Chapter 15 D.C. CIRCUITS

Content

- Circuit symbols and diagrams
- Series and parallel arrangements
- Potential divider
- Balanced potentials

Learning Outcomes

Candidates should be able to:

(a) recall and use appropriate circuit symbols as set out in the ASE publication *Signs, Symbols and Systematics* (*The ASE Companion to 16–19 Science, 2000*).

(b) draw and interpret circuit diagrams containing sources, switches, resistors, ammeters, voltmeters, and/or any other type of component referred to in the syllabus.

(c) solve problems using the formula for the combined resistance of two or more resistors in series.

(d) solve problems using the formula for the combined resistance of two or more resistors in parallel.

(e) solve problems involving series and parallel circuits for one source of e.m.f.

(f) show an understanding of the use of a potential divider circuit as a source of variable p.d.

(g) explain the use of thermistors and light-dependent resistors in potential dividers to provide a potential difference which is dependent on temperature and illumination respectively.

(h) recall and solve problems by using the principle of the potentiometer as a means of comparing potential differences.

15.1 Introduction

Direct current & Alternating current

Direct current (d.c.) is the *unidirectional* flow of electric charge. This flow of electric charge in a constant direction distinguishes it from the alternating current (a.c.). Such direct currents are produced by sources such as batteries and solar cells.

Direct currents may also be obtained from an alternating current supply by the use of a rectifier, which allows current to flow only in one direction. Similarly, direct currents can also be converted into alternating currents with the use of an inverter.

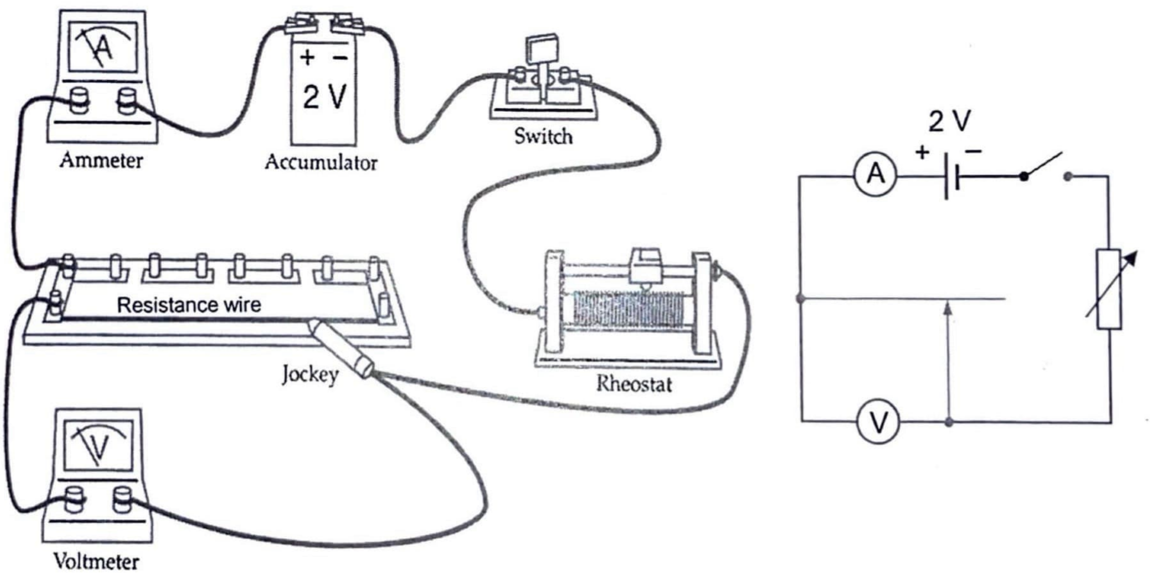
The first commercial electric power transmission that was developed by Thomas Edison in the late 19^{th} century used direct current. Today, direct current is used in rail transport, transmission of large amounts of power from remote generation sites, as well as to charge batteries.

15.2 Practical Circuits

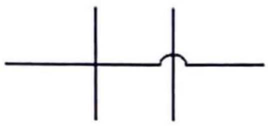
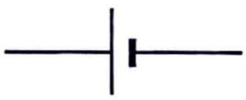
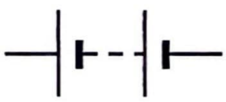
Circuit Diagrams

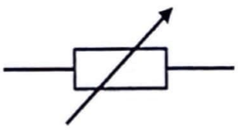
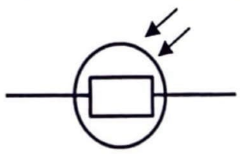
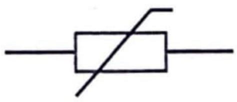
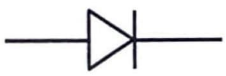
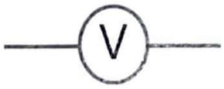
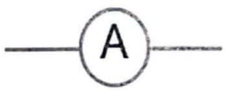
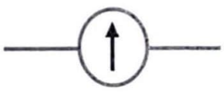
All circuit diagrams are drawn with wires and electrical components. Only essential features of a practical circuit are drawn and all electrical devices are represented by standard symbols.

The figure below shows an actual circuit arrangement with its circuit diagram. Note that inter-connected wires must be indicated with a dot (•).



Circuit Symbols The following is a list of some common circuit symbols used in the A-level syllabus.

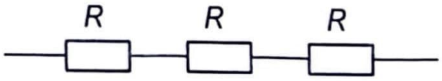
Category	Component	Circuit Symbol	Function of Component
Wires and Connections	Wire		To pass current easily from one part of a circuit to another.
	Wires that are connected		This symbol is used in circuit diagrams where wires cross to show that they are connected (joined).
	Wires that are not connected		In complex circuit diagrams it is often necessary to draw wires crossing even though they are not connected. You may prefer to use the 'hump' symbol shown on the right, because the simple crossing on the left looks like connection.
Power Supplies	Cell		Supplies electrical energy. Single cells are often wrongly called batteries; strictly, a battery comprises two or more cells joined together. The side with the longer line is the positive terminal.
	Battery		Supplies electrical energy. A battery is more than one cell joined together. The side with the longer line is the positive terminal.
	D.C. supply		Supplies electrical energy.
	A.C. supply		Supplies electrical energy.
	Earth (ground)		A connection to earth. For many electronic circuits, this is the 0 V (zero volt) of the power supply, but for mains electricity and some radio circuits, it really means the earth. It is also known as ground.
Lamps	Lamp		A transducer which converts electrical energy to light. This symbol is used for a lamp providing illumination, e.g. a car headlamp or torch bulb.
	Indicator or light source		

Category	Component	Circuit Symbol	Function of Component
Switches	On-Off switch		An on-off switch allows current to flow only when it is in the closed (on) position.
Resistors	Resistor		A resistor restricts the flow of current, for example to limit the current passing through an LED.
	Variable resistor		This type of variable resistor (e.g. a rheostat) is usually used to control current.
	Light Dependent Resistor (LDR)		An LDR is a semiconductor device. Its resistance decreases as the brightness (illumination) of light falling on the LDR increases.
	Thermistor		A thermistor is a semiconductor device. The resistance of most thermistors decreases as their temperature increases (negative temperature coefficient or N.T.C. type).
Diodes	Diode		A device which only allows current to flow in one direction.
	Light Emitting Diode (LED)		A transducer which converts electrical energy to light.
	Voltmeter		A voltmeter is used to measure potential difference.
	Ammeter		An ammeter is used to measure current.
	Galvanometer		A galvanometer is a very sensitive meter which is used to detect tiny currents, usually 1 mA or less.
	Ohmmeter		An ohmmeter is used to measure resistance. Most multimeters have an ohmmeter setting.
	Oscilloscope		An oscilloscope is used to display the shape of electrical signals and it can be used to measure their p.d. and period.

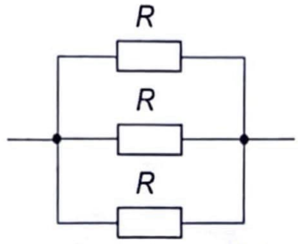
15.3 Series and Parallel Arrangements

Combination of Resistors

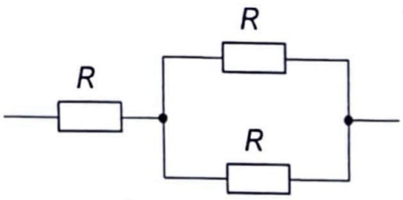
The following are four possible combinations of three resistors:

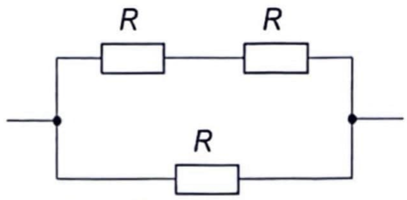


resistors in **series**



resistors in **parallel**





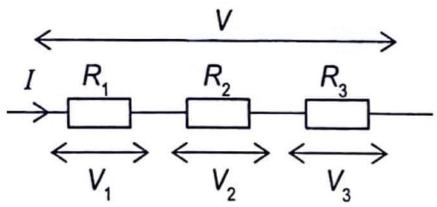
resistors in a **combination of series and parallel** arrangement

equivalent/ effective resistance

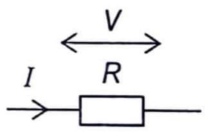
It is possible to replace a combination of resistors in any given circuit with a single resistor without altering the p.d. and current across the terminals of the combination. The resistance of this single resistor is called the *equivalent resistance* of the combination.

