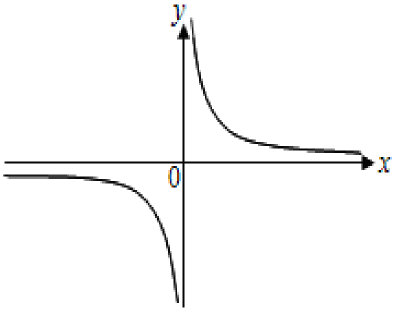


a < 0

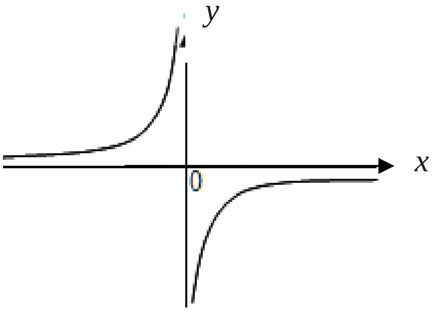
Reciprocal Graphs

$y = ax^{-1},\ when\ n = -1,\ a \neq 0$

a > 0

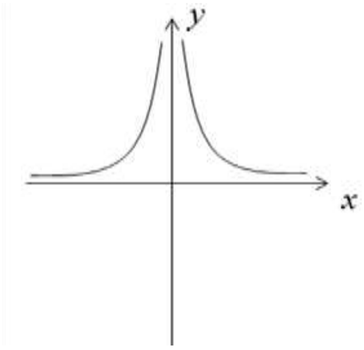


a < 0

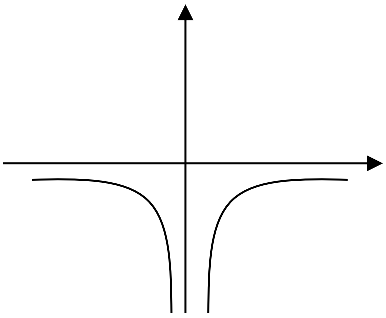


$y = ax^{-2},\ when\ n = -2,\ a \neq 0$

a > 0

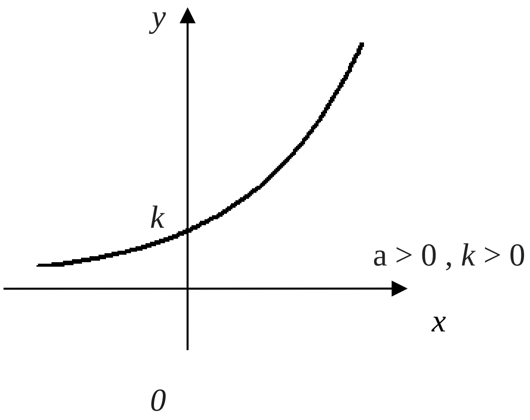


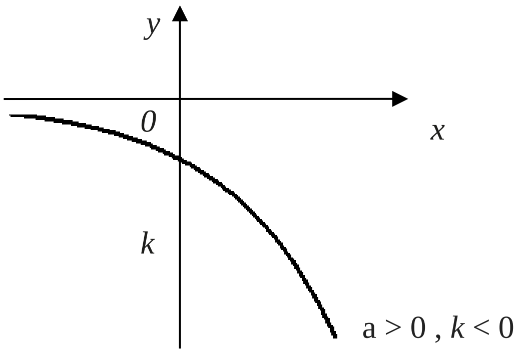
a < 0



Exponential Graphs

$y = ka^x$, where a is a positive integer

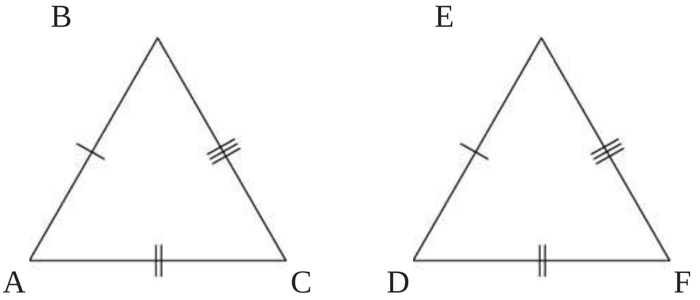




Chapter 6: Congruence and Similarity

Tests of Congruency

1. Side – Side – Side

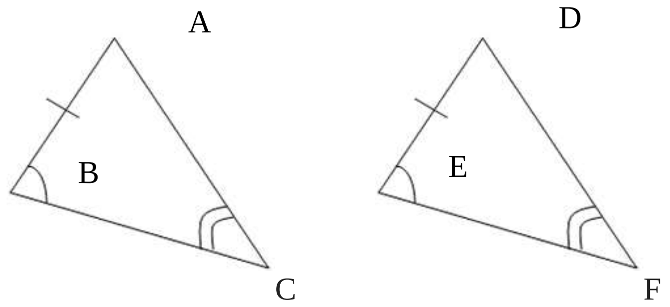


$\angle A = \angle D$
$\angle B = \angle E$
$\angle C = \angle F$

$AB = DE$
$BC = EF$
$AC = DF$

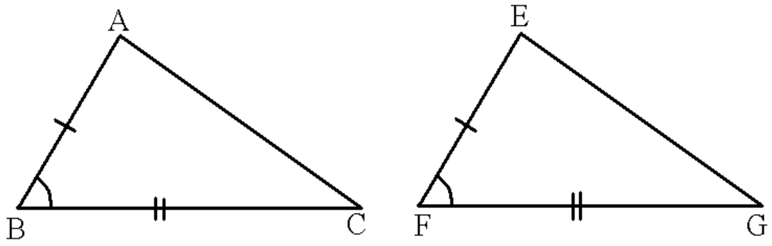
So $\Delta ABC \cong \Delta DEF$ (SSS Rule)

2. Angle – Side – Angle



Let $BC = EF$, $\angle B = \angle E$, $AB = DE$, $\angle C = \angle F$
So $\Delta ABC \cong \Delta DEF$ (ASA Rule)

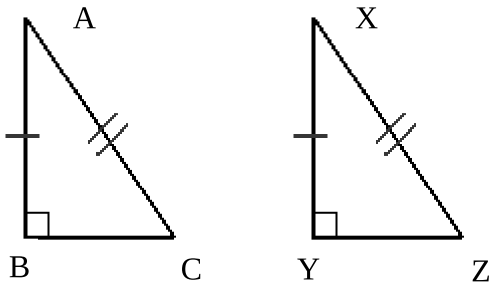
3. Side – Angle – Side



$AB = EF$
$\angle ABC = \angle EFG$
$BC = FG$

So $\Delta ABC \cong \Delta EFG$ (SAS Rule)

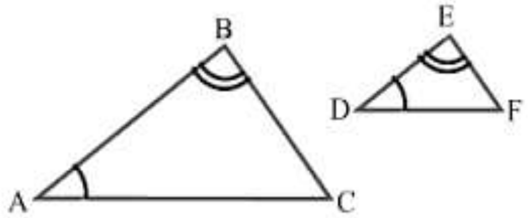
4. Right angled – Hypotenuse – Side



$\angle B = \angle Y = 90^{o}$
AB = XY
AC = XZ
So $\Delta ABC \cong \Delta XYZ$ (SAS Rule)

Tests for Similarity

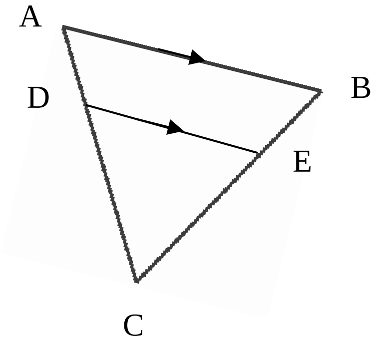
ΔABC and ΔDEF are similar if,



1. 2 pairs of corresponding angles are equal

i.e. $\angle A = \angle D$, $\angle B = \angle E$ (AA)

Example



In ΔABC and ΔDEC,

$\angle ACB = \angle DCE$ (common)

$\angle ABC = \angle DEC$ (corresponding angles)

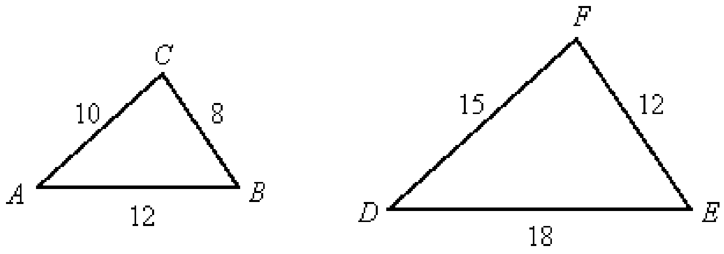
$\angle BAC = \angle EDC$ (corresponding angles)

$\therefore$ ΔABC is similar to ΔDEC (AA Similarity)

2. 3 pairs of corresponding sides are in the same ratio

i.e. $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$ (SSS)

Example



In ΔABC and ΔDEF,

$\frac{AB}{DE} = \frac{12}{18} = \frac{2}{3}$, $\frac{BC}{EF} = \frac{8}{12} = \frac{2}{3}$, $\frac{AC}{DF} = \frac{10}{15} = \frac{2}{3}$

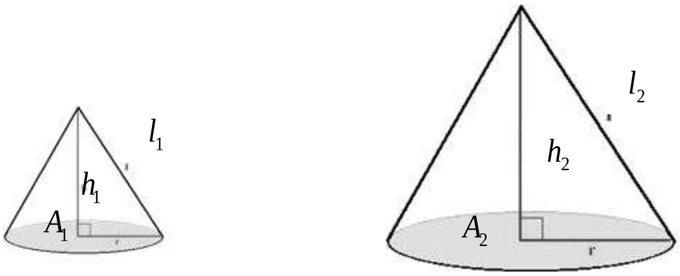
$\therefore$ ΔABC is similar to ΔDEF (SSS Similarity)

