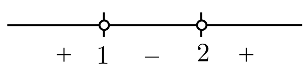
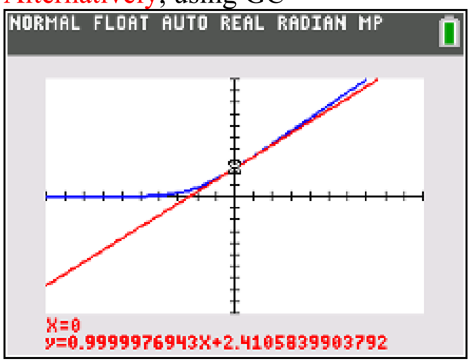
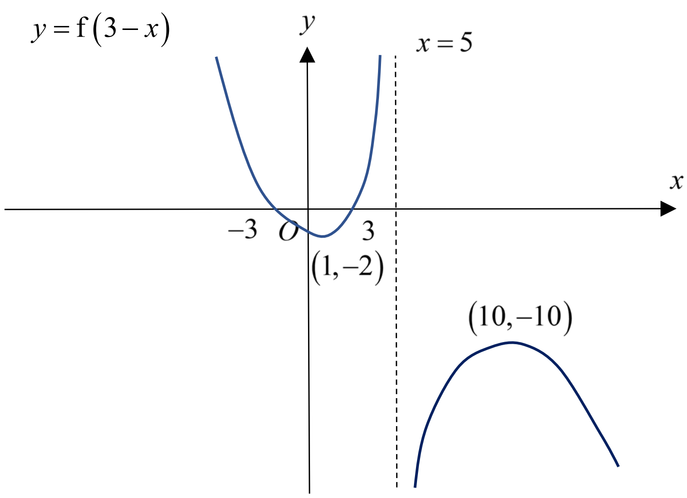
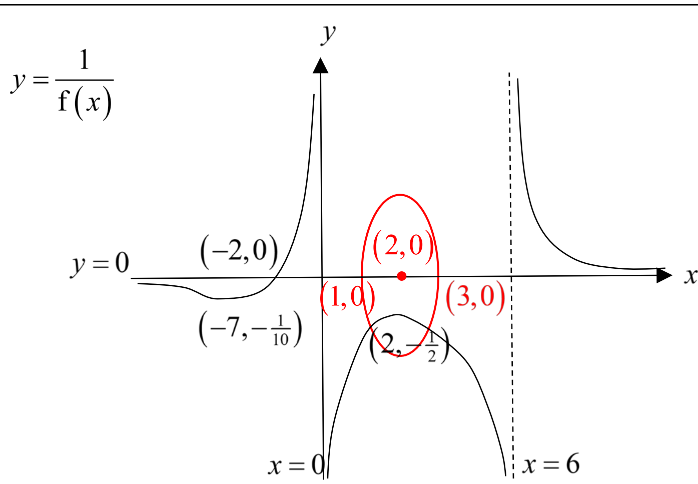
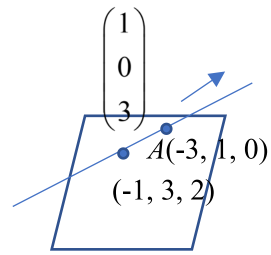
2023 J2 Common Test SOLUTION (For Students)

No.	Solution	Comments
1	$\dfrac{x-7}{x^2-3x+2}-1=\dfrac{x-7-x^2+3x-2}{x^2-3x+2}$ $=-\dfrac{x^2-4x+9}{x^2-3x+2}$ $\dfrac{x-7}{x^2-3x+2}<1$ $-\dfrac{x^2-4x+9}{x^2-3x+2}<0$ $\dfrac{x^2-4x+9}{x^2-3x+2}>0$ $\dfrac{(x-2)^2+5}{(x-2)(x-1)}>0$ Since $(x-2)^2+5>0$ for all real values of x, $(x-2)(x-1)>0$  $x<1$ or $x>2$	Multiplying -1 on both sides of an inequality will cause the inequality sign to change. Since **no GC is allowed**, you need to show that x^2-4x+9 is always positive. In this case, we complete the square to show the expression is always positive. Showing $x^2-4x+9=0$ has no roots does not allow you to conclude $(x-2)(x-1)>0$. You need to also mention that coefficient of $x^2>0$ and hence, $x^2-4x+9>0$ for all real x.
2(a)	Differentiating $\mathrm{e}^y=1+3^{x-a}$ w.r.t. x, $\mathrm{e}^y\dfrac{\mathrm{d}y}{\mathrm{d}x}=(3^{x-a})\times\ln 3$ $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\left(\dfrac{3^{x-a}}{\mathrm{e}^y}\right)\ln 3$ $=\left(\dfrac{3^{x-a}}{1+3^{x-a}}\right)\times\dfrac{3^{-(x-a)}}{3^{-(x-a)}}\times\ln 3$ $=\dfrac{3^0\ \ln 3}{3^{-(x-a)}+3^0}$ $=\dfrac{\ln 3}{1+3^{a-x}}$, $k=3$ (shown) Alternatively, $y=\ln\left(1+3^{x-a}\right)$ $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{(\ln 3)3^{x-a}}{1+3^{x-a}}$ $=\left(\dfrac{\ln 3}{1+3^{x-a}}\right)\left(\dfrac{1}{3^{a-x}}\right)\ln 3$ $=\dfrac{\ln 3}{3^{a-x}+3^0}$ $=\dfrac{\ln 3}{1+3^{a-x}}$, $k=3$ (shown)	**To remember:** For $c>0$, $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(c^x\right)=c^x\ln c$ **To remember:** You can derive it from scratch. Let $u=c^x$. $\ln u=x\ln c$ Differentiate w.r.t. x $\dfrac{1}{u}\left(\dfrac{\mathrm{d}u}{\mathrm{d}x}\right)=\ln c$ $\dfrac{\mathrm{d}u}{\mathrm{d}x}=u\ln c$ $=c^x\ln c$ This is a "show" question, so provide clear working on how $\dfrac{3^{x-a}}{1+3^{x-a}}$ can be simplified to $\dfrac{1}{1+3^{a-x}}$