Resistors in Series

Consider 3 resistors of resistances R_1, R_2 and R_3 connected in series.
Current I through each resistor is <u>the same</u>.



$\Leftrightarrow$



$V_1 = IR_1 \qquad V_2 = IR_2 \qquad V_3 = IR_3$

$$V = V_1 + V_2 + V_3$$
$$= IR_1 + IR_2 + IR_3$$
$$\frac{V}{I} = R_1 + R_2 + R_3 \quad \cdots\cdots(1)$$

$V = IR$

$$R = \frac{V}{I} \quad \cdots\cdots(2)$$

Comparing equations (1) & (2), the equivalent resistance $R = R_1 + R_2 + R_3$

Hence for a case of n resistors in series, the equivalent resistance is

$$\boxed{R = R_1 + R_2 + \ldots + R_n}$$

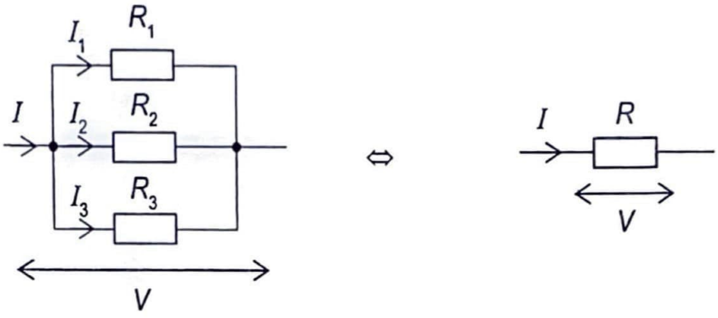
Resistors in Parallel

Consider 3 resistors of resistances R_1, R_2 and R_3 connected in parallel.
Potential difference *V* across each resistor is <u>the same</u>.

$$I_1 = \frac{V}{R_1}$$

$$I_2 = \frac{V}{R_2}$$

$$I_3 = \frac{V}{R_3}$$



$$I = I_1 + I_2 + I_3$$

$$I = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

$$\frac{I}{V} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \quad \text{......(1)}$$

$$V = IR$$

$$\frac{1}{R} = \frac{I}{V} \quad \text{......(2)}$$

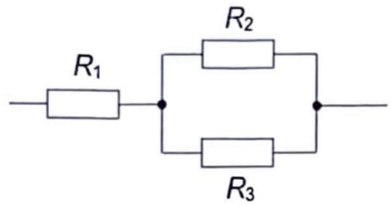
Comparing equations (1) & (2), the equivalent resistance R can be expressed as

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Hence for a case of n resistors in parallel, the equivalent resistance R can be expressed as

$$\boxed{\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n}}$$

Example 1 Three resistors with R_1 = 2.0 Ω, R_2 = 4.0 Ω and R_3 = 6.0 Ω are connected as shown below.



Calculate

(a) the equivalent resistance of the combination,

(b) the current that passes through the combination if a potential difference of 8.0 V is applied to the terminals,

(c) the potential difference across each resistor and the current flowing through each resistor.

[Solution]

(a) $$\frac{1}{R_{23}} = \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{4.0} + \frac{1}{6.0}$$

$$\Rightarrow R_{23} = 2.4\ \Omega$$

$$R_{eq} = R_1 + R_{23} = 2.0 + 2.4 = 4.4\ \Omega$$

(b) $$I = \frac{V}{R_{eq}} = \frac{8.0}{4.4} = 1.818 = 1.8\ \text{A}$$

(c)

current flowing through R_1 = 1.818 A
using $V = IR$
p.d. across R_1 = 1.818 × 2.0 = 3.64 V,

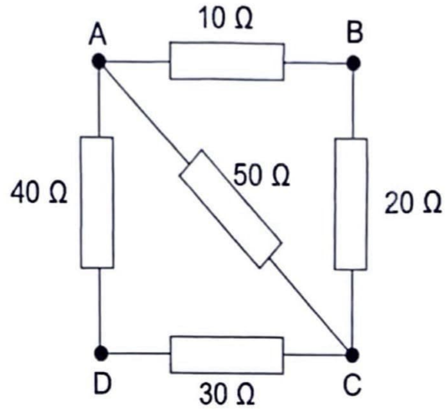
p.d. across R_2 = p.d. across R_3 = p.d. across R_{23} = 8.0 – 3.64 = 4.36 V
OR
p.d. across R_2 = p.d. across R_3 = p.d. across R_{23} = 1.818 × 2.4 = 4.36 V

current in $R_1 = I = 1.8$ A

current in $R_2 = \frac{4.36}{4.0} = 1.1$ A

current in $R_3 = \frac{4.36}{6.0} = 0.73$ A

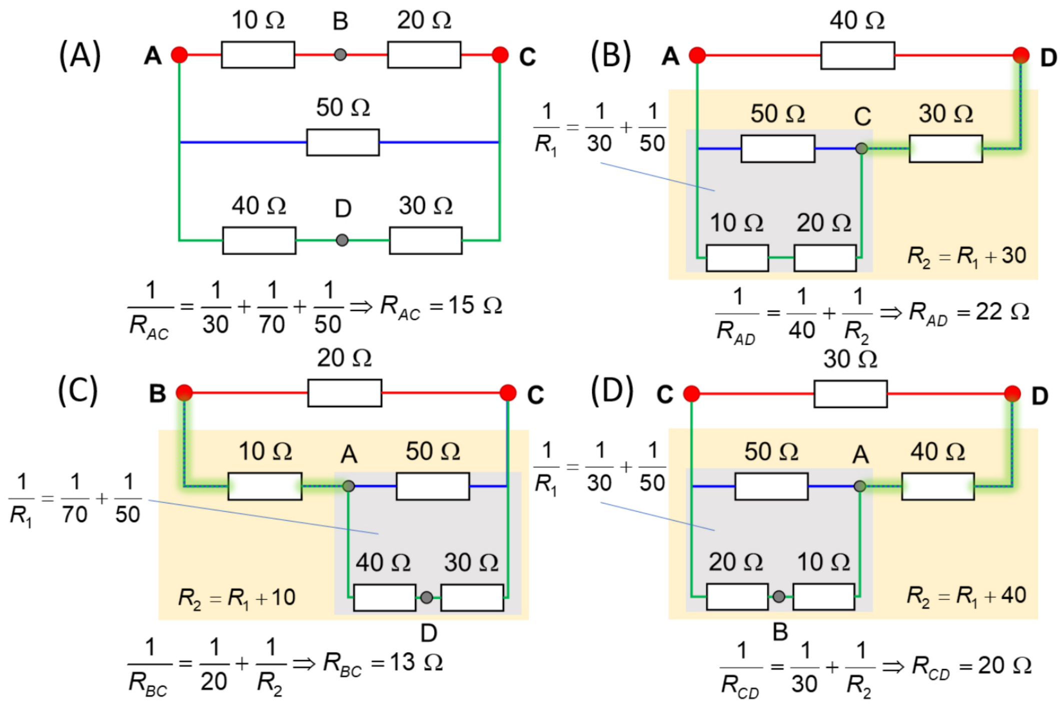
Example 2 Five resistors are connected as shown.



Between which two points is the resistance of the combination a maximum?

A A and C **B** A and D
C B and C **D** C and D

[Solution]



(A) $\frac{1}{R_{AC}} = \frac{1}{30} + \frac{1}{70} + \frac{1}{50} \Rightarrow R_{AC} = 15\ \Omega$

(B) $\frac{1}{R_1} = \frac{1}{30} + \frac{1}{50}$, $R_2 = R_1 + 30$, $\frac{1}{R_{AD}} = \frac{1}{40} + \frac{1}{R_2} \Rightarrow R_{AD} = 22\ \Omega$

(C) $\frac{1}{R_1} = \frac{1}{70} + \frac{1}{50}$, $R_2 = R_1 + 10$, $\frac{1}{R_{BC}} = \frac{1}{20} + \frac{1}{R_2} \Rightarrow R_{BC} = 13\ \Omega$

(D) $\frac{1}{R_1} = \frac{1}{30} + \frac{1}{50}$, $R_2 = R_1 + 40$, $\frac{1}{R_{CD}} = \frac{1}{30} + \frac{1}{R_2} \Rightarrow R_{CD} = 20\ \Omega$

In ascending order of equivalent resistance: C < A < D < B

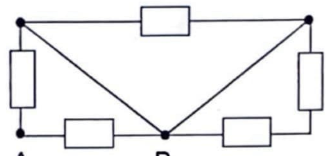
The resistance is maximum between A and D. (Option B)

Simplification of D.C. Circuits

Strategy for simplfying D.C circuits:

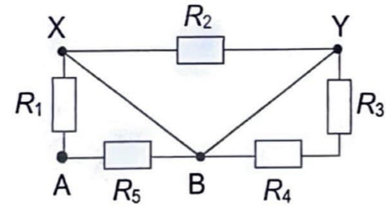
1. Identify and label junctions or points in the circuit that are of the same potential
2. Remove redundant circuit components i.e. where
   - $I_{component} = 0$ or
   - $\Delta V_{component} = 0$
3. Redraw equivalent circuit.