Ratio of Areas of Similar Figures

If 2 figures are similar, then the ratio of their areas is given by

$$\frac{A_1}{A_2}=\left(\frac{h_1}{h_2}\right)^2=\left(\frac{l_1}{l_2}\right)^2$$

Ratio of Volumes of Similar Solids



For 2 geometrically similar solids,

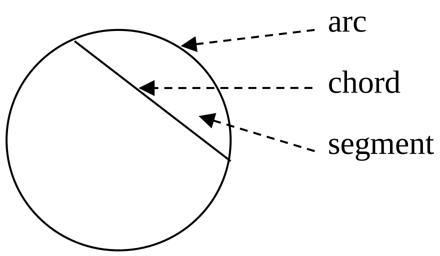
the ratio of their length or heights are $l_1 : l_2$ and $h_1 : h_2$

then the ratio of their areas is given by $\frac{A_1}{A_2}=\left(\frac{h_1}{h_2}\right)^2=\left(\frac{l_1}{l_2}\right)^2$

and the ratio of their volumes is given by $\frac{V_1}{V_2}=\left(\frac{h_1}{h_2}\right)^3=\left(\frac{l_1}{l_2}\right)^3$

Chapter 7: Properties of Circles

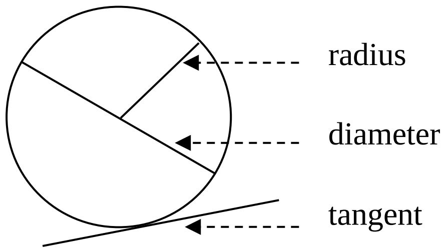
Parts of a circle



Arc : A segment along the circumference of a circle.

Chord : A line segment with both endpoints on a circle.

Segment: The area bounded by a chord and the included arc.



Radius : A line segment that joins the centre of a circle with any point on its circumference.

Diameter: A straight line segment passing through the centre of a circle and has its endpoints on the circle.

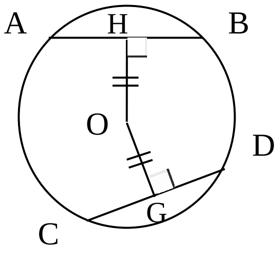
Circumference : is the distance around a circle.

Tangent : A line that touches a circle at only one point.

Symmetry properties

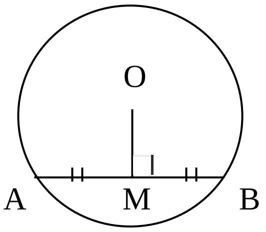
1. Equal chords are equidistant from the centre

Chords AB and CD are equal,
$AB = CD \Leftrightarrow OH = OG$



2. The perpendicular bisector of a chord passes through the centre

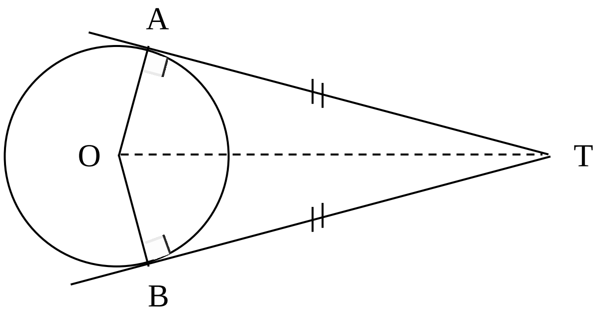
$AM = MB \Leftrightarrow OM \perp AB$



3. Tangents from an external point are equal in length

   If TA and TB are tangents from T to a circle centre O

   then TA = TB

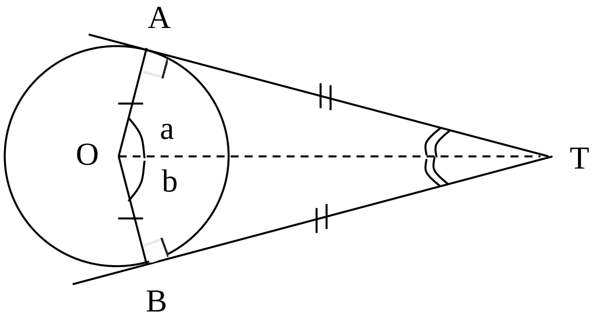


4. The line joining an external point to the centre of the circle bisects the angle between the tangents

   If TA and TB are tangents from T to a circle centre O

   then OT bisects ∠ATB

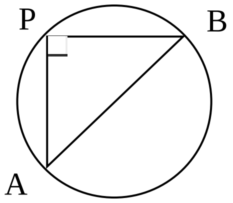
   and ∠a = ∠b



Angle properties

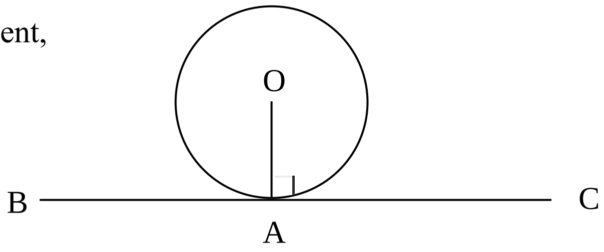
1. Angle in a semicircle is a right angle

   AB is the diameter ⇔ ∠APB = 90º



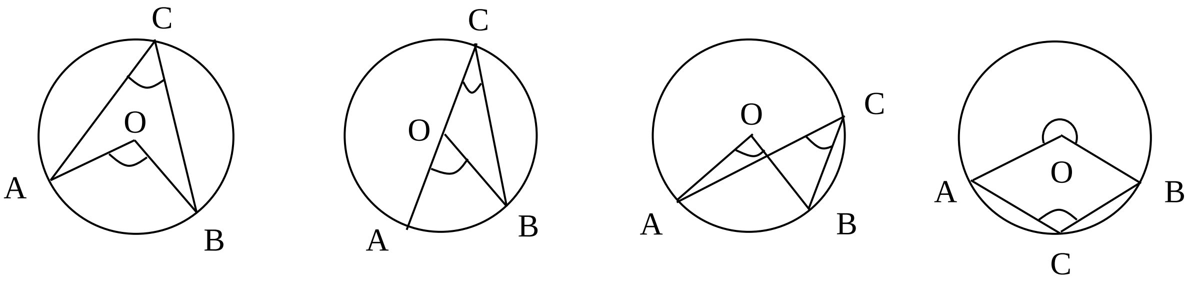
2. Angle between tangent and radius of a circle is a right angle

   If OA is the radius, BC is the tangent,
   A is the point of contact,
   then OA is perpendicular to BC



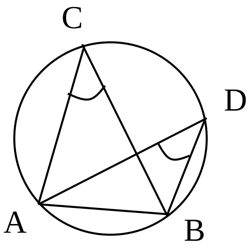
3. Angle at the centre is twice the angle at the circumference subtended by the same arc.

$\angle AOB = 2 \times \angle ACB$



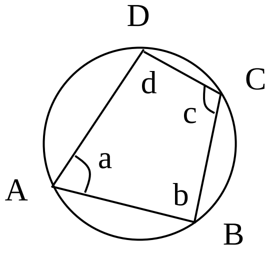
4. Angles in the same segment are equal

$\angle ACB = \angle ADB$



5. Angles in opposite segments are supplementary

$\angle a + \angle c = 180^{o}$, $\angle b + \angle d = 180^{o}$

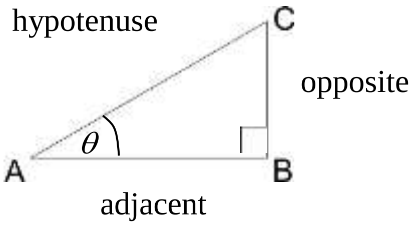


Chapter 8: Trigonometry

Trigonometric Ratios of Acute Angles

For right–angled triangle,

1. Sine $\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$ SOH
2. Cosine $\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$ CAH
3. Tangent $\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$ TOA



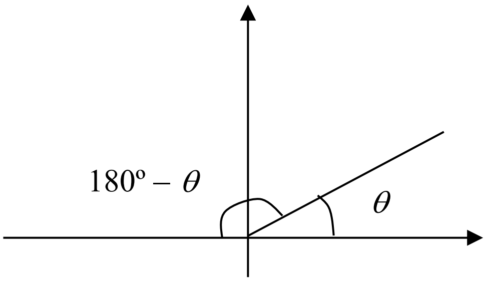
Sine and Cosine of Obtuse Angle

Definition of obtuse angle :
An **obtuse angle** is an angle more than 90 degrees and less than 180 degrees.
The range of an acute angle is between 90° and 180°.

For any acute angle θ ,

$\sin(180^\circ - \theta) = \sin\theta$

$\cos(180^\circ - \theta) = -\cos\theta$



Example

Express in terms of trigonometric ratios of acute angle and find the value.
Give your answers correct to 4 significant figures.

(a) sin 160° (b) cos 118°

(a) $\sin 160^\circ = \sin(180^\circ - 20^\circ)$

$= \sin 20^\circ$

$= 0.3420$

(b) $\cos 118^\circ = \cos(180^\circ - 62^\circ)$

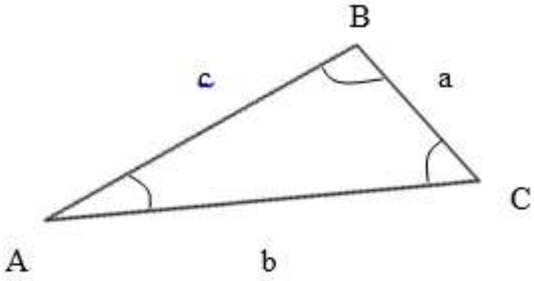
$= -\cos 62^\circ$

$= -0.4695$

Area of Triangle

For non–right angled triangle with any 2 given sides and an included angle,

$$\begin{aligned}\text{Area of }\Delta\text{ABC} &= \frac{1}{2}ab\sin C\\ &= \frac{1}{2}ac\sin B\\ &= \frac{1}{2}bc\sin A\end{aligned}$$



Sine rule

For any triangle ABC,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

where A, B and C are the interior angles
a, b and c are length of their opposite sides respectively.