(b)	Tangent to C at $x = 0$ makes an angle of 45° with the positive x-axis. $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\ln 3}{1+3^{a-0}} = \tan 45^\circ = 1$ $\ln 3 = 1 + 3^a$ $3^a = (\ln 3) - 1$ $a \ln 3 = \ln((\ln 3) - 1)$ $a = -2.1086 = -2.11$ At $x = 0$, $\mathrm{e}^y = 1 + 3^{0-(-2.1086)}$ $y = \ln(1 + 3^{2.1086})$ Equation of T: $y - \ln(1+3^{2.1086}) = 1(x - 0)$ $y = x + 2.41$ Alternatively, using GC  Equation of T: $y = x + 2.41$	**To remember:** Grad of line = $\tan\theta$ where θ is the angle which the line makes with the positive x-axis. Working should be in 5 s.f. and final answer can be given in 3 s.f. Alternatively, Use GC to find equation of tangent is allowed. Sketch $y = \ln\left(1+3^{x+2.1086}\right)$, then on the graphing screen, press [2nd] [prgm] and select 5: Tangent. Press 0. Notice that the gradient of the tangent given by GC is 1.00. But from the question, you know it should be 1. Hence, equation of T: $y = x + 2.41$
3	$\ln(\sec x)$ $= \ln\left(\frac{1}{\cos x}\right)$ $= -\ln(\cos x)$ $= -\ln\left(1 - \frac{x^2}{2} + \frac{x^4}{24} + \ldots\right)$ (from MF26) $= -\ln\left(1 + \left(-\frac{x^2}{2} + \frac{x^4}{24}\right) + \ldots\right)$ $= -\left[\left(-\frac{x^2}{2} + \frac{x^4}{24}\right) - \frac{1}{2}\left(-\frac{x^2}{2} + \frac{x^4}{24}\right)^2 + \ldots\right]$ (from MF26) $= -\left[-\frac{x^2}{2} + \frac{x^4}{24} - \frac{1}{2}\left(\frac{x^4}{4} + \ldots\right) + \ldots\right]$ $= -\left[-\frac{x^2}{2} + \frac{x^4}{24} - \frac{x^4}{8} + \ldots\right]$ $= \frac{x^2}{2} + \frac{x^4}{12} + \ldots$ (shown)	**Follow the given instruction**: Use standard series from MF26. Repeated differentiation is NOT allowed. Standard series which are found in MF26 are: • $(1+x)^n$ • e^x • $\sin x$ and $\cos x$ • $\ln(1+x)$ We do not have standard series of sec x so use trigo identity and law of logarithm to change $\ln\left(\frac{1}{\cos x}\right)$ to $-\ln(\cos x)$. When expanding $-\ln\left(1+\left(-\frac{x^2}{2}+\frac{x^4}{24}\right)+\ldots\right)$, treat $\left(-\frac{x^2}{2}+\frac{x^4}{24}\right)$ as a single term and apply MF26 formula.