Example 3 Five resistors, each of resistance *R*, are connected as shown. Find the equivalent resistance between points A and B.



[Solution]

Step 1:

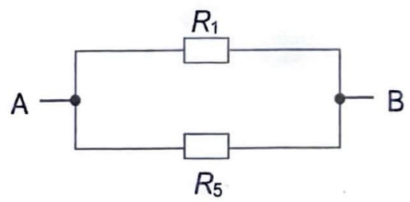


$$V_X = V_B = V_Y$$

Step 2:

- $\Delta V_{XY} = 0$ => Remove
- R_3 and R_4 are in series and $\Delta V_{BY} = 0$=> Remove
- $\Delta V_{AX} = V_X - V_A = V_B - V_A = \Delta V_{AB}$

Step 3:



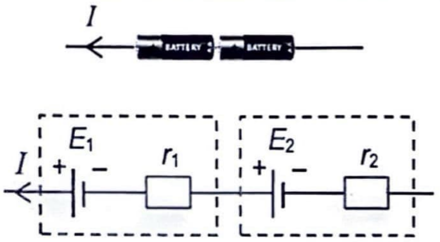
$$R_{eq} = \frac{R_1 R_5}{R_1 + R_5} = \frac{R^2}{2R} = \frac{R}{2}$$

Combination of E.M.F. Cells

In many devices that use batteries, such as portable radios and flashlights, it is insufficient to use just one cell at a time. Cells are usually grouped together in a serial arrangement to increase the voltage or in a parallel arrangement to increase current.

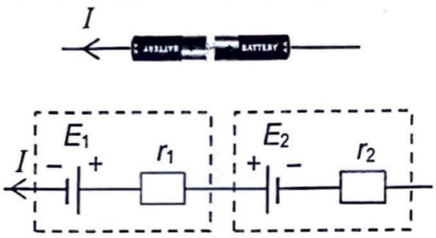
1) in series

For two e.m.f. cells in series, there are two possible combinations:



Total e.m.f. $E = E_1 + E_2$

Total internal resistance $r = r_1 + r_2$

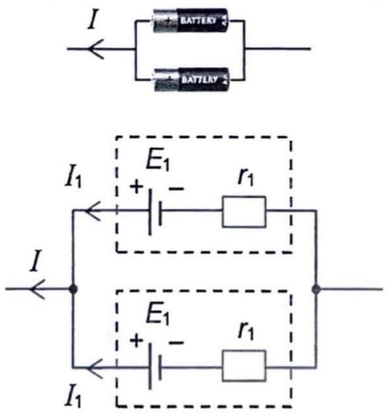


Total e.m.f. $E = |E_2 - E_1|$

Total internal resistance $r = r_1 + r_2$

2) in parallel

For two identical cells with e.m.f. E_1 and internal resistance r_1 connected in parallel:



Total e.m.f. $E = E_1$
(p.d. across both cells are the same)

Total internal resistance r

$$\frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_1} = \frac{2}{r_1}$$

$$r = \left(\frac{2}{r_1}\right)^{-1} = \frac{1}{2}r_1$$

*[**Note:** When unlike cells are connected in parallel, there are no simple relationships and Kirchhoff's laws (see Appendix) must be applied.]*

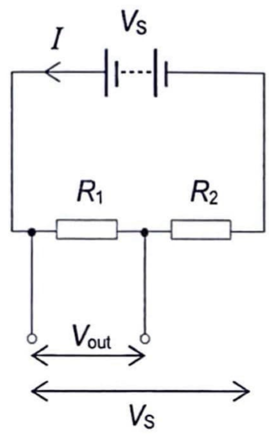
15.4 Potential Dividers

Principle of Potential Dividers

A ***potential divider*** is an arrangement of two or more resistors connected in *series* across an applied potential difference (p.d.), with the output p.d. V_{out} being taken from across one of the resistors.

The potential divider is used to produce an output p.d. V_{out} that is a ***fraction*** of the voltage supply V_S. The output p.d. V_{out} may then be connected to a load, such as a lamp.

The circuit below shows a simple potential divider, which consists of two known resistors of resistances R_1 and R_2 connected in series to a voltage supply V_S.
The output p.d. V_{out} is taken across resistor R_1.



Current $I = \dfrac{V_S}{R_1 + R_2}$

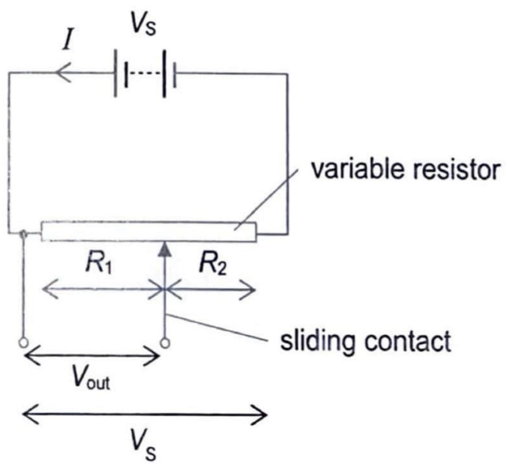
p.d. across R_1, $V_{out} = IR_1 = \left(\dfrac{V_S}{R_1 + R_2}\right)R_1$

Rearranging the RHS of the equation,

$$V_{out} = \left(\frac{R_1}{R_1 + R_2}\right)V_S$$

This shows that V_0 is a fraction of V_s.

To supply a **continuously variable potential difference** from zero to the full potential difference of the supply V_S, a resistor with a sliding contact may be used as shown below.



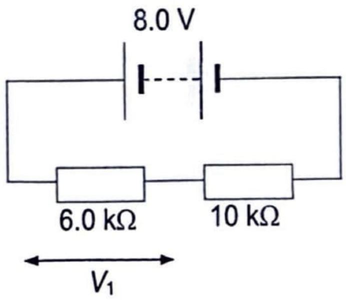
This variable resistor is also known as a potentiometer.

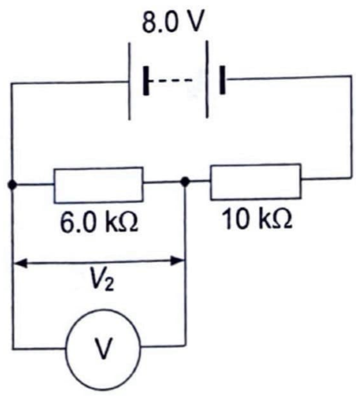
Example 4 A 10 kΩ and a 6.0 kΩ resistor are connected in series to a 8.0 V battery. The potential difference across the 6.0 kΩ resistor is measured by connecting a voltmeter of resistance 75 kΩ across it. Calculate

(a) the p.d. V_1 across the 6.0 kΩ resistor without the voltmeter,

(b) the p.d. V_2 across the 6.0 kΩ resistor in the presence of the voltmeter,

(c) the resulting percentage error in the measurement taken by the voltmeter.





[Solution]

(a) $V_1 = \dfrac{6.0}{16} \times 8.0 = 3.0 \text{ V}$

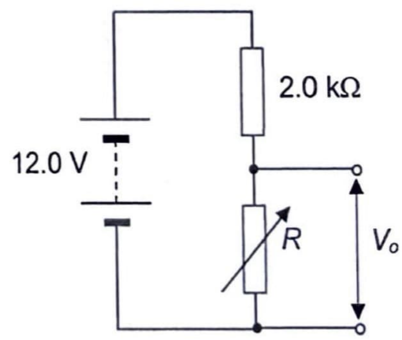
(b) Let the equivalent resistance of the 6.0 kΩ resistor and voltmeter be R.

$$\frac{1}{R} = \frac{1}{6.0} + \frac{1}{75} \quad \Rightarrow \quad R = 5.556 \text{ k}\Omega$$

$$V_2 = \frac{5.556}{5.556 + 10} \times 8.0 = 2.857 = 2.86 \text{ V}$$

(c) Percentage error = $\left|\dfrac{V_2 - V_1}{V_1}\right| \times 100\% = \left|\dfrac{2.857 - 3.0}{3.0}\right| \times 100\% = 4.8\%$

Example 5 A potential divider circuit which can be used to provide a variable output potential difference V_o is as shown in the figure. The resistance R of the variable resistor can be varied from 1.0 kΩ to 10 kΩ. Calculate the range of output potential difference V_o that can be produced.