Cosine rule

For any triangle ABC,

$a^2 = b^2 + c^2 - 2bc\ \cos A$ $\quad$ $\cos A$ is an included angle

$b^2 = a^2 + c^2 - 2ac\ \cos B$ $\quad$ $\cos B$ is an included angle

$c^2 = a^2 + b^2 - 2ab\ \cos C$ $\quad$ $\cos C$ is an included angle

or

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

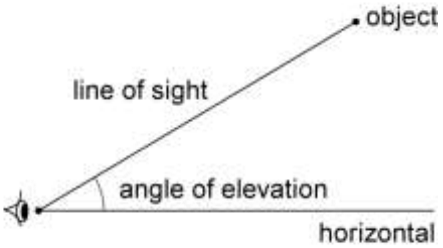
$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

where A, B and C are the interior angles
a, b and c are length of their opposite sides respectively.

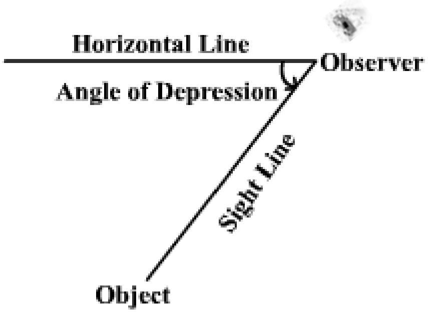
Chapter 9: Applications of Trigonometry

Angles of Elevation and Depression

1. When you look at an object above your location, the angle formed between the horizontal ground and the line of sight is called the **angle of elevation**.



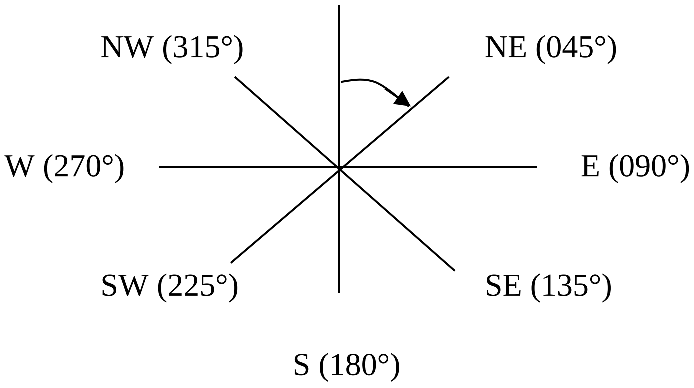
2. When you look at an object below your location, the angle formed between the imaginary horizontal ground and the line of sight is called the **angle of depression**.



Bearing

3 rules of bearings

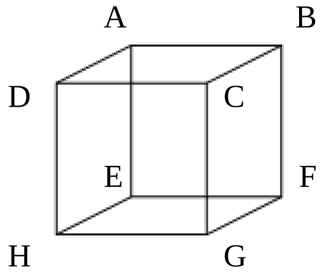
1. Always measured from the North (N).
2. Measure in a clockwise direction.
3. Always express in 3-digit number.



3-D Problems

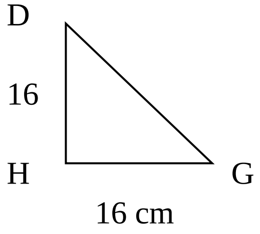
The basic technique used to solve a 3-D problem is to reduce it to a problem in a plane (2-D).

The diagram below shows a 16 cm – cube.
Find
(i) DG
(ii) DF
(iii) $\angle$ FDG



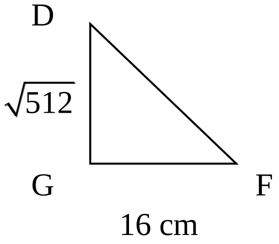
(i)

$$DG^2 = DH^2 + HG^2 \text{ (Pythagoras Theorem)}$$
$$DG^2 = 16^2 + 16^2$$
$$DG = \sqrt{16^2 + 16^2}$$
$$= \sqrt{512}$$
$$= 22.63$$
$$\approx 22.6\ cm \text{ (3 s.f.) } (ans)$$



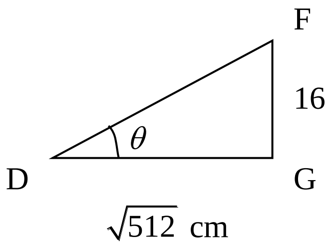
(ii)

$$DF^2 = DG^2 + GF^2 \text{ (Pythagoras Theorem)}$$
$$DF^2 = 512 + 16^2$$
$$DF = \sqrt{512 + 16^2}$$
$$= \sqrt{768}$$
$$= 27.71$$
$$\approx 27.7\ cm \text{ (3 s.f.) } (ans)$$



(iii)

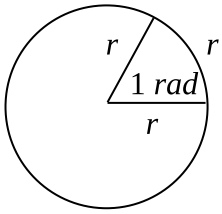
$$\tan \angle FDG = \frac{16}{22.63}$$
$$\angle FDG = 35.26°$$
$$\approx 35.3° \text{ (1 d.p.) } (ans)$$



Chapter 10: Circular Measure

Radian is another unit of measure for angles, similar to degrees.

One radian is the angle made at the center of a circle by an arc whose length is equal to the radius of the circle.



Conversion from radians and degrees

$$2\pi \ radians = 360°$$

$$\pi \ radians = \frac{360°}{2} = 180°$$

$$1 \ radian = \frac{180°}{\pi}$$

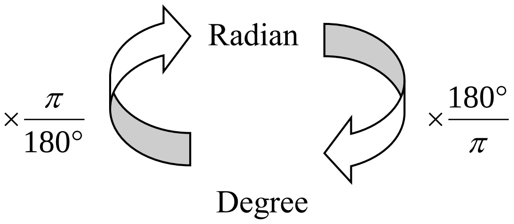
Conversion from degree to radians

$$360° = 2\pi \ radians$$

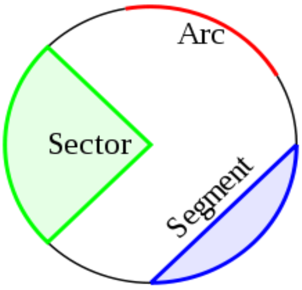
$$180° = \pi \ radians$$

$$1° = \frac{\pi}{180} rad$$

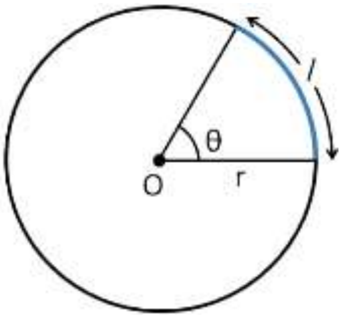
Conversion between radians and degrees



Parts of a circle



Arc Length



If θ is measured in degree then arc length $= \dfrac{\theta}{360°} \times 2\pi r$

If θ is measured in radian then arc length $= \dfrac{\theta}{2\pi} \times 2\pi r = r\theta$

Area of Sector

$$\frac{\text{Area of sector}}{\text{Area of circle}} = \frac{\text{Central angle, }\theta}{360°}$$

Area of circle $= \pi r^2$

If θ is measured in degree then area of sector $= \dfrac{\theta}{360°} \times \pi r^2$

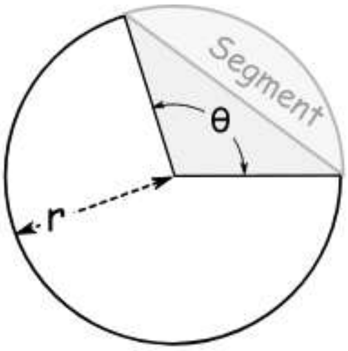
If θ is measured in radian then area of sector $= \dfrac{\theta}{2\pi} \times \pi r^2 = \dfrac{1}{2} r^2\theta$

Area of Segment

Area of segment = Area of Sector – Area of Triangle

$$= \frac{\theta°}{360°} \times \pi r^2 - \frac{1}{2} \mathrm{r}^2 \ \sin\theta \quad (if \ \ \theta \text{ is in degree})$$

$$= \frac{1}{2} r^2\theta - \frac{1}{2} \mathrm{r}^2 \ \sin\theta \quad (if \ \ \theta \text{ is in radian})$$



Important: If angles are in radian, change the calculator to radian mode.

For Sec 4 Students

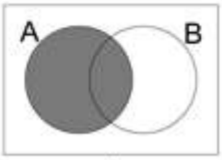
Chapter 1: Sets and Venn Diagrams

Set Language and Notations

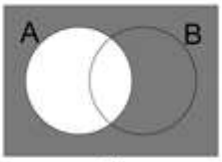
Set Language	Notation
'… is an element of …'	$\in$
'… is not an element of …'	$\notin$
The number of elements in set A	$n(A)$
Universal set	ξ
The empty set	$\varnothing$
Equal sets	=
A is a (proper) subset of B	$A \subset B$
A is not a (proper) subset of B	$A \not\subset B$
Union of sets A and B	$A \cup B$
Intersection of sets A and B	$A \cap B$
Complement of set A	A'
Disjoint Sets	$A \cap B = \varnothing$

Venn Diagrams

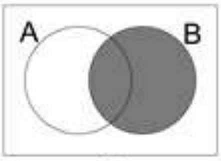
A Venn diagram is a pictorial representation of the relationships between sets.