	Putting $x=\frac{\pi}{4}$, $\ln\left(\sec\frac{\pi}{4}\right)\approx\frac{1}{2}\left(\frac{\pi}{4}\right)^2+\frac{1}{12}\left(\frac{\pi}{4}\right)^4$ $\ln\left(\frac{1}{\frac{1}{\sqrt{2}}}\right)\approx\frac{\pi^2}{2(16)}+\frac{\pi^4}{12(256)}$ $\ln\sqrt{2}\approx\frac{\pi^2}{2(16)}+\frac{\pi^4}{12(256)}$ $\frac{1}{2}\ln 2\approx\frac{\pi^2}{2(16)}+\frac{\pi^4}{12(256)}$ $\ln 2\approx\frac{\pi^2}{16}+\frac{\pi^4}{1536}$, $m=16$ and $n=1536$ (Shown)	**Follow the given instruction**: You are asked to substitute $x=\frac{\pi}{4}$.
	$\ln(\sec x)=\frac{x^2}{2}+\frac{x^4}{12}+\ldots$ From MF26, $\int\tan x\,\mathrm{d}x=\ln(\sec x)+C$. Differentiating w.r.t. *x*, $\tan x=x+\frac{x^3}{3}+\ldots$	**Make full use of MF26** You need to "Deduce" the series expansion for tan *x*. You need to use the result $\ln(\sec x)=\frac{x^2}{2}+\frac{x^4}{12}+\ldots$
4(a) (i)		Method 1: $y=\mathrm{f}(x)\rightarrow y=\mathrm{f}(x+3)\rightarrow y=\mathrm{f}(-x+3)$ Step 1: Replace *x* with *x* + 3 Translate graph 3 units in the negative *x*-direction Step 2: Replace *x* with -*x* Reflect the graph in the *y*-axis Method 2: $y=\mathrm{f}(x)\rightarrow y=\mathrm{f}(-x)\rightarrow y=\mathrm{f}(-(x-3))$ $y=\mathrm{f}(3-x)$ Step 1: Replace *x* with -*x* Reflect the graph in the *y*-axis Step 2: Replace *x* with *x* – 3 Translate graph 3 units in the positive *x*-direction
(a) (ii)		You need to sketch the tail-end behaviour correctly. As $x\rightarrow\pm\infty$, $\mathrm{f}(x)\rightarrow\pm\infty$ and $\frac{1}{\mathrm{f}(x)}\rightarrow 0$. Label the features of your graph as stated in the question. Note that $(2,-\frac{1}{2})$ should be positioned lower than $(-7,-\frac{1}{10})$.

4(b)	$a(x-2)^2 = a - \frac{1}{[f(x)]^2}$ $a(x-2)^2 = a - y^2$ $(x-2)^2 + \frac{y^2}{a} = 1$ According to the sketch on the diagram in part**(a)(ii)**, $a > \left(\frac{1}{2}\right)^2 = \frac{1}{4}$ for there to be 2 distinct real roots.	In **(a)(ii)**, $y = \frac{1}{f(x)}$. So replace $\frac{1}{[f(x)]^2}$ as y^2 $(x-2)^2 + \frac{y^2}{a} = 1$ is an ellipse with centre (2, 0)
5(a)	$x+1 = \frac{z-2}{3}$, $y = 3$ can be written as $r = \begin{pmatrix} -1 \\ 3 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix}$, $\lambda \in \mathbb{R}$ Normal to π_1 : $\left[\begin{pmatrix} -1 \\ 3 \\ 2 \end{pmatrix} - \begin{pmatrix} -3 \\ 1 \\ 0 \end{pmatrix}\right] \times \begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix}$ $= \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix} \times \begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix}$ $= \begin{pmatrix} 6 \\ -4 \\ -2 \end{pmatrix} = 2\begin{pmatrix} 3 \\ -2 \\ -1 \end{pmatrix}$ Hence, π_1 is perpendicular to $\begin{pmatrix} 3 \\ -2 \\ -1 \end{pmatrix}$ $\begin{pmatrix} 3 \\ -2 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} -3 \\ 1 \\ 0 \end{pmatrix} = -9-2 = -11$ Equation of π_1 : $3x - 2y - z = -11$	 Note the difference between "Show" and "Verify". To show, we are not allowed to use the given result. You need to show the given result. Hence, you should not be verifying $\begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -2 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -2 \\ -1 \end{pmatrix}$ equal to 0. This is a "show" question. You should write $\begin{pmatrix} 6 \\ -4 \\ -2 \end{pmatrix} = 2\begin{pmatrix} 3 \\ -2 \\ -1 \end{pmatrix}$ to illustrate that the normal is // to $\begin{pmatrix} 3 \\ -2 \\ -1 \end{pmatrix}$ and provide a conclusion.
(b)	**Method 1** Eq of perpendicular which pass through *B*: $\mathbf{r} = \begin{pmatrix} 5 \\ -6 \\ 10 \end{pmatrix} + \beta \begin{pmatrix} 3 \\ -2 \\ -1 \end{pmatrix}$, $\beta \in \mathbb{R}$ $\left[\begin{pmatrix} 5 \\ -6 \\ 10 \end{pmatrix} + \beta \begin{pmatrix} 3 \\ -2 \\ -1 \end{pmatrix}\right] \cdot \begin{pmatrix} 3 \\ -2 \\ -1 \end{pmatrix} = -11$ $15 + 9\beta + 12 + 4\beta - 10 + \beta = -11$ $14\beta = -28$ $\beta = -2$	Method 1: See *N* as the intersection of line and plane Method 2: See $\overrightarrow{BN}$ as projection of $\overrightarrow{BA}$ on normal of plane and $\overrightarrow{ON} = \overrightarrow{OB} + \overrightarrow{BN}$ (or $\overrightarrow{NB}$ as projection of $\overrightarrow{AB}$ on normal of plane)