[Solution]

When R = 1.0 kΩ,

$$V_o = \frac{1.0}{1.0 + 2.0} \times 12.0 = 4.0 \ V$$

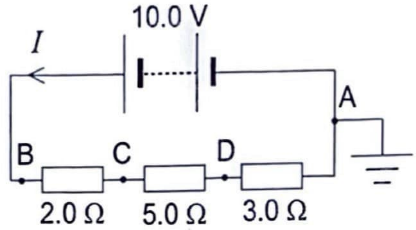
When R = 10 kΩ,

$$V_o = \frac{10}{10.0 + 2.0} \times 12.0. = 10.0 \ V$$

Range of V_o is 4.0 V to 10 V.

Potential at a Point

If the potential at a point in an electric circuit is known (similar to specifying a point of reference for GPE = *mgh* with *h* = 0), the potential at any other point in the circuit can be found. This is usually done by connecting a point to earth, which assigns 0 V to that point. In the circuit below, point A is earthed.



To find the potential at different points in the circuit,

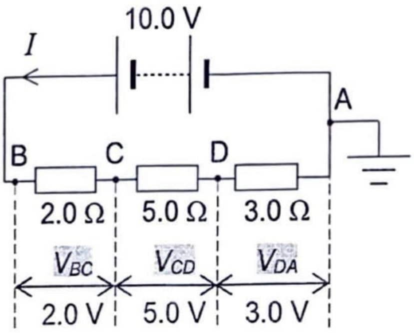
1) **Find the potential difference across each resistor.**

Using the potential divider principle,

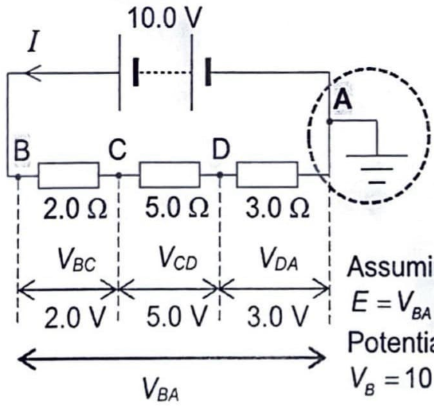
$$V_{BC} = \left(\frac{2.0}{2.0+5.0+3.0}\right)(10.0) = 2.0\text{ V}$$

$$V_{CD} = \left(\frac{5.0}{2.0+5.0+3.0}\right)(10.0) = 5.0\text{ V}$$

$$V_{DA} = \left(\frac{3.0}{2.0+5.0+3.0}\right)(10.0) = 3.0\text{ V}$$



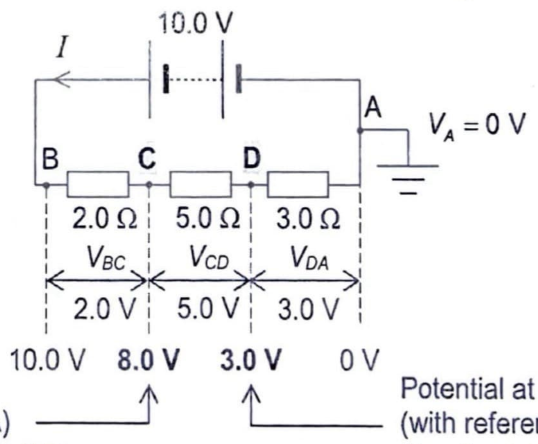
2) **Identify the known potentials**



Since A is **earthed**, it is assigned a potential of **0 V** i.e. $V_A = 0$ V

Assuming that the cell is ideal,
$E = V_{BA} = 10$ V
Potential at B (with reference to A)
$V_B = 10$ V

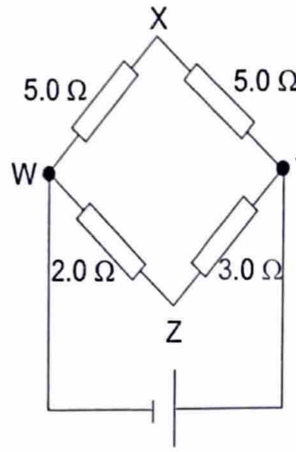
3) **Determine the potentials at the other points** based on the p.d. calculated in step 1 and the identified potentials of B and A as determined in step 2.



Potential at **C** (with reference to A)
$V_C = 0 + 3.0 + 5.0 = 8.0$ V

Potential at **D** (with reference to A)
$V_D = 0 + 3.0 = 3.0$ V

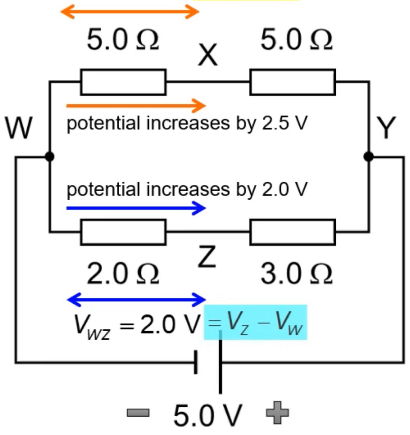
Example 6 The potential difference between points W and Y is 5.0 V.
What is the potential of X with respect to Z?



A − 2.50 V
B − 0.50 V
C + 0.50 V
D + 2.50 V

[Solution]

$V_{WX} = 2.5\text{ V} = V_X - V_W$

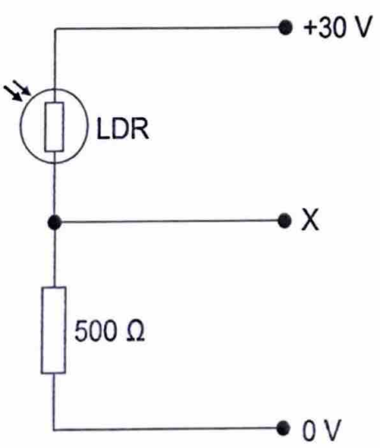


$V_{WZ} = 2.0\text{ V} = V_Z - V_W$

p.d. between X and Z is 0.5 V.

Use of a Light Dependent Resistor (LDR) in a potential divider circuit A light-dependent resistor (LDR) can be used in a potential divider to provide a potential difference which is dependent on the **illumination** that it is exposed to.

Example 7 A LDR and a 500 Ω resistor form a potential divider between potential lines held at +30 V and 0 V as shown. The resistance of the LDR is 1.0 kΩ in the dark, but drops to 100 Ω in bright light. Determine the corresponding change in the potential at X.



[Solution]

In the dark, p.d. across 500 Ω resistor $= \dfrac{500}{500+1000} \times 30 = 10\text{ V}$

Potential at X = 10 V

In bright light, p.d. across 500 Ω resistor $= \dfrac{500}{500+100} \times 30 = 25\text{ V}$

Potential at X = 25 V

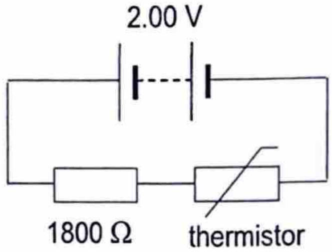
Thus
$\Delta V_X = 25 - 10 = 15\text{ V}$

Use of a Thermistor in a potential divider circuit

In a similar manner, a thermistor can be used in a potential divider to provide a potential difference which is dependent on **temperature**.

Example 8

The temperature change due to an expansion of a gas can be detected using a small thermistor. The thermistor is connected in series with a fixed resistor of resistance 1800 Ω and a 2.00 V battery of negligible internal resistance, as shown.



Initially, the thermistor has resistance 1800 Ω and after the gas expands, its resistance is 1910 Ω. Calculate the potential difference across the thermistor before and after the gas has expanded.

[Solution]

Before gas expands, p.d. across thermistor $= \dfrac{1800}{1800+1800} \times 2.00 = 1.00\ \text{V}$

After gas expands, p.d. across thermistor $= \dfrac{1910}{1800+1910} \times 2.00 = 1.03\ \text{V}$

15.5 The Potentiometer

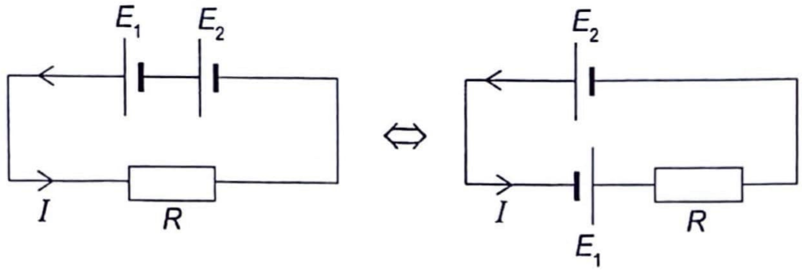
Principle of the potentiometer

As mentioned earlier, the term *potentiometer* can be used for any variable resistor. It is an instrument that can be used to measure the e.m.f. of a source without drawing any current from the source.