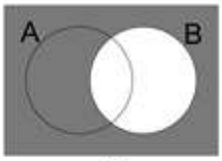
A



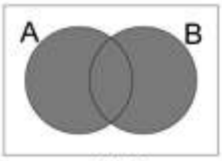
A'



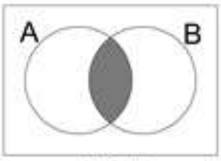
B



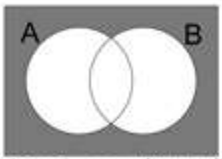
B'



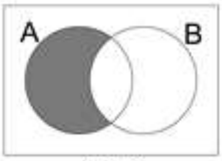
$A \cup B$



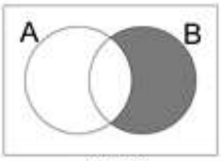
$A \cap B$



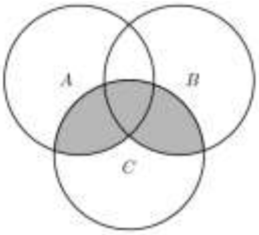
$A' \cap B'$ or $(A \cup B)'$



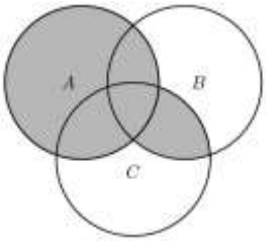
$A \cap B'$



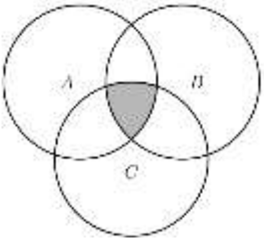
$A' \cap B$



(A U B) ∩ C



A U (B ∩ C)



A ∩ B ∩ C

Chapter 2: Probability

Probability Using Set Notations

If the sample space of an experiment has m equally likely outcomes with k ways for event E to occur, then the probability of an event E (or a specific outcome) happening is

$$\mathrm{P}(E)=\frac{\text{number of favourable outcomes for event } E}{\text{total number of possible outcomes}}=\frac{k}{m}=\frac{\mathrm{n}(E)}{\mathrm{n}(S)}=\frac{\mathrm{n}(E)}{\mathrm{n}(E)+\mathrm{n}(E')}$$

Recall that for any event E, $\mathrm{P}(\text{not } E)=1-\mathrm{P}(E)$. Therefore,

$$\mathrm{P}(E')=1-\mathrm{P}(E).$$

Example

Suppose A is the event that a letter is chosen from the word ‘INTENSE’ is a vowel. Using set notations, find P(A).

Solution

$S=\{\mathrm{I},\mathrm{N}_1,\mathrm{T},\mathrm{E}_1,\mathrm{N}_2,\mathrm{S},\mathrm{E}_2\}$

$\mathrm{n}(S)=7$

$A=\{\mathrm{I},\mathrm{E}_1,\mathrm{E}_2\}$

$\mathrm{n}(A)=3$

$\mathrm{P}(A)=\frac{\mathrm{n}(A)}{\mathrm{n}(S)}=\frac{3}{7}$

Combined Events, Possibility and Tree Diagrams

We can also list all the possible outcomes in an experiment using a possibility diagram or a tree diagram.

In Summary		
Number of stages or components in an experiment	**Example**	**Representation of Sample Space**
1	Choosing 1 ball from a bag	List of outcomes in a set
2	Choosing 2 balls from a bag, one after another	Possibility diagram or tree diagram
3 or more	Choosing 5 balls from a bag, one after another	Tree diagram

Example – Possibility Diagram

A pair of fair dice are rolled. Let B denote the integer sum of the two numbers shown on both dice. Find the probability that

a) $B=10$; b) B is odd; c) $B<13$; d) $B>12$.

Solution

A possibility diagram is drawn to represent the sample space for rolling two dice.

First Die

	+	**1**	**2**	**3**	**4**	**5**	**6**
Second Die	**1**	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

a) $$\mathrm{P}(B=10)=\frac{3}{36}$$
$$=\frac{1}{12}$$

b) $$\mathrm{P}(B \text{ is odd})=\frac{18}{36}$$
$$=\frac{1}{2}$$

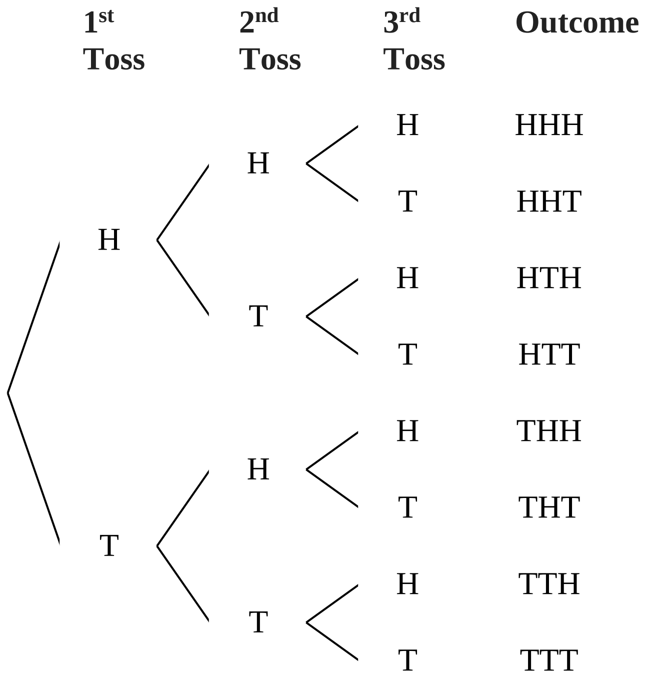
c) $$\mathrm{P}(B<13)=\frac{36}{36}$$
$$=1$$

d) $$\mathrm{P}(B>12)=\frac{0}{36}$$
$$=0$$

Example – Tree Diagram

Three fair coins are tossed. Find the probability of getting
a) 3 heads;
b) at least one tail.

Solution
A tree diagram is drawn to represent the sample space for tossing three coins.



a) $$\text{P(3 heads)} = \text{P(HHH)} = \frac{1}{8}$$

b) Method 1

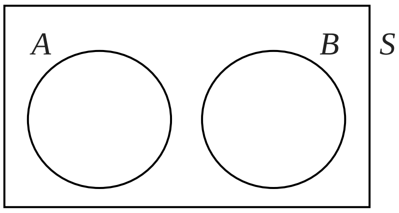
$$\begin{aligned}\text{P(at least 1 tail)} &= 1 - \text{P(no tail)}\\ &= 1 - \text{P(3 heads)}\\ &= 1 - \frac{1}{8}\\ &= \frac{7}{8}\end{aligned}$$

Method 2
Count the number of outcomes with at least 1 'T'.

$$\text{P(at least 1 tail)} = \frac{7}{8}$$

Addition Law of Probability and Mutually Exclusive Events

If two sets of events A and B cannot occur at the same time, then A and B are mutually exclusive. For example, when tossing a coin, the event A: 'a head appears' and event B: 'a tail appears' are mutually exclusive events.



In general,

- If A and B are mutually exclusive events,
  - $\text{P}(A \text{ or } B) = \text{P}(A \cup B) = \text{P}(B \cup A) = \text{P}(A) + \text{P}(B)$
  - $\text{P}(A \cap B) = \text{P}(\varnothing) = 0$
- **To put it simply, add when you see "or".**

Example

A fair die is tossed. Find the probability of obtaining

i) either 1 or 4; ii) neither 1 nor 4; iii) 1 and 4.

Solution

i) *A dice cannot show both 1 or 4 at the same time so obtaining 1 and obtaining 4 from a die throw are mutually exclusive events.*

$$\begin{aligned}\text{P}(1 \text{ or } 4) &= \text{P}(1) + \text{P}(4) \\ &= \frac{1}{6} + \frac{1}{6} \\ &= \frac{1}{3}\end{aligned}$$

ii) $$\begin{aligned}\text{P}(\text{neither } 1 \text{ nor } 4) &= 1 - \text{P}(1 \text{ or } 4) \\ &= 1 - \frac{1}{3} \\ &= \frac{2}{3}\end{aligned}$$

iii) *There is only one number on each face of a die so this is an impossible outcome and event.*

$$\text{P}(1 \text{ and } 4) = 0$$

Multiplication Law of Probability and Independent Events

Independent Events

If two events A and B are **independent**, the occurrence or non-occurrence of A **does not affect** the probability of B occurring or not occurring.

Examples of Independent Events

1. event A: 'picking a Queen from a deck of cards' and event B: 'getting a tail from a coin toss'.
2. randomly picking a Queen from a deck of cards, *replacing it* and then picking a King from the same deck.

Dependent Events

If the occurrence or non-occurrence of A **affects** the probability of B occurring or not occurring, then the two events are **dependent**.

For example, randomly picking a Queen from a deck of cards, and then picking a Queen again from the same deck.

In general,

- If events A and B are independent events, $\mathrm{P}(A \text{ and } B) = \mathrm{P}(A \cap B) = \mathrm{P}(B \cap A) = \mathrm{P}(A) \times \mathrm{P}(B)$
- To put it simply, multiply when you see "and".

Example – Independent Events

A bag contains 3 white and 2 black balls. Two balls are drawn at random with replacement (i.e. the first ball is drawn and replaced before the second is drawn). Find the probability that

a) the first ball drawn is black;
b) the first ball drawn is black, the second ball drawn is white;
c) both balls drawn are of the same colour.

a) $\mathrm{P}(\text{first ball is black}) = \frac{2}{5}$

b) $\mathrm{P}(\text{first ball is black, second is white}) = \frac{2}{5} \times \frac{3}{5} = \frac{6}{25}$

c) $\mathrm{P}(\text{both black}) = \frac{2}{5} \times \frac{2}{5} = \frac{4}{25}$

$$\mathrm{P}(\text{both white}) = \frac{3}{5} \times \frac{3}{5} = \frac{9}{25}$$

$$\begin{aligned}\mathrm{P}(\text{both same colour}) = \mathrm{P}(\text{both black}) + \mathrm{P}(\text{both white}) &= \frac{4}{25} + \frac{9}{25} \\ &= \frac{13}{25}\end{aligned}$$

Chapter 3: Statistics

Cumulative Frequency Table and Curve

Cumulative frequency is the running total of frequencies. The cumulative frequency of a value is obtained by adding up the frequencies which are less than or equal to that value.