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$\overrightarrow{ON}=\begin{pmatrix}5\\-6\\10\end{pmatrix}-2\begin{pmatrix}3\\-2\\-1\end{pmatrix}=\begin{pmatrix}-1\\-2\\12\end{pmatrix}$

Method 2

$\overrightarrow{BA}=\begin{pmatrix}-3\\1\\0\end{pmatrix}-\begin{pmatrix}5\\-6\\10\end{pmatrix}=\begin{pmatrix}-8\\7\\-10\end{pmatrix}$

$\overrightarrow{BN}=\left[\begin{pmatrix}-8\\7\\-10\end{pmatrix}\cdot\frac{\begin{pmatrix}3\\-2\\-1\end{pmatrix}}{\sqrt{9+4+1}}\right]\frac{\begin{pmatrix}3\\-2\\-1\end{pmatrix}}{\sqrt{9+4+1}}$

$=\frac{-24-14+10}{14}\begin{pmatrix}3\\-2\\-1\end{pmatrix}=-2\begin{pmatrix}3\\-2\\-1\end{pmatrix}$

$\overrightarrow{ON}=\overrightarrow{OB}+\overrightarrow{BN}=\begin{pmatrix}5\\-6\\10\end{pmatrix}+\begin{pmatrix}-6\\4\\2\end{pmatrix}=\begin{pmatrix}-1\\-2\\12\end{pmatrix}$

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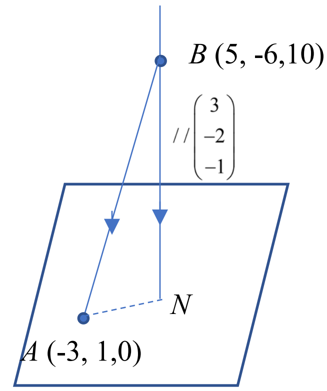
Sketch a diagram with crucial information to help you visualize what is going on.



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Method 1

Let *A'* be the reflection of *A* in line *BN*

$\overrightarrow{ON}=\frac{\overrightarrow{OA}+\overrightarrow{OA'}}{2}$

$\overrightarrow{OA'}=2\begin{pmatrix}-1\\-2\\12\end{pmatrix}-\begin{pmatrix}-3\\1\\0\end{pmatrix}=\begin{pmatrix}1\\-5\\24\end{pmatrix}$

$\overrightarrow{BA'}=\begin{pmatrix}1\\-5\\24\end{pmatrix}-\begin{pmatrix}5\\-6\\10\end{pmatrix}=\begin{pmatrix}-4\\1\\14\end{pmatrix}$

Equation of line *BA'*: $\mathbf{r}=\begin{pmatrix}5\\-6\\10\end{pmatrix}+\delta\begin{pmatrix}-4\\1\\14\end{pmatrix},\ \delta\in\mathbb{R}$

Method 2

Let *A'* be the reflection of *A* in line *BN*

$\overrightarrow{BN}=\frac{\overrightarrow{BA}+\overrightarrow{BA'}}{2}$

$\overrightarrow{BA'}=2\begin{pmatrix}-6\\4\\2\end{pmatrix}-\begin{pmatrix}-8\\7\\-10\end{pmatrix}=\begin{pmatrix}-4\\1\\14\end{pmatrix}$

Equation of line *BA'*: $\mathbf{r}=\begin{pmatrix}5\\-6\\10\end{pmatrix}+\delta\begin{pmatrix}-4\\1\\14\end{pmatrix},\ \delta\in\mathbb{R}$

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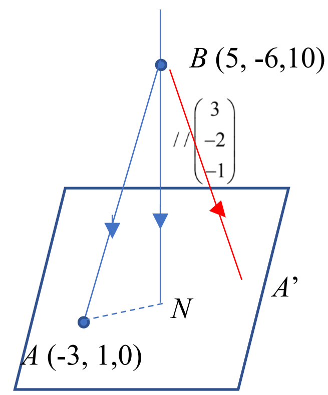