Essentially it balances an unknown potential difference against an adjustable, measurable potential difference.

Arrangement 1

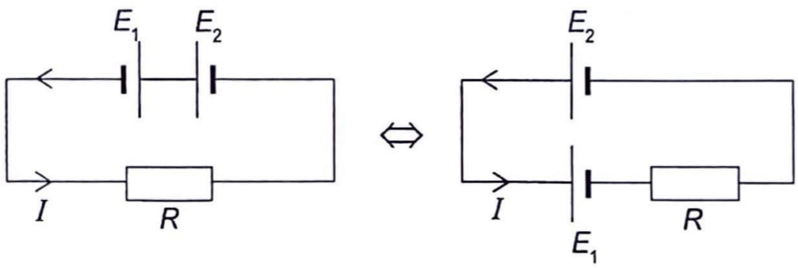
When two cells are connected in series as shown below,



the current in the circuit, $I = \frac{E_1 + E_2}{R}$

Arrangement 2

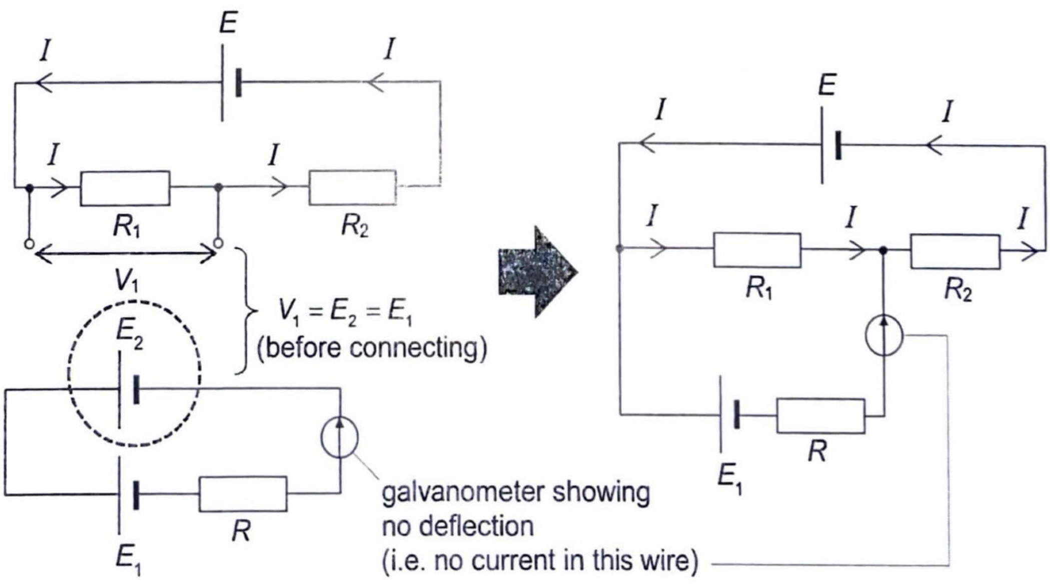
When two cells are connected in parallel as shown below,



If $E_1 = E_2$, there will be <u>no current in the circuit</u> (i.e. $I = 0$).

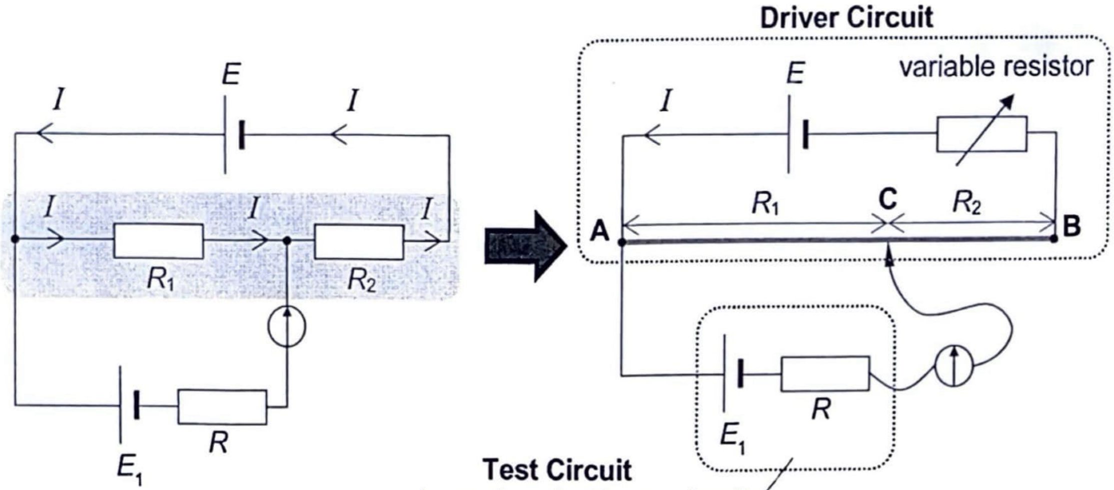
E_2 can be replaced with a potential divider circuit with another e.m.f. source E and resistors R_1 and R_2, such that V_1 is equal to E_2 and also E_1.

i.e. $V_1 = \frac{R_1}{R_1 + R_2} E = E_2 = E_1$



If the value of E_1 was not known earlier, it can now be determined accurately using the potential divider principle.

A more practical form of this instrument is to use a resistance wire AB in place of the two resistors, R_1 and R_2, with a sliding contact C connected through a galvanometer to the e.m.f. source that is to be measured.



As the contact C is moved along the resistance wire, the resistance R_{AC} varies (If the resistance wire is uniform, R_{AC} will be proportional to the length L_{AC}).

Null / Balance Point Contact C is moved from A to a position such that the galvanometer shows no deflection, which means that there is no current passing through the galvanometer. This point is known as the ***null point*** or ***balance point***.

Balance Length The length and the length L_{AC} is known as the ***balance length***. At the null point, the p.d. V_{AC} across L_{AC} is equal to the p.d. being measured E_1.

Assumptions

- Wire has uniform cross-sectional area
- Potential difference across wire remains constant with time

Applying Potential Divider Principle

For a uniform wire of length L, $R = \frac{\rho L}{A} \quad \Rightarrow R \propto L$

Since the same current flows through the entire wire when the potentiometer is at balance,

$$V_{AC} = \frac{R_{AC}}{R_{AB}} \times V_{AB} = \frac{L_{AC}}{L_{AB}} \times V_{AB}$$

$$\therefore \frac{V_{AC}}{V_{AB}} = \frac{R_{AC}}{R_{AB}} = \frac{L_{AC}}{L_{AB}}$$

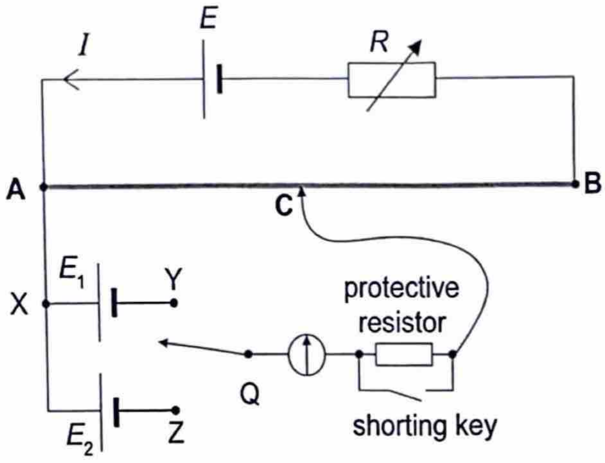
Hence, the p.d. being measured $= V_{AC} = kL_{AC}$

where $k = \frac{V_{AB}}{L_{AB}} =$ p.d. per unit length of the wire

[*TIP:* *In solving problems involving the potentiometer, it is always useful to first determine the total length (L_{AB}) and resistance (R_{AB}) of the resistance wire and potential difference (V_{AB}) across the resistance wire.*]

Comparing e.m.f.s

The circuit below is used to compare the e.m.f.s of two cells.

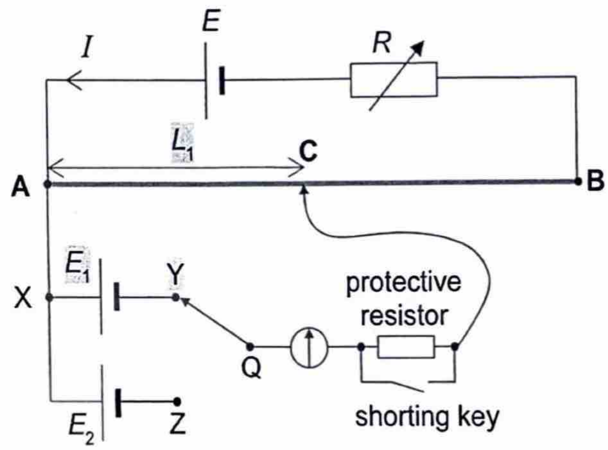


Determining balance length for E_1

With E_1 in the circuit (i.e. key Q connected to point Y), the balance point is located by *tapping* the jockey on the slide wire until the galvanometer registers zero current.