We use the data in a cumulative frequency table to draw a cumulative frequency curve.

Example 1

The table below shows the frequency table of the scores obtained by 32 students in a Math test.

Score, *x*	**Frequency**
$0 < x \le 10$	2
$10 < x \le 20$	5
$20 < x \le 30$	13
$30 < x \le 40$	9
$40 < x \le 50$	3

(a) Construct a cumulative frequency table for the given data.

(b) Using the table in (a), find the number of students who
   (i) scored less than or equal to 30 marks,
   (ii) scored more than 20 marks but not more than 40 marks.

(c) Using a scale of 2 cm to represent 10 marks on the horizonal axis and 2 cm to represent 5 students on the vertical axis, draw the cumulative frequency curve for the given data.

Solution

(a)

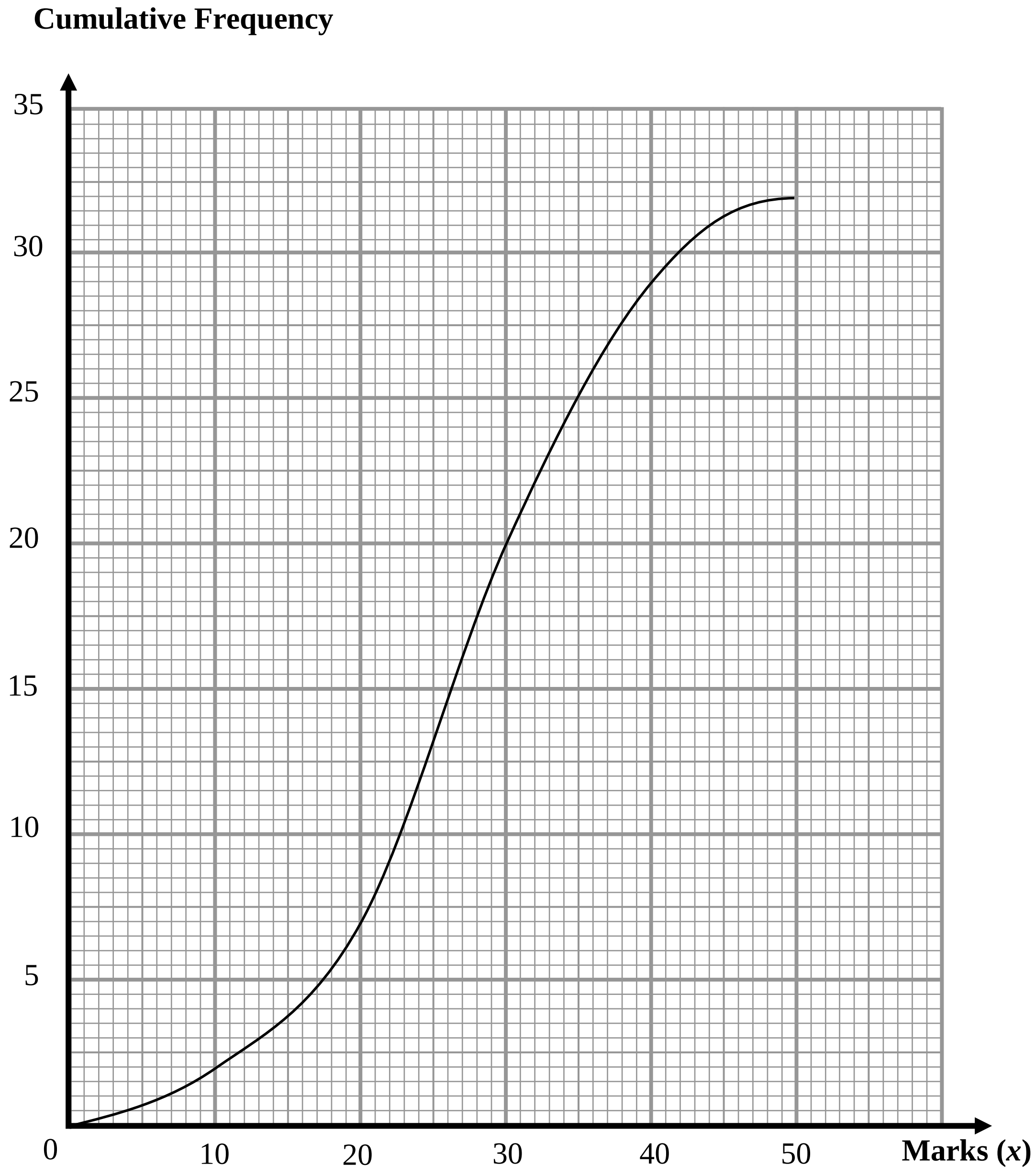
Score, *x*	**Cumulative Frequency**
$x \le 10$	2
$x \le 20$	2 + 5 = 7
$x \le 30$	7 + 13 = 20
$x \le 40$	20 + 9 = 29
$x \le 50$	29 + 3 = 32

Notice that the cumulative frequency for the last class in the table ($x \le 50$) is the total number of students in the class.

(b) (i) number of students who scored less than or equal to 30 marks = 20

(ii) number of students who scored more than 20 marks but not more than 40 marks
$= 29 - 7$
$= 22$

(c)



Median, Quartiles, Percentiles, Range and Interquartile Range

When a set of data is arranged in ascending order, **quartiles** divide the data into four equal parts:

- **Median** (Q_2) – the middle value of a set of data and it divides that set of data into two equal halves,
- Lower quartile (Q_1) – middle of the lower half of a set of data,
- Upper quartile (Q_3) – middle of the upper half of a set of data.

The **range** measures the spread of data. There are two measures of spread to show the degree of variation i.e. how spread out the data values are.

For a set of data,
1. **Range** = Largest value – Smallest value
2. **Interquartile range** = Upper quartile – Lower quartile = $Q_3 - Q_1$

Example

For each of the following sets of data find

- Q_1, Q_2 and Q_3,
- the range,
- the interquartile range.

(a) $\{13, 3, 11, 8, 9, 5, 10, 3, 17, 1, 20\}$

(b) $\{10, 30, 13, 5, 12, 9, 24, 28\}$

(a) Arranging the data set in ascending order, we get:

$$\underbrace{1,\ \ 3,\ \ \overset{\substack{\text{lower quartile}\\(Q_1)\\\downarrow}}{3},\ \ 5,\ \ 8}_{\text{lower half}},\ \ \overset{\substack{\text{median}\\(Q_2)\\\downarrow}}{9},\ \ \underbrace{10,\ \ 11,\ \ \overset{\substack{\text{upper quartile}\\(Q_3)\\\downarrow}}{13},\ \ 17,\ \ 20}_{\text{upper half}}.$$

$Q_1 = 3$

$Q_2 = 9$

$Q_3 = 13$

Range = $20 - 1$
= 19

Interquartile Range = $13 - 3$
= 10

(b) Arranging the data set in ascending order, we get:

$$\underbrace{5,\quad 9,\quad \overset{\substack{\text{lower quartile}\\(Q_1)\\\downarrow}}{}\quad 10,\quad 12,}_{\text{lower half}}\quad \overset{\substack{\text{median}\\(Q_2)\\\downarrow}}{}\quad \underbrace{13,\quad 24,\quad \overset{\substack{\text{upper quartile}\\(Q_3)\\\downarrow}}{}\quad 28,\quad 30.}_{\text{upper half}}$$

$$Q_1 = \frac{9+10}{2}$$
$$= 9.5$$

$$Q_2 = \frac{12+13}{2}$$
$$= 12.5$$

$$Q_3 = \frac{24+28}{2}$$
$$= 26$$

Range = $30-5$
= 25

Interquartile Range = $26-9.5$
= 16.5

Percentile

A cumulative frequency curve is used to estimate the quartiles for grouped data by dividing the total frequency or data set into four equal parts:

- Lower quartile (Q_1) is estimated using $\frac{1}{4} \times \text{total frequency}$,
- Median (Q_2) is estimated using $\frac{1}{2} \times \text{total frequency}$,
- Upper quartile (Q_3) is estimated using $\frac{3}{4} \times \text{total frequency}$.

We can also divide the total frequency or data set on a cumulative frequency curve into 100 equal parts. Each part is called a **percentile**.

- Lower quartile (Q_1) = 25th percentile = P_{25},
- Median (Q_2) = 50th percentile = P_{50},
- Upper quartile (Q_3) = 75th percentile = P_{75}.

Box-and-Whisker Plots

A box-and-whisker plot, also called a box plot, is a way of summarising a set of data using five numbers – **minimum, lower quartile, median, upper quartile and maximum values of data set contained in one diagram**. The box shows the range of the middle 50% of the data.

Example

Draw a box-and-whisker plot for the following set of 24 km run timings in minutes: 10, 16, 10, 7, 11, 14, 9.

Arranging the data set in ascending order, we get: 7, 9, 10, 10, 11, 14, 16.

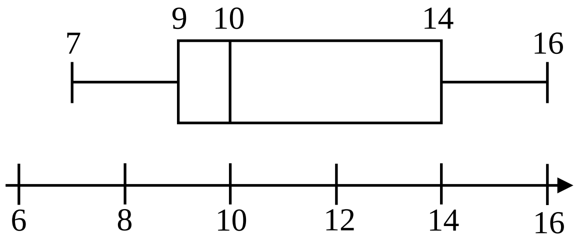
$Q_1 = 9$

$Q_2 = 10$

$Q_3 = 14$

Min. = 7

Max. = 16



Time (min)

Standard Deviation

Interquartile range measures the spread of a set of **data from the median**

Standard deviation measures the spread of a set of **data from the mean**.