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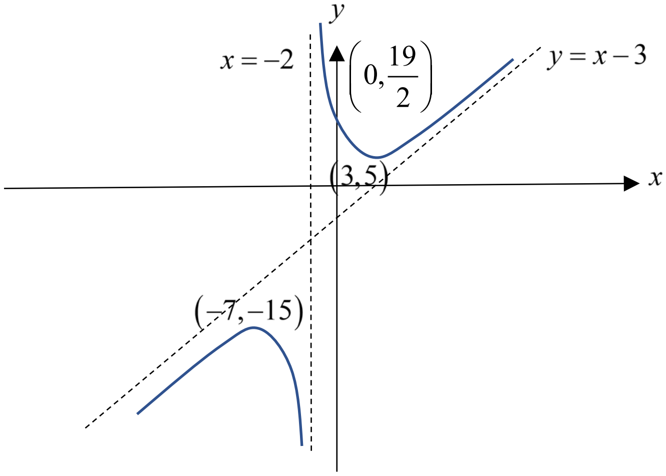
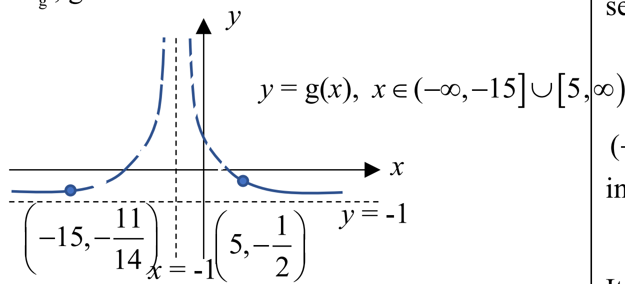
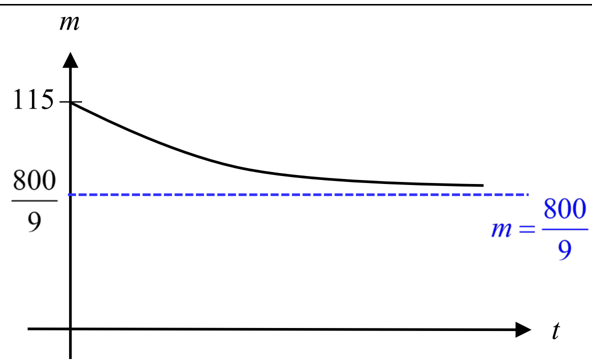
6(a)	$\frac{x^2-x+19}{x+2}=x-3+\frac{25}{x+2}$  	You should do long division or compare coefficient to make $\frac{x^2-x+19}{x+2}$ into a proper fraction to find the equation of oblique asymptote. You need to set $y_{\min}$ to be negative enough to see the lower half of the graph on your GC. Read question carefully and ensure you label the turning points, axial intercepts and equations of asymptotes
(b)	$R_f=(-\infty,-15]\cup[5,\infty)$ and $D_g=\mathbb{R}\setminus\{-1\}$ Since $R_f\subset D_g$, gf exist.  Let $R_f=(-\infty,-15]\cup[5,\infty)$ be the new Dg, $R_{gf}=(-1,-\frac{1}{2}]$	**Presentation:** Use correct interval notation OR set notation. You need to state both R_f and D_g. $(-\infty,-15],[5,\infty)$ or $(-\infty,-15]$ or $[5,\infty)$ are incorrect. You need to use correct notation $\cup$. It is easier to use R_f as the new domain of g to find R_{gf}. It is not easy to read the R_{gf} from the graph of gf
(c)	**Method 1** $gg(x)=\left\|\frac{3}{\left\|\frac{3}{x+1}\right\|-1+1}\right\|-1$ $=\|x+1\|-1$ $=\begin{cases}-x-2 & \text{if } x<-1\\ x & \text{if } x>-1\end{cases}$ Hence, $p(x)=-x-2$; $q(x)=x$ and $a=-1$	

	Method 2 $g(x)=\begin{cases}-\dfrac{3}{x+1}-1, & x<-1\\ \dfrac{3}{x+1}-1, & x>-1\end{cases}$ $gg(x)=\begin{cases}\dfrac{3}{-\dfrac{3}{x+1}-1+1}-1, & x<-1\\ \dfrac{3}{\dfrac{3}{x+1}-1+1}-1, & x>-1\end{cases}$ $=\begin{cases}-x-2, & x<-1\\ x, & x>-1\end{cases}$ Hence, $p(x)=-x-2$; $q(x)=x$ and $a=-1$	Note that for both $x>-1$ and $x<-1$, $g(x)>-1$.
7(a)	Let m kg be Mr Eccles' mass $\dfrac{dm}{dt}=k(C-36m)$ Mass is maintained constant at 115 kg, $\dfrac{dm}{dt}=0$ $k(C-36m)=0$ $C=36(115)=4140$ Alternatively, Mr Eccles needs $36(115)=4140$ calories to meet his daily energy needs. If he consumes 4140 calories a day, there will be no excess calories and hence his weight will be constant. Hence, $C=4140$.	It is not enough to write $C=36(115)=4140$. You need to justify your answer.
(b)	Calorie intake = 3200 $\dfrac{dm}{dt}=k(3200-36m)$ $-\dfrac{1}{36}\int\dfrac{-36}{3200-36m}\,dm=\int k\,dt$ $\ln\lvert 3200-36m\rvert=-36kt+c$ $\lvert 3200-36m\rvert=Ae^{-36kt}$ (where $A=e^{c}$) $3200-36m=Be^{-36kt}$ $36m=3200-Be^{-36kt}$ $m=\dfrac{800}{9}-De^{-36kt}$ (where $D=\dfrac{B}{36}$)	**Read and unpack question carefully**: "The rate of change of the body mass is proportional to the number of excess calories per day which is calculated by subtracting the calories to meet his daily energy needs from his calorie intake per day." $\dfrac{dm}{dt}=k(3200-36m)$ Calories to meet his daily energy needs = $36m$ Replace modulus first before substituting in the conditions.