The balance length L_1 is thus determined.

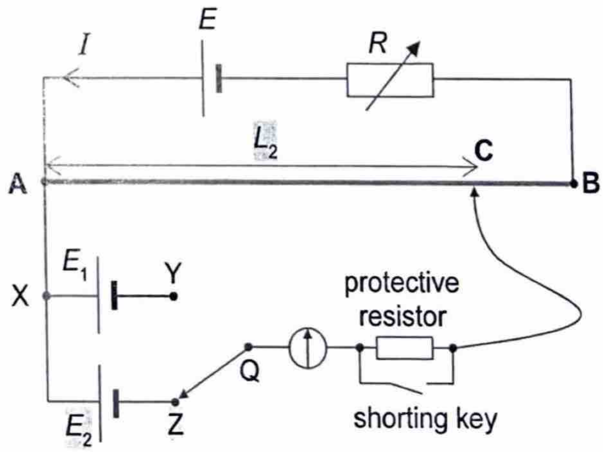
$$E_1 = \frac{L_1}{L_{AB}} \times V_{AB}$$



Determining balance length for E_2

With E_2 in the circuit (i.e. key Q is connected to Z), the new balance length L_2 is similarly determined.

$$E_2 = \frac{L_2}{L_{AB}} \times V_{AB}$$



Since at balance point, no current is drawn from either cell, the p.d. across their terminals is their e.m.f.s.

$$\frac{E_1}{E_2} = \frac{L_1}{L_2}$$

If the e.m.f. of one of the cells is to be *measured*, the other cell should be a standard cell which has an accurately known e.m.f. The standard Weston cell, which has an e.m.f. of 1.0183 V, is often used.

Potentiometer vs Voltmeter

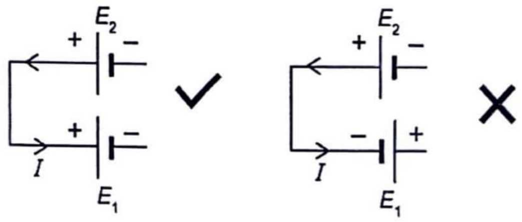
When a potentiometer is used to compare the e.m.f.s of cells, no errors are introduced by the internal resistances of the cells because no current flows through the cells at balance point.

A real voltmeter has a finite resistance and draws a current from the cell, thus lowers the terminal p.d. of the cell when it is connected across the cell (see Example 4).

In contrast, since no current flows from the cell when a balance point is found, the potentiometer may be considered to be a voltmeter with an infinitely high resistance, which is the ideal voltmeter.

Practical Details

1. The positive terminals of the cells being compared, E_1 and E_2, must be connected in the same direction as the driver cell E. Otherwise, no balance point can be found.



2. The accuracies of the measurements of balance lengths are greatest when L_1 and L_2 are large. A preliminary measurement should be made to determine which of L_1 and L_2 is the larger. The rheostat (variable resistor) in the driver circuit should then be adjusted so that the larger of the two balance lengths is close to B. The rheostat setting must be the same for the measurement for both balance lengths.
3. The current through the potentiometer wire must be steady throughout the experiment.
4. To ensure that the balance point of a potentiometer can be found with high *precision*, a very sensitive galvanometer is used.

   Such a galvanometer should be connected in series with a protective resistor (with a high resistance). This prevents relatively high currents from flowing through the galvanometer when in off-balance situations.

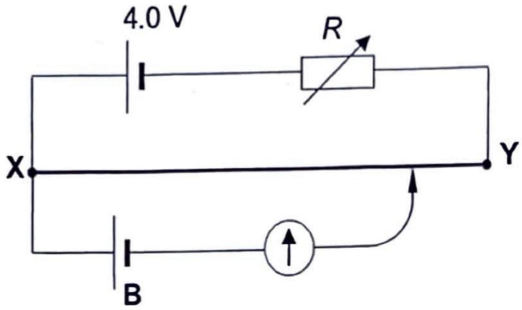
   The shorting key is left open until an approximate balance point has been found.

   The key is then closed, to short out the protective resistance and allow the full current to flow through the galvanometer, so that an accurate balance point can be found.

Example 9 A simple potentiometer circuit is set up as shown using a uniform wire XY, 1.00 m long, which has a resistance of 2.2 Ω. The internal resistance of the 4.0 V battery is negligible. If the variable resistor R is given a value of 2.8 Ω, calculate

(a) the p.d. per unit length of the wire XY.

(b) the e.m.f. of the cell B, if the length XJ is 0.85 m when the galvanometer shows zero deflection.



Solution

(a)

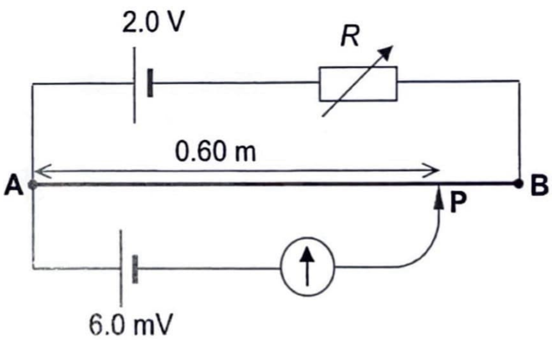
$$V_{XY} = \frac{R_{XY}}{R_{XY}+R} \times E_{driver} = \frac{2.2}{2.2+2.8} \times 4.0 = 1.76 \text{ V}$$

$$\text{p.d. per unit length} = \frac{V_{XY}}{L_{XY}} = \frac{1.76}{1.00} = 1.76 \text{ V m}^{-1}$$

(b) $$\text{e.m.f. of cell B} = V_{XJ} = \frac{L_{XJ}}{L_{XY}} V_{XY} = L_{XJ}\left(\frac{V_{XY}}{L_{XY}}\right) = 0.85(1.76) = 1.496 = 1.5 \text{ V}$$

Example 10 The figure on the right shows a simple potentiometer circuit for measuring a small e.m.f. source.

The wire AB has a length of 1.0 m and a resistance of 5.0 Ω. The driver cell has an e.m.f. of 2.0 V.



(a) If a balance point, P, is obtained 0.60 m along AB when measuring an e.m.f. of 6.0 mV, determine the value of the resistance R.

(b) A 10 Ω resistor is added in series to the 6.0 mV battery. Determine the new balance length.

[Solution]

(a)

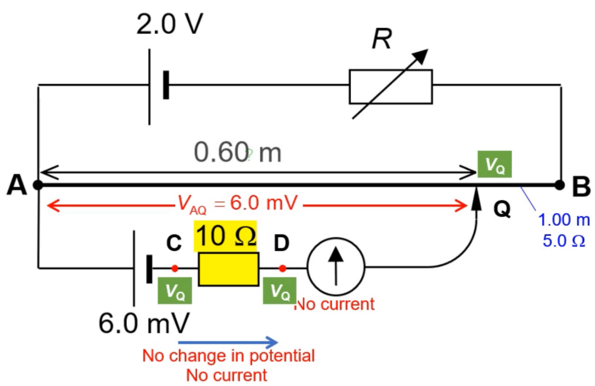
$$V_{AB} = \frac{L_{AB}}{L_{AP}} \times V_{AP} = \frac{1.0}{0.60} \times 6.0$$

$$= 10 \text{ mV}$$

$$= \frac{R_{AB}}{R_{AB}+R} \times E$$

$$10 \times 10^{-3} = \frac{5.0}{5.0+R} \times 2.0$$

$$R = 995 \text{ Ω}$$

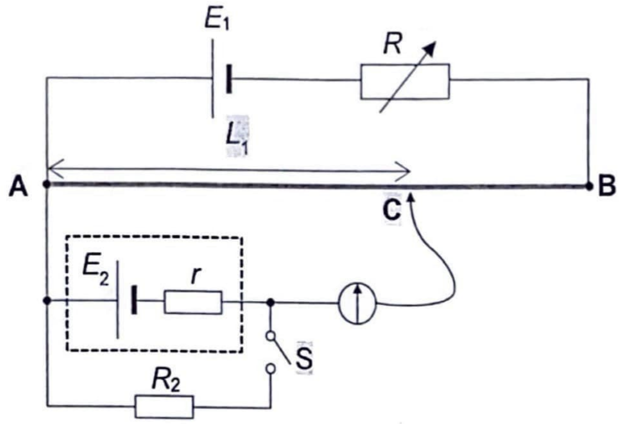


b) new balance length = 0.60 m

Measuring e.m.f. & internal resistance of a cell

The circuit below is used to determine the e.m.f. of a cell and its internal resistance.