To find the standard deviation of an **ungrouped data** set $\{x_1, x_2, x_3, \ldots, x_n\}$, where n is the number of data in the set:

Formula 1

$$\text{Standard Deviation} = \sqrt{\frac{(x_1-\bar{x})^2+(x_2-\bar{x})^2+(x_3-\bar{x})^2+\ldots+(x_n-\bar{x})^2}{n}}$$

$$= \sqrt{\frac{\sum(x-\bar{x})^2}{n}} \quad , \text{ where } \bar{x} = \frac{\sum x}{n}$$

Formula 2 (easier)

$$\text{Standard Deviation} = \sqrt{\frac{(x_1)^2+(x_2)^2+(x_3)^2+\ldots+(x_n)^2}{n}-(\bar{x})^2}$$

$$= \sqrt{\frac{\sum x^2}{n}-(\bar{x})^2} \quad , \text{ where } \bar{x} = \frac{\sum x}{n}$$

Example

The heights of five people in centimetres (cm) are: 155, 176, 182, 149, 166. Find the mean and the standard deviation of their heights.

$$\bar{x} = \frac{155+176+182+149+166}{5} = 165.6\text{ cm}$$

$$\sum x^2 = 155^2+176^2+182^2+149^2+166^2$$
$$= 137882$$

$$\text{Standard deviation} = \sqrt{\frac{\sum x^2}{n}-(\bar{x})^2}$$
$$= \sqrt{\frac{137882}{5}-165.6^2}$$
$$= \sqrt{153.04}$$
$$= 12.4\text{ cm (3 s.f.)}$$

Standard Deviation for Grouped Data

To find the standard deviation of a **grouped data** set:

Formula 1

Standard Deviation $=\sqrt{\dfrac{\sum\left[f\left(x-\bar{x}\right)^2\right]}{\sum f}}$, where f = frequency and x = mid-value of the group/class and $\bar{x}=\dfrac{\sum fx}{\sum f}$.

Formula 2 (easier)

Standard Deviation $=\sqrt{\dfrac{\sum\left[f\left(x^2\right)\right]}{\sum f}-\left(\bar{x}\right)^2}$, where f = frequency and x = mid-value of the group/class and $\bar{x}=\dfrac{\sum fx}{\sum f}$.

Example

The table below shows the ages of 60 participants in an archery competition.

Age (z years)	$15<x\leq 20$	$20<x\leq 25$	$25<x\leq 30$	$30<x\leq 35$
Frequency	24	19	10	7

Calculate the mean and the standard deviation of the age of participants.

z	f	Mid-value, x	fx	$f\left(x^2\right)$
$15<x\leq 20$	24	$\dfrac{15+20}{2}=17.5$	420	7350
$20<x\leq 25$	19	22.5	427.5	9618.75
$25<x\leq 30$	10	27.5	275	7562.5
$30<x\leq 35$	7	32.5	227.5	7393.75
	$\sum f=60$		$\sum fx=1350$	$\sum f\left(x^2\right)=31925$

$$\begin{aligned}\text{mean age}&=\frac{\sum fx}{\sum f}\\&=\frac{1350}{60}\\&=22.5\end{aligned}$$

$$\text{Standard Deviation}=\sqrt{\frac{\sum\left[f\left(x^2\right)\right]}{\sum f}-\left(\bar{x}\right)^2}\quad=\sqrt{\frac{31925}{60}-22.5^2}$$

$$=5.08\,(3\text{ s.f.})$$

Chapter 4: Matrices

1 Introduction to Matrices

- A matrix (plural: matrices) is a rectangular array of numbers belonging to the same set.
- Each number in a matrix is called an element.
- The order of a matrix tells us how many rows and columns it has.
  An $m \times n$ matrix has m rows and n columns.

Examples of Matrices	Order	No. of Rows	No. of Columns	Special Name
$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$	2×2	2	2	**Square matrix**: Matrix has the same number of rows and columns.
$\begin{pmatrix} 5 \\ 6 \\ 7 \end{pmatrix}$	3×1	3	1	**Column matrix**: Matrix has only one column.
$\begin{pmatrix} 8 & 9 & 10 & 11 \end{pmatrix}$	1×4	1	4	**Row matrix**: Matrix has only one row.
$\begin{pmatrix} 1 & 0 \\ 3 & -2 \\ 5 & 2 \end{pmatrix}$	3×2	3	2	-
$\begin{pmatrix} 1 \end{pmatrix}$	1×1	1	1	-
$\begin{pmatrix} 0 \end{pmatrix}$				**Zero matrix/Null matrix**: All elements in the matrix (of any order) is zero. Commonly denoted by **0**.
$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	2×3	2	3	
$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	2×2	2	2	**Identity matrix**: A square matrix with elements in the diagonal equal to 1 and the remaining elements are zero. Commonly denoted by **I**.
$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	3×3	3	3	

Equal matrices have the same order and their corresponding elements are the same.

2 Addition and Subtraction of Matrices

We can only add or subtract matrices of the same order.

In general, given **A** = $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ and **B** = $\begin{pmatrix} e & f \\ g & h \end{pmatrix}$, then

$$\mathbf{A} + \mathbf{B} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} + \begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{pmatrix} a+e & b+f \\ c+g & d+h \end{pmatrix}$$

$$\mathbf{A} - \mathbf{B} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} - \begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{pmatrix} a-e & b-f \\ c-g & d-h \end{pmatrix}$$

Note:

- **A** + **B** = **B** + **A**

  e.g. $\begin{pmatrix} 9 \\ 10 \end{pmatrix} + \begin{pmatrix} 7 \\ 8 \end{pmatrix} = \begin{pmatrix} 7 \\ 8 \end{pmatrix} + \begin{pmatrix} 9 \\ 10 \end{pmatrix} = \begin{pmatrix} 16 \\ 18 \end{pmatrix}$

- **A** + **B** + **C** = (**A** + **B**) + **C** = **A** + (**B** + **C**)

  e.g. $$\begin{aligned} \mathbf{A} + \mathbf{B} + \mathbf{C} &= \begin{pmatrix} 1 & 2 \end{pmatrix} + \begin{pmatrix} 3 & 4 \end{pmatrix} + \begin{pmatrix} 5 & 6 \end{pmatrix} \\ &= \begin{pmatrix} 1+3+5 & 2+4+6 \end{pmatrix} \\ &= \begin{pmatrix} 9 & 12 \end{pmatrix} \end{aligned}$$

  $$\begin{aligned} (\mathbf{A} + \mathbf{B}) + \mathbf{C} &= \left[\begin{pmatrix} 1 & 2 \end{pmatrix} + \begin{pmatrix} 3 & 4 \end{pmatrix}\right] + \begin{pmatrix} 5 & 6 \end{pmatrix} \\ &= \begin{pmatrix} 4 & 6 \end{pmatrix} + \begin{pmatrix} 5 & 6 \end{pmatrix} \\ &= \begin{pmatrix} 9 & 12 \end{pmatrix} \end{aligned}$$

  $$\begin{aligned} \mathbf{A} + (\mathbf{B} + \mathbf{C}) &= \begin{pmatrix} 1 & 2 \end{pmatrix} + \left[\begin{pmatrix} 3 & 4 \end{pmatrix} + \begin{pmatrix} 5 & 6 \end{pmatrix}\right] \\ &= \begin{pmatrix} 1 & 2 \end{pmatrix} + \begin{pmatrix} 8 & 10 \end{pmatrix} \\ &= \begin{pmatrix} 9 & 12 \end{pmatrix} \end{aligned}$$

3 Matrix Multiplication

Multiplication by a Scalar

If a matrix is multiplied by a scalar k, every element in the matrix is multiplied by k.

$$\text{If } \mathbf{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}, \text{ then } k\mathbf{A} = \begin{pmatrix} ka & kb \\ kc & kd \end{pmatrix}.$$

Multiplication by another Matrix

Multiplication of a matrix is only possible if the number of **columns in the first/left** matrix is **equal to** the number of **rows in the second/right** matrix.

$$\underset{m\times n}{\mathbf{A}} \quad \times \quad \underset{n\times r}{\mathbf{B}} \quad = \quad \underset{m\times r}{\mathbf{C}}$$

MUST be equal

We **multiply** the **rows of the first**/left matrix **with** the **columns in the second**/right matrix.

Example

It is given that $\mathbf{A} = \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 9 \\ 10 \end{pmatrix}$, $\mathbf{C} = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$, $\mathbf{D} = \begin{pmatrix} -1 & 0 \\ 3 & 11 \end{pmatrix}$ and $\mathbf{E} = \begin{pmatrix} 7 \end{pmatrix}$.

(a) Find the following.

  (i) **AD**,

  (ii) **DA**,

(a) (i) $$\mathbf{AD} = \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix}\begin{pmatrix} -1 & 0 \\ 3 & 11 \end{pmatrix}$$
$$= \begin{pmatrix} 5(-1)+6(3) & 5(0)+6(11) \\ 7(-1)+8(3) & 7(0)+8(11) \end{pmatrix}$$
$$= \begin{pmatrix} 13 & 66 \\ 17 & 88 \end{pmatrix}$$

(ii) $$\mathbf{DA} = \begin{pmatrix} -1 & 0 \\ 3 & 11 \end{pmatrix}\begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix}$$
$$= \begin{pmatrix} -1(5)+0(7) & -1(6)+0(8) \\ 3(5)+11(7) & 3(6)+11(8) \end{pmatrix}$$
$$= \begin{pmatrix} -5 & -6 \\ 92 & 106 \end{pmatrix}$$

Compare this answer with the answer to (a)(i). Notice that **AD≠DA** *so unlike multiplication of real numbers, multiplication by matrices is not commutative.*

4 Application of Matrices

We can solve word problems with the use of matrices.