	Given that when $t = 0$, $m = 115$ $115 = \frac{800}{9} - De^{0}$ $D = -\frac{235}{9} \approx -26.111$ Given that when $t = 20$, $m = 115 - 4 = 111$ $111 = \frac{800}{9} - \left(-\frac{235}{9}\right)e^{-36k(20)}$ $\frac{199}{235} = e^{-720k}$ $k = -\frac{1}{720}\ln\frac{199}{235} \approx 2.3094 \times 10^{-4}$ Hence $m = \frac{800}{9} + \frac{235}{9}e^{\left(\frac{1}{20}\ln\frac{199}{235}\right)t}$ (i.e. $m = \frac{800}{9} + \frac{235}{9}\left(\frac{199}{235}\right)^{\frac{t}{20}}$, $m = 88.9 + 26.1e^{-0.00831t}$)	
(c)	 Mr Eccles is <u>not able to meet</u> his target of 85 kg since from the graph, it is observed that Mr Eccles' mass will <u>decrease with time to approach but not go below</u> $\frac{800}{9} = 88.9$ (3 sf) .	**Answer in context:** Sketch graph for $t \geq 0$ only and label equation of the asymptote. You need to explain why Mr Eccles is not able to meet his target.
(d)	Let the greatest daily calorie intake be $C_{\max}$ $C_{\max} - 36m = Be^{-36kt}$ $36m = C_{\max} - Be^{-36kt}$ $m = \frac{C_{\max}}{36} - De^{-36kt}$, where $D = \frac{B}{36}$ The equation of the horizontal asymptote is $m = \frac{C_{\max}}{36}$. To reach his target of 85 kg $\frac{C_{\max}}{36} < 85$ $C_{\max} < 3060$ Hence the greatest daily calorie intake to reach the 85 kg target is **3059**.	If he consumes 3060 calories, his weight will tend to 85 kg, but never reached 85 kg. Hence, his greatest daily calories intake to meet his target is 3059.

8(a)	**<u>Method 1: Union</u>** **Case 1**: No. of codes that start and end with the same letter $\text{P(case 1)} = \frac{1}{26}$ **Case 2**: No. of codes with exactly 2 digits the same $\text{P(case 2)} = \frac{1}{9} \times \frac{8}{9} \times \frac{3!}{2!} = \frac{8}{27}$ **Case 3**: No. of codes that start and end with the same letter and have exactly 2 digits the same $\text{P(case 3)} = \frac{1}{26} \times \frac{1}{9} \times \frac{8}{9} \times \frac{3!}{2!} = \frac{4}{351}$ Hence, required probability $= \frac{1}{26} + \frac{8}{27} - \frac{4}{351} = \frac{227}{702} \approx 0.323$ **<u>Method 2: Union</u>** **Case 1**: No. of codes that start and end with the same letter $= 26 \times 1 \times 9^3 = 18954$ **Case 2**: No. of codes with exactly 2 digits the same $= 26 \times 26 \times 9 \times 8 \times \frac{3!}{2!}$ (**OR** $26^2 \times {}^9C_2 \times \frac{3!}{2!} \times 2$) $= 146016$ **Case 3**: No. of codes that start and end with the same letter and have exactly 2 digits the same $= 26 \times 9 \times 8 \times \frac{3!}{2!}$ (**OR** ${}^{26}C_1 \times {}^9C_2 \times \frac{3!}{2!} \times 2$) $= 5616$ Required probability $= \frac{18954 + 146016 - 5616}{26^2 \times 9^3} = \frac{227}{702} \approx 0.323$ **<u>Method 3: Direct approach</u>** **Case 1**: No. of codes that start and end with the same letter and any 3 digits $\text{P}(\text{Case 1}) = \frac{26 \times 9^3}{26^2 \times 9^3} = \frac{1}{26}$ **Case 2**: No. of codes with both letters different and exactly 2 digits the same $\text{P}(\text{Case 2}) = \frac{26 \times 25 \times 9 \times 8 \times \frac{3!}{2!}}{26^2 \times 9^3} = \frac{100}{351}$ Required probability = $= \frac{1}{26} + \frac{100}{351} = \frac{227}{702} \approx 0.323$	Let A: Event which codes that start and end with the same letter and B be event which codes with exactly 2 digits the same. Probability required $= \text{P}(A \cup B)$ $= \text{P}(A) + \text{P}(B) - \text{P}(A \cap B)$ Note that in this case, A and B are independent. Hence, $\text{P}(A \cap B) = \text{P}(A) \times \text{P}(B)$.