Determining balance length for e.m.f. E_2



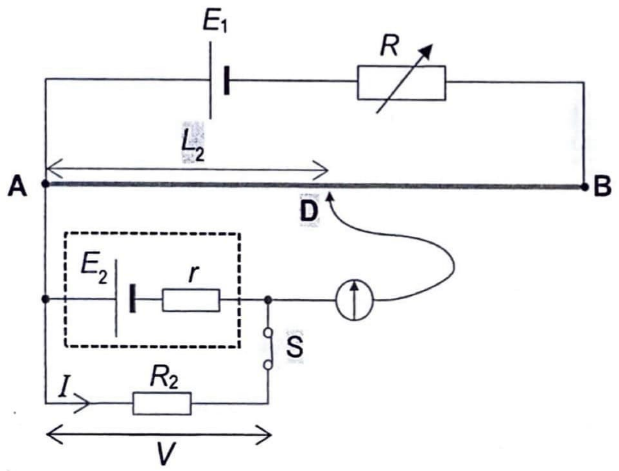
With switch S **open**, no current passes through the loop containing the resistor of resistance R_2.

The **balance point** is at point C and the **balance length** is L_1.

The **p.d. across AC**

$$V_{AC} = E_2 = \frac{L_1}{L_{AB}} \times V_{AB}$$

Determining balance length for terminal p.d. V



With switch S **closed**, current passes through the loop containing the resistor of resistance R_2.

The **balance point** is now at point D and the **balance length** is L_2.

The **p.d. across AD**

$$V_{AD} = V = \frac{L_2}{L_{AB}} \times V_{AB}$$

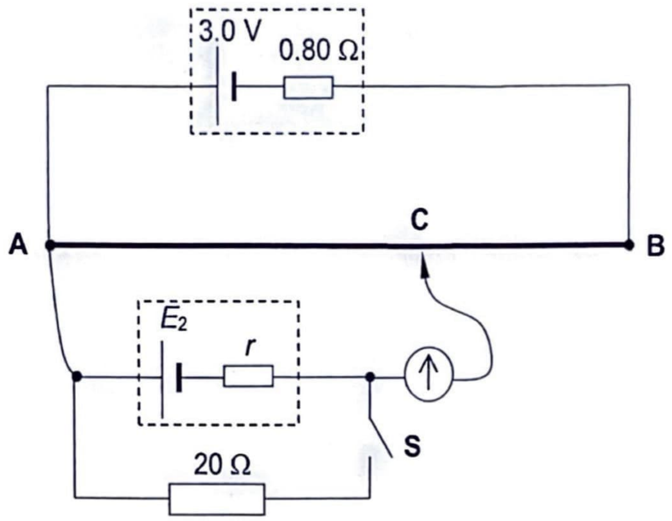
Step 1: Use the balance length L_1 to determine the e.m.f. E_2.

Step 2: Use the balance length L_2 to determine the terminal p.d. V across E_2 (when there is a current flowing though E_2)

Step 3: Use $V = IR$ to determine the current I flowing through R_2.

Step 4: Use $V = E_2 - Ir$ to determine the internal resistance r.

Example 11 A potentiometer circuit for measuring the internal resistance of a cell is shown. The uniform wire AB has a length of 100 cm and a resistance of 1.6 Ω. The driver cell is of e.m.f. 3.0 V and internal resistance 0.80 Ω. A cell of unknown e.m.f. E_2 and internal resistance r and a resistor of 20 Ω is connected to the circuit.



The movable contact C can be connected to any point along the wire AB. When switch S is opened, a balance length of 61.0 cm is obtained. When switch S is closed, a balance length of 49.5 cm is obtained. Determine the e.m.f. E_2 and the internal resistance, r.

[Solution]

$$V_{AB} = \left(\frac{1.6}{1.6+0.8}\right)3.0$$
$$= 2.0\text{ V}$$

When switch S is opened, $$E_2 = \frac{L_1}{L_{AB}} \times V_{AB} = \frac{61}{100} \times 2.0 = 1.22\text{ V}$$

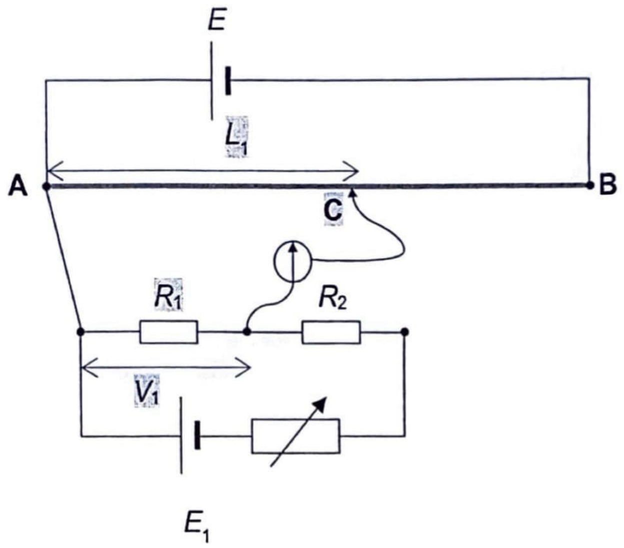
When switch S is closed, $$V_S = \frac{L_2}{L_{AB}} \times V_{AB} = \frac{49.5}{100} \times 2.0 = 0.99\text{ V}$$

$$E_2 = V_S + Ir$$
$$1.22 = 0.99 + \left(\frac{0.99}{20}\right)r \quad \Rightarrow r = 4.65\ \Omega$$

Comparing resistance of two resistors

The circuit below is used to compare the resistance of two resistors connected in series in the test circuit. By keeping the settings of both rheostats constant, the same current I flows through R_1 and R_2.

Determining balance length for V_1

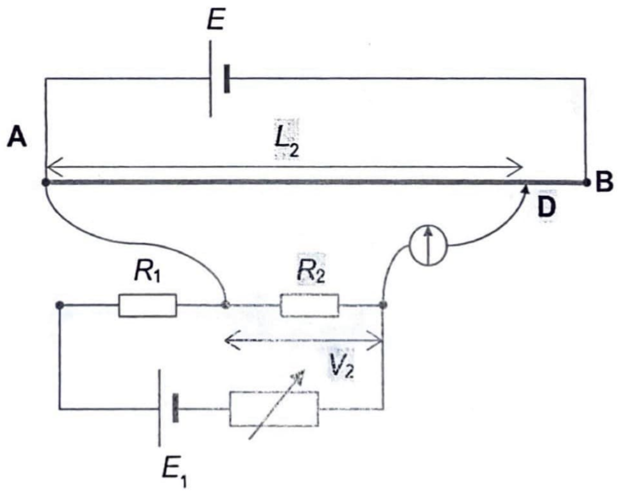


To compare the resistances R_1 and R_2, the p.d. across R_1 is first connected to the potentiometer circuit.

The **balance point** is at point C and the **balance length** is L_1.
The **p.d. across AC**

$$V_{AC} = V_1 = \text{p.d. across } R_1$$
$$= \frac{L_1}{L_{AB}} \times V_{AB}$$
$$= IR_1$$

Determining balance length for V_2



The p.d. across R_2 is next connected to the potentiometer circuit.

The **balance point** is at point D and the **balance length** is L_2.
The **p.d. across AD**

$$V_{AD} = V_2 = \text{p.d. across } R_2$$
$$= \frac{L_2}{L_{AB}} \times V_{AB}$$
$$= IR_2$$

By comparing the two balance lengths,

$$\frac{R_1}{L_1} = \frac{R_2}{L_2} \Rightarrow \frac{R_1}{R_2} = \frac{L_1}{L_2}$$

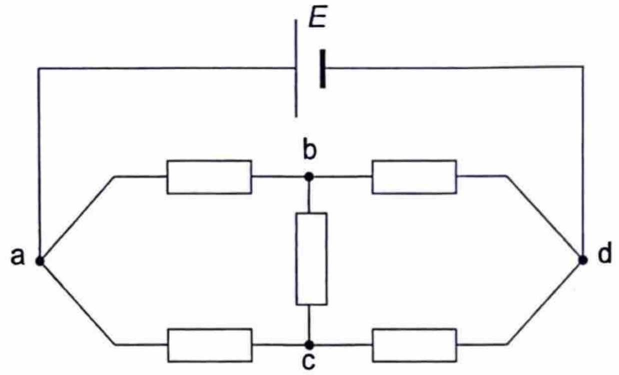
If the resistance of either R_1 or R_2 is known, the resistance of the other unknown resistor can be determined.

APPENDIX

Kirchhoff's Laws

Kirchhoff's Laws are useful in solving circuit problems. Kirchhoff's First Law, also known as the Point Rule, is applied to junctions in a circuit. A branch point (junction) is a point of a network at which three or more conductors are joined. Kirchhoff's Second Law, also known as the Loop Rule, is applied to loops in a circuit. A loop is any closed conducting path.

In the circuit below, the junctions are: a, b, c and d
The loops are: abca , bdcb , abdca , a*E*dba, a*E*dca.