Example

A factory makes two types of confectionary – Chocolate Dream and Strawberry Wonder. The number of packets of ingredients required to make a box of each is as follows:

- Chocolate Dream needs 3 packets of sugar, 3 packets of flour and 2 packets of yeast.
- Strawberry Wonder needs 2 packets of sugar, 4 packets of flour and 2 packets of yeast.

The cost of a packet of sugar, flour and yeast is \$1.20, \$8.25 and \$3 respectively.

It is given that $\mathbf{E} = \begin{pmatrix} 3 & 3 & 2 \\ 2 & 4 & 2 \end{pmatrix}$ and $\mathbf{F} = \begin{pmatrix} 1.20 \\ 8.25 \\ 3 \end{pmatrix}$.

(i) Evaluate **EF**.
(ii) Explain what your answer in (a) represents.
(iii) Bala's Sweets orders 10 boxes of Chocolate Dream and 5 boxes of Strawberry Wonder every week.
  (a) Write down matrix **D** such that its product with matrix **EF**, matrix **DEF** will give the total cost of ingredients incurred by the factory to fulfil Bala's Sweet's weekly order.
  (b) Hence, find the total cost of ingredients to fulfil Bala's Sweet's weekly order.

Solution

(i) **EF**

$$= \begin{pmatrix} 3 & 3 & 2 \\ 2 & 4 & 2 \end{pmatrix}\begin{pmatrix} 1.20 \\ 8.25 \\ 3 \end{pmatrix}$$

$$= \begin{pmatrix} 3(1.2)+3(8.25)+2(3) \\ 2(1.2)+4(8.25)+2(3) \end{pmatrix}$$

$$= \begin{pmatrix} 34.35 \\ 41.40 \end{pmatrix}$$

(ii) The total cost of ingredients to make one box of Chocolate Dream and one box of Strawberry Wonder.

(iii) (a) $\mathbf{D} = \begin{pmatrix} 10 & 5 \end{pmatrix}$

(b) $\mathbf{DEF} = \begin{pmatrix} 10 & 5 \end{pmatrix}\begin{pmatrix} 34.35 \\ 41.40 \end{pmatrix}$

$$= \begin{pmatrix} 10(34.35)+5(41.40) \end{pmatrix}$$

$$= \begin{pmatrix} 550.50 \end{pmatrix}$$

$\therefore$ total cost of ingredients to fulfil Bala's Sweet's weekly order = \$550.50.

Chapter 5: Vectors

1 Vectors in Two Dimensions

Scalars and Vectors

A vector has both magnitude and direction but a scalar only has magnitude.

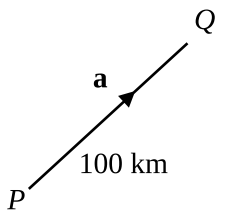
For example, a car has a speed of 90 km/h but a velocity of 90 km/h northwest.
(Speed is a scalar but velocity is a vector. Speed is the magnitude of the velocity.)

Other examples:

Scalar	**Vector**
Distance	Displacement
Mass	Weight
Temperature	

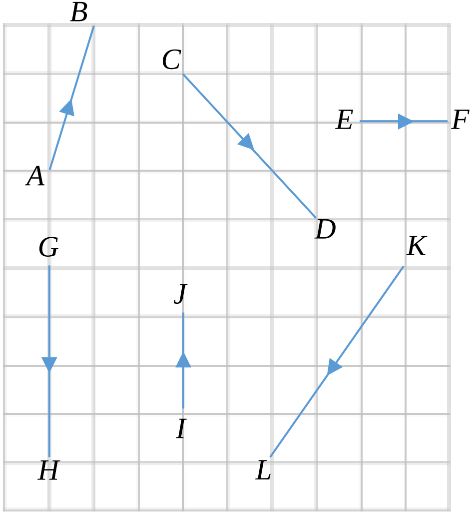
Representation of Vectors

- A vector is represented by a line segment.
  - The length of the line segment represents the magnitude of the vector.
  - The arrows on the line segment indicates the direction of the vector.
- The vector on the right is denoted by $\overrightarrow{PQ}$ or **a**.
  (In your workings, instead of **a** you can write $\underset{\sim}{a}$.)



Column Vectors

Vectors on a Cartesian plane can be expressed as a column vector. Column vectors are written in the form $\begin{pmatrix} x \\ y \end{pmatrix}$, where x is the horizontal component and y is the vertical component.



Examples:

$$\overrightarrow{AB} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} \qquad \overrightarrow{CD} = \begin{pmatrix} 3 \\ -3 \end{pmatrix}$$

$$\overrightarrow{EF} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \qquad \overrightarrow{GH} = \begin{pmatrix} 0 \\ -4 \end{pmatrix}$$

$$\overrightarrow{IJ} = \begin{pmatrix} 0 \\ 2 \end{pmatrix} \qquad \overrightarrow{KL} = \begin{pmatrix} -3 \\ -4 \end{pmatrix}$$

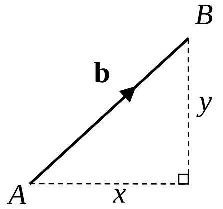
Magnitude of Vectors

Magnitude of $\overrightarrow{PQ}$ = length of $\overrightarrow{PQ}$ = $\left|\overrightarrow{PQ}\right|$ = $|\mathbf{a}|$ = $|\underset{\sim}{a}|$.

For the vector $\overrightarrow{PQ}$ on page 1, $\left|\overrightarrow{PQ}\right| = 100$.

The magnitude of a column vector $\overrightarrow{AB} = \mathbf{b} = \begin{pmatrix} x \\ y \end{pmatrix}$ is given by

$$\left|\overrightarrow{AB}\right| = |\mathbf{b}| = \sqrt{x^2 + y^2}\,.$$



Equal Vectors

Two vectors are equal if they have the same magnitude and same direction.

i.e. $\overrightarrow{AB} = \overrightarrow{CD}$ so AB is parallel to CD and $\left|\overrightarrow{AB}\right| = \left|\overrightarrow{CD}\right|$.

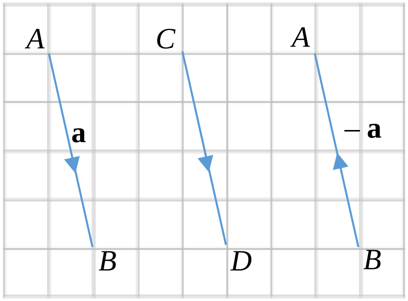
Negative Vectors

Two vectors are negative vectors of each other is they have the same magnitude but opposite direction.

i.e. $\overrightarrow{AB} = -\overrightarrow{BA}$ OR $\overrightarrow{BA} = -\overrightarrow{AB}$

$\mathbf{a} = -(-\mathbf{a})$ $\quad$ $-\mathbf{a} = -(\mathbf{a})$

Regardless, $\left|\overrightarrow{AB}\right| = \left|\overrightarrow{BA}\right|$.



2 Addition and Subtraction of Vectors

Vector Addition

The principle of adding two vectors can be extended to more than two vectors.

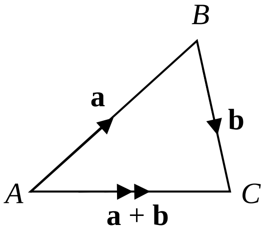
Method 1: Triangle Law of Vector Addition

$$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC} = \mathbf{a} + \mathbf{b}$$

same point

start *end*

$\overrightarrow{AC}$ is called the **vector sum** or **resultant vector**, indicated by a double arrow in the diagram.

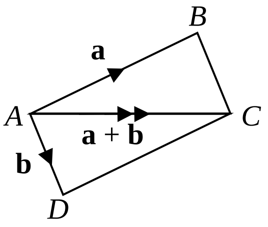


Method 2: Parallelogram Law of Vector Addition

$$\overrightarrow{AB} + \overrightarrow{AD} = \overrightarrow{AC} = \mathbf{a} + \mathbf{b}$$

same point

Notice **a** and **b** are the sides of the parallelogram and $\overrightarrow{AC}$ = **a** + **b** (the resultant vector) is the diagonal.



Method 3: Resultant by Column Vectors

For column vectors,

$$\begin{pmatrix} p \\ q \end{pmatrix} + \begin{pmatrix} r \\ s \end{pmatrix} = \begin{pmatrix} p+r \\ q+s \end{pmatrix}$$

In general,

- Vector addition is commutative: **a** + **b** = **b** + **a**.
- Vector addition is associative: **a** + **b** + **c** = (**a** + **b**) + **c** = **a** + (**b** + **c**).
- It is usually easier to use method 1 and 3 unless the parallelogram has been drawn.

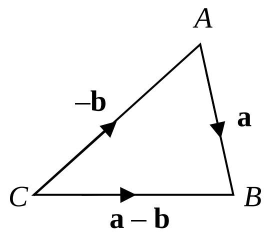
Vector Subtraction

The principle of subtracting two vectors can be extended to more than two vectors.

Method 1: Addition of Negative Vectors

$\mathbf{a} - \mathbf{b} = \mathbf{a} + (-\mathbf{b})$ OR $\overrightarrow{AB} - \overrightarrow{AC} = \overrightarrow{AB} + \left(-\overrightarrow{AC}\right)$

$$= \overrightarrow{AB} + \overrightarrow{CA}$$
$$= \overrightarrow{CA} + \overrightarrow{AB}$$
$$= \overrightarrow{CB}$$



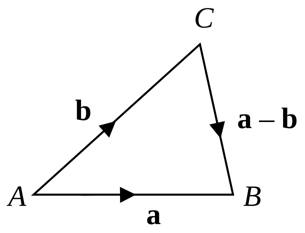
Method 2: Triangle Law of Subtraction

From addition, $\overrightarrow{AC} + \overrightarrow{CB} = \overrightarrow{AB}$.