(b)	**Method 1** $\text{Required probability} = \frac{1}{26}\times\frac{5}{9}\times\frac{4}{9}\times\frac{4}{9}\times\frac{3!}{2!}\times\frac{25}{26}\times 2$ $= \frac{12000}{26^2 9^3}$ ≈ 0.0244 **Method 2** **Case 1: no repeated digits** $\text{No. of ways} = {}^{25}C_1 \times 2! \times {}^{5}C_1 \times {}^{4}C_2 \times 3! = 9000$ Out of 25 letters (excl E) choose 1; E can be first or last letter 5 odd digits choose 1, 4 even digits choose 2, arrange all 3 digits **Case 2: one repeated even digit** $\text{No. of ways} = {}^{25}C_1 \times 2! \times {}^{5}C_1 \times {}^{4}C_1 \times \frac{3!}{2!} = 3000$ Out of 25 letters (excl E) choose 1; E can be first or last letter 5 odd digits choose 1, 4 even digits choose 1 only (since it is repeated), arrange all 3 digits $\text{Hence, required probability} = \frac{9000+3000}{26^2 \times 9^3} \approx 0.0244$	
9(a)	The binomial distribution is appropriate in modelling the number of coffee-flavoured cookies in a box because the binomial conditions are justifiable: 1. Each box contains 12 selected cookies (i.e. 12 trials). 2. Each cookie is either 'coffee-flavoured' or 'not coffee-flavoured' (i.e. 2 possible outcomes). 3. The probability that a cookie is coffee-flavoured cookie is constant at 0.2. 4. The flavour of a cookie is independent of that of any other cookie since selections are made from large quantities of cookies	The question is asking for the 4 conditions to make B.D. an appropriate model, it is not asking for assumptions.
(b)	Let X be the number of coffee-flavoured cookies in a box of 12 cookies. $X \sim B(12, 0.2)$ $P(4 \le X \le 8) = P(X \le 8) - P(X \leqslant 3)$ $= 0.20537 \approx 0.205$	You need to define your variable and state its distribution before proceeding with the calculation. "no more than 8" means ≤ 8

(c)	$E(\overline{X}) = E(X)$ $= 12 \times 0.2$ $= 2.4$ $\text{Var}(\overline{X}) = \dfrac{\text{Var}(X)}{50}$ $= \dfrac{12 \times 0.2 \times 0.8}{50}$ $= \dfrac{1.92}{50}$ Since $n = 50$ is large, by Central Limit Theorem, $\overline{X} \sim N\left(2.4, \dfrac{1.92}{50}\right)$ approximately $P(\overline{X} \geq 2.5) = 0.30492 \approx 0.305$	Clear presentation is expected. You need to write down "Since $n = 50$ is large, by Central Limit Theorem, $\overline{X} \sim N\left(2.4, \dfrac{1.92}{50}\right)$ approximately" before calculating $P(\overline{X} \geq 2.5)$
(d)	Let M be the number of matcha-flavoured cookies in a box of 12. $M \sim B(12, p)$ $P(M = 2) < P(M = 3)$ $\binom{12}{2} p^2 (1-p)^{10} < \binom{12}{3} p^3 (1-p)^9$ $66(1-p) < 220p$ since $p > 0$ and $1 - p > 0$ $p > \dfrac{3}{13}$ **And** $P(M = 3) > P(M = 4)$ $\binom{12}{3} p^3 (1-p)^9 > \binom{12}{4} p^4 (1-p)^8$ $220(1-p) > 495p$ since $p > 0$ and $1 - p > 0$ $p < \dfrac{4}{13}$ $\therefore \dfrac{3}{13} < p < \dfrac{4}{13}$	If 3 is the modal number, then $P(M = 3) > P(M = 2)$ **and** $P(M = 3) > P(M = 4)$. Read question, answer should be in exact form and reduced to simplest form.
10(a)	$\dfrac{1}{6} + a + \dfrac{2}{5} + b = 1$ $a + b = \dfrac{13}{30}$ -----------(1) Given mean number = $\dfrac{43}{10}$ $2\left(\dfrac{1}{6}\right) + 3a + 5\left(\dfrac{2}{5}\right) + 8b = \dfrac{43}{10}$ $3a + 8b = \dfrac{59}{30}$ -----------(2) Solving, $a = \dfrac{3}{10}$, $b = \dfrac{2}{15}$	

(b)

X	**2**	**3**	**5**	**8**
2	0	1	3	6
3	1	0	2	5
5	3	2	0	3
8	6	5	3	0

$$q = \mathrm{P}(X=0) = \left(\frac{1}{6}\right)^2 + \left(\frac{3}{10}\right)^2 + \left(\frac{2}{5}\right)^2 + \left(\frac{2}{15}\right)^2$$

$$= \frac{133}{450}$$

$$r = \mathrm{P}(X=2) = 2\left(\frac{3}{10}\right)\left(\frac{2}{5}\right) = \frac{6}{25} \text{ (or 0.24)}$$

$$s = \mathrm{P}(X=3) = 2\left(\frac{1}{6}\right)\left(\frac{2}{5}\right) + 2\left(\frac{2}{5}\right)\left(\frac{2}{15}\right) = \frac{6}{25} \text{ (or 0.24)}$$

Probability Distribution of X:

x	0	1	2	3	5	6
$\mathrm{P}(X=x)$	$q=\frac{133}{450}$	$\frac{1}{10}$	$r=\frac{6}{25}$	$s=\frac{6}{25}$	$\frac{2}{25}$	$\frac{2}{45}$

You can check your answer by adding up the probabilities to make sure it equals to 1.