Therefore, $\overrightarrow{AB} - \overrightarrow{AC} = \overrightarrow{CB}$.

end *start*

same point



Method 3: Resultant by Column Vectors/Coordinates

For column vectors,

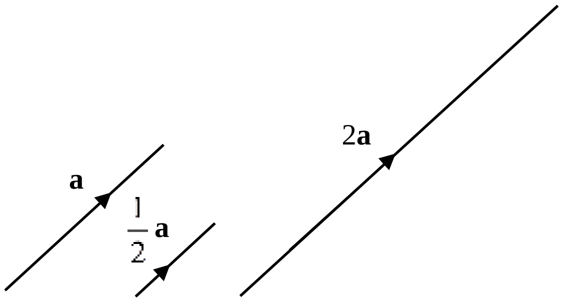
$$\begin{pmatrix} p \\ q \end{pmatrix} - \begin{pmatrix} r \\ s \end{pmatrix} = \begin{pmatrix} p-r \\ q-s \end{pmatrix}$$

3 Scalar Multiplication and Parallel Vectors

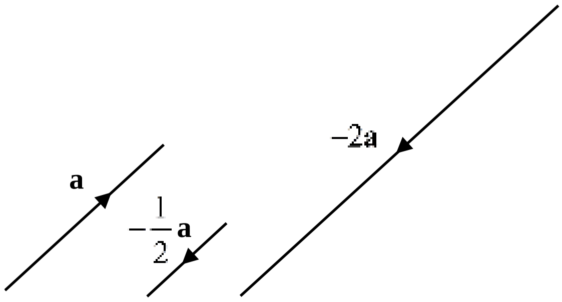
Scalar Multiple

When vector **a** is multiplied by scalar *k*, the resulting vector has magnitude *k* times of **a**, $|k\mathbf{a}| = k|\mathbf{a}|$.

- When $k > 0$, $k\mathbf{a}$ is a vector with the <u>same</u> direction as **a** and a magnitude that is *k* times of **a**.



- When $k < 0$, $k\mathbf{a}$ is a vector with the <u>opposite</u> direction as **a** and a magnitude that is *k* times of **a**.



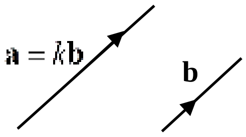
- When $k = 0$, $k\mathbf{a}$ is a zero vector.

In general, for any vector **a** and **b**, and real numbers *m* and *n*, then

- $m(n\mathbf{a}) = n(m\mathbf{a}) = (mn)\mathbf{a}$
- $m\mathbf{a} + n\mathbf{a} = \mathbf{a}(m+n)$
- $m\mathbf{a} + m\mathbf{b} = m(\mathbf{a}+\mathbf{b})$

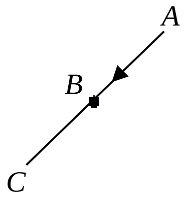
Parallel Vectors

- If $\mathbf{a} = k\mathbf{b}$, where *k* is a scalar and $k \neq 0$, then **a** is parallel to **b** and $|\mathbf{a}| = k|\mathbf{b}|$.
- If **a** is parallel to **b**, then $\mathbf{a} = k\mathbf{b}$ where *k* is a scalar and $k \neq 0$.
- If $m\mathbf{a} + n\mathbf{b} = p\mathbf{a} + q\mathbf{b}$, where *m*, *n*, *p* and *q* are scalars and **a** is <u>not</u> parallel to **b**, then $m = p$ and $n = q$.



Collinear Points

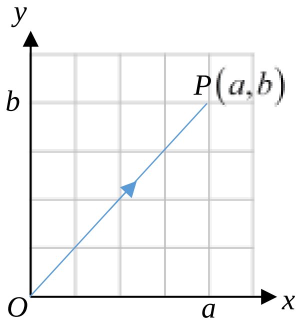
If points A, B and C lie in a straight line (i.e. they are collinear), then $\overrightarrow{AB} = k\overrightarrow{BC}$.



4 Position Vectors

The **position vector** of any point is the vector from the origin to that point.
For example, $\overrightarrow{OP}$ is the position vector of *P* relative to *O*.

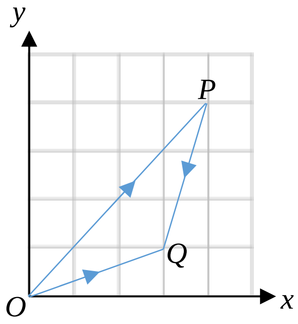
If point *P* has coordinates (a,b), then the position vector of *P*, $\overrightarrow{OP}$ is written as $\overrightarrow{OP} = \begin{pmatrix} a \\ b \end{pmatrix}$.



For any two points *P* and *Q*, $\overrightarrow{PQ}$ is the position vector of *Q* relative to *P*.

$$\overrightarrow{PQ} = \overrightarrow{PO} + \overrightarrow{OQ} = \overrightarrow{OQ} - \overrightarrow{OP}$$

$\overrightarrow{PQ}$ is also known as a **translation vector** because the vector can be regarded as a movement from *P* to *Q*.



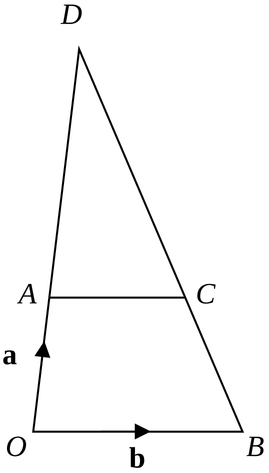
5 Applications of Vectors

Solving Geometric Problems Involving Vectors

Example

In the diagram, *OACB* is a trapezium where *AC* is parallel to *OB*.

The line *OA* is produced to the point *D* such that $\frac{OA}{AD}=\frac{1}{2}$.



(i) Given that $\overrightarrow{OA}=\mathbf{a}$ and $\overrightarrow{OB}=\mathbf{b}$, express as simply as possible, in terms of **a** and/or **b**,
  (a) $\overrightarrow{BD}$,
  (b) $\overrightarrow{OC}$.

(ii) Given that $\overrightarrow{OE}=3\mathbf{a}+2\mathbf{b}$,
  (a) state the name of the quadrilateral *ODEB*,
  (b) explain why *O*, *C* and *E* lie in a straight line.

(iii) Find, giving your answers as fractions in the simplest form,
  (a) $\frac{\text{area of } \Delta ADC}{\text{area of } \Delta ODB}$,
  (b) $\frac{\text{area of } \Delta ADC}{\text{area of quadrilateral } ODEB}$.

Solution

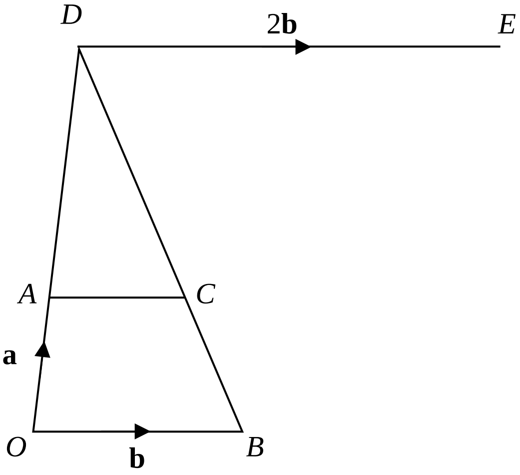
(i) (a) $\overrightarrow{BD}=3\mathbf{a}-\mathbf{b}$

(b) *Since AC is parallel to OB, triangle DAC and DOB are similar. Since* $\frac{OA}{AD}=\frac{1}{2}$,

then $\frac{BC}{CD}=\frac{1}{2}$ *so* $\frac{DB}{DC}=\frac{2}{3}$.

$$\overrightarrow{OC}=\overrightarrow{OD}+\overrightarrow{DC}$$
$$=3\mathbf{a}+\frac{2}{3}(\mathbf{b}-3\mathbf{a})$$
$$=\mathbf{a}+\frac{2}{3}\mathbf{b}$$

(ii) (a)Since DE is parallel to OB, ODEB is a Trapezium



(b) $\overrightarrow{OE} = 3\mathbf{a} + 2\mathbf{b}$

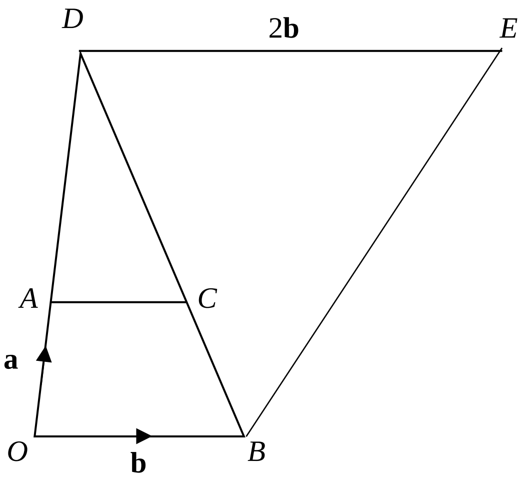
$$= 3\left(\mathbf{a} + \frac{2}{3}\mathbf{b}\right)$$

$$= 3\overrightarrow{OC}$$

Thus $\overrightarrow{OC}$ is parallel to $\overrightarrow{OE}$. Hence the O, C and E are collinear.

(iii) (a) $\dfrac{\text{area of } \Delta ADC}{\text{area of } \Delta ODB} = \left(\dfrac{2}{3}\right)^2 = \dfrac{4}{9}$

(b) Δ ADC → 4u
Δ ODB → 9u



Δ BDE → 9u × 2 = 18u
Quadrilateral ODEB → 9u + 18u = 27u

$$\frac{\text{area of } \Delta ADC}{\text{area of quadrilateral } ODEB} = \frac{4}{27}$$

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