(c)

$$\mathrm{P}(X<3.5) = 1-\mathrm{P}(X=5)-\mathrm{P}(X=6)$$

$$=1-\left(\frac{2}{25}+\frac{2}{45}\right)$$

$$=\frac{197}{225} \text{ or } 0.876$$

Alternatively,

$$\mathrm{P}(X<3.5) = \mathrm{P}(X=0)+\mathrm{P}(X=1)+\mathrm{P}(X=2)+\mathrm{P}(X=3)$$

$$=\frac{197}{225} \text{ or } 0.876$$

It is safer to use $\mathrm{P}(X<3.5)$ as $1-\mathrm{P}(X=5)-\mathrm{P}(X=6)$. The values of $\mathrm{P}(X=5)$ and $\mathrm{P}(X=6)$ have been given in the question.

(last part)

$$\mathrm{E}(X) = \frac{1}{10}+2\left(\frac{6}{25}\right)+3\left(\frac{6}{25}\right)+5\left(\frac{2}{25}\right)+6\left(\frac{2}{45}\right)$$

$$=\frac{59}{30} \approx 1.96667$$

$\mathrm{E}(2X) > k$

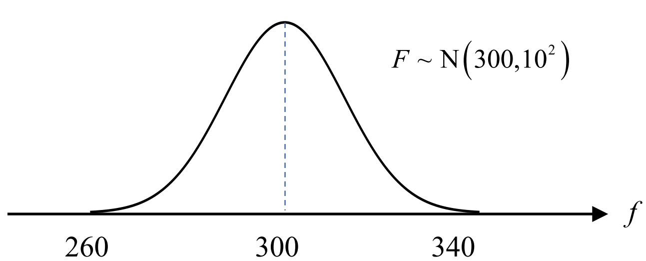
$2\mathrm{E}(X) > k$

$\therefore 0 < k < 3.93$

Final answer in 2 d.p. for money

11(a)

Let F g be the mass of a fruit, J g be the mass of a juice, T g be the total mass of a fruit and juice



Follow the given instructions: The bell-shaped curve must be labelled with both tails almost touching axis when near 260 and 340

(b)	$F \sim \mathrm{N}\left(300, 10^2\right)$ $\overline{F} \sim \mathrm{N}\left(300, \frac{10^2}{n}\right)$ $\mathrm{P}(\overline{F} > 301) = 0.233$ $\mathrm{P}\left(Z > \frac{301-300}{\frac{10}{\sqrt{n}}}\right) = 0.233$ $\frac{301-300}{\frac{10}{\sqrt{n}}} \approx 0.72900$ $\sqrt{n} \approx 7.2900$ $n \approx 53.1441$ $n = 53$	**Presentation:** You have been explicitly told to state **clearly** the values of the parameters of any normal distribution you use. You need to state distribution of $\overline{F}$. Do normal rounding off here.
(c)	$F \sim \mathrm{N}\left(300, 10^2\right)$, $T \sim \mathrm{N}\left(400, 8^2\right)$ $F_1 + F_2 + F_3 + F_4 - 3T \sim \mathrm{N}\left(4(300) - 3(400), 4(10^2) + 3^2(8^2)\right)$ $K = F_1 + F_2 + F_3 + F_4 - 3T \sim \mathrm{N}\left(0, 976\right)$ $\mathrm{P}(\lvert K \rvert \le 10) = P\left(-10 \le K \le 10\right)$ $= 0.25110 \approx 0.251$	Read question carefully Note that $F_1 + F_2 + F_3 + F_4 \ne 4F$ "Differ" implies the difference is between -10 and 10.
(d)	$J = T - F$ $J \sim \mathrm{N}\left(400 - 300, 8^2 + 10^2\right)$ (i.e. $J \sim \mathrm{N}\left(100, 164\right)$) $P(J > 110) = 0.21744 \approx 0.217$	Note that $\mathrm{Var}\left(T - F\right) = \mathrm{Var}(T) + \mathrm{Var}(F)$
	The total mass of fruit and juice in a randomly chosen tin and the mass of fruit in a randomly chosen tin are independent. i.e. T and F are independent.	In (c), we calculate var($F_1 + F_2 + F_3 + F_4 - 3T$) and in (d), we calculate var(T – F). For both cases, we need to assume T and F to be independent.