<u>Kirchhoff's First Law (Point Rule)</u> states that:

The algebraic sum of the currents toward any branch point of a network is zero.

At a junction, $\Sigma I = 0$

This law is based on the observation that no charge accumulates at a junction and hence total charge per unit time entering a junction equals total charge per unit time leaving the same junction.

<u>Kirchhoff's Second Law (Loop Rule)</u> states that:

The algebraic sum of e.m.f.s in any loop of a network equals the algebraic sum of the *iR* products in the same loop.

$$\Sigma E = \Sigma IR$$

This law is based on the Principle of Conservation of Energy. Recall that e.m.f. and p.d. are terms used for energy per unit charge. A source of e.m.f. is a source of electrical energy whereas p.d. represents a sink of electrical energy where electrical energy is converted to other forms of energy. Imagine the journey of a coulomb of charge round a circuit loop. The total energy it delivers to the components in the loop (i.e. the sum of p.d.s round the loop) is equal to the total electrical energy which is supplied to it (i.e. the net e.m.f.).

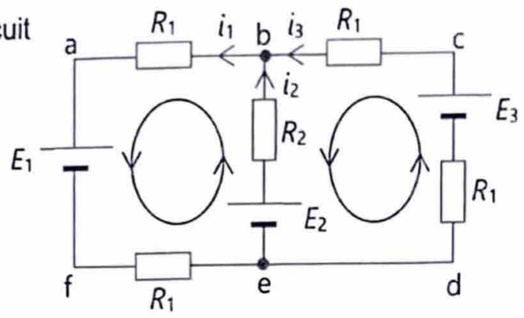
<u>Application of Kirchhoff's Laws:</u>

If there are *n* junctions in a circuit, Kirchhoff's First Law can be applied to $(n - 1)$ junctions to obtain equations that are distinct from each other. Similarly, if there are *n* loops, Kirchhoff's Second Law can be applied to $(n - 1)$ loops to form equations. Then, if there are *N* unknowns to be solved, the number of equations needed is *N* and are formulated using Kirchhoff's First and Second Laws.

Example:

Determine the three currents in the circuit given that:

$R_1 = 1.0\ \Omega$
$R_2 = 2.0\ \Omega$
$E_1 = 2.0$ V
$E_2 = E_3 = 4.0$ V



The circuit has only two junctions: b and e.

Applying Kirchhoff's Point Rule to junction b:

total current entering the junction = total current leaving the junction

$i_2 + i_3 = i_1 \quad \cdots (1)$

[Note: Application of Kirchhoff's Point Rule to junction e will also yield equation (1).]

Since there are three unknowns to be solved, another two equations are needed and these can be obtained by applying Kirchhoff's Loop Rule to two out of the following three possible loops: afeba, bedcb and afedcba.

Consider loop afeba:

E_2 (acting alone) will supply current in the direction of the loop while E_1 (acting alone) will supply a current in the opposite direction to the loop. Hence net e.m.f. in the direction of the loop is given by:

$\Sigma E = E_2 - E_1 = 4.0 - 2.0 = 2.0$ V

The currents i_1 and i_2 are both in the direction of the loop and the iR products due to each of the currents flowing through the resistors are assigned positive values. Hence,

$\Sigma iR = i_1R_1 + i_1R_1 + i_2R_2 = 2.0\ i_1 + 2.0\ i_2$

Equating ΣE to ΣiR for this loop:

$2.0\ i_1 + 2.0\ i_2 = 2.0$

$i_1 + i_2 = 1.0 \quad \cdots (2)$

Consider loop bedcb:

E_3 (acting alone) will supply current in the direction of the loop while E_2 (acting alone) will supply a current opposite in direction to the loop. Hence net e.m.f. in the direction of the loop is given by:

$\Sigma E = E_3 - E_2 = 4.0 - 4.0 = 0$

The current i_3 is in the direction of the loop and the iR products due to the current i_3 passing through the resistors are assigned positive values. However, the current i_2 is opposite in direction to that of the loop, and so the iR products due to i_2 are assigned negative values.

$\Sigma iR = i_3R_1 + i_3R_1 - i_2R_2 = 2.0\ i_3 - 2.0\ i_2$

Hence, $\Sigma E = \Sigma iR$ for this loop leads to:

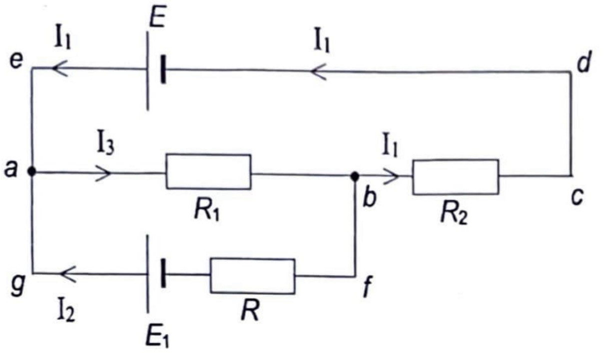
$i_3 - i_2 = 0 \cdots (3)$

Solving equations (1), (2) and (3):

$i_1 = 0.67$ A , $i_2 = 0.33$ A and $i_3 = 0.33$ A

Potentiometer circuit in more details

Consider the following circuit for a potentiometer.



Kirchhoff's first law applied to junction *a* yields the equation

$$I_1 + I_2 = I_3 \quad (1)$$

Kirchhoff's second law applied to the driver circuit *acdea* yields the equation

$$E - I_3R_1 - I_1R_2 = 0 \quad (2)$$

and to the circuit *abfga* yields

$$E_1 - I_3R_1 - I_2R = 0 \quad (3)$$

In principle, one can solve the above three simultaneous equations and obtain expressions for the three currents in terms of the emf's and resistances. But for our current purpose, it suffices to consider the following three scenarios:

Scenario 1: $I_3R_1 < E_1$

This happens when R_1 is too small (or if the jockey is in contact with a point to the left of the balance point if a resistance wire is used instead of two resistors). From Eq. (3):

$$I_2R = E_1 - I_3R_1 > 0$$

Hence $I_2 > 0$, which means the current is flowing from *f* to *g*, just as illustrated in the circuit diagram above.

Scenario 2: $I_3R_1 > E_1$

This happens when R_1 is too large (or if the jockey is in contact with a point to the right of the balance point if a resistance wire is used instead of two resistors). Again from Eq. (3):

$$I_2R = E_1 - I_3R_1 < 0$$

Hence $I_2 < 0$, which means the current is flowing from *g* to *f*, opposite to the direction illustrated in the circuit diagram.

Scenario 3: $I_3R_1 = E_1$

This happens when R_1 is of just the right resistance (or if the jockey is in contact with the balance point). From Eq. (3), one obtains

$$I_2R = E_1 - I_3R_1 = 0$$

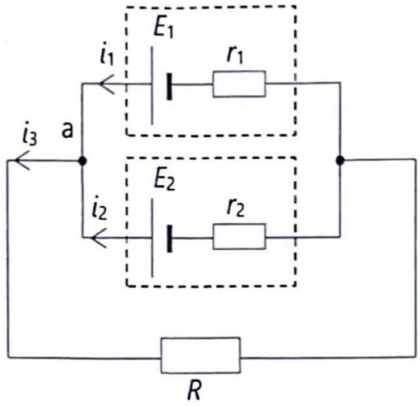
Hence $I_2 = 0$: there is no current in the branch containing E_1 and R in this case! Eq. (1) then says $I_1 = I_3$. Substitute E_1 for I_3R_1 in Eq. (2) yields

$$I_1 = I_3 = \frac{E - E_1}{R_2} = \frac{E_1}{R_1} = \frac{E}{R_1 + R_2},$$

where the second last equality comes from $I_3R_1 = E_1$, and the last comes from substituting I_3 for I_1 in Eq. (2).

Total e.m.f. and total internal resistance for unlike cells connected in parallel

Consider two cells, one with e.m.f. E_1 and internal resistance r_1 and the other with e.m.f. E_2 and internal resistance r_2, connected in parallel. The combination is connected to an external resistance R, as shown below.



Applying Kirchhoff's Point Rule to junction a,

$$i_3 = i_1 + i_2 \quad \cdots (1)$$

Applying Kirchhoff's Loop Rule for the loop containing E_1, r_1 and R,

$$E_1 = i_1r_1 + i_3R \quad \cdots (2)$$

Applying Kirchhoff's Loop Rule for the loop containing E_2, r_2 and R,

$$E_2 = i_2r_2 + i_3R \quad \cdots (3)$$

Multiplying equation (2) by r_2 and equation (3) by r_1 and then adding them:

$$E_1r_2 + E_2r_1 = (i_1 + i_2)\,r_1r_2 + i_3R(r_1 + r_2)$$

Since $i_3 = i_1 + i_2$, the above equation becomes:

$$E_1r_2 + E_2r_1 = i_3\,[r_1r_2 + R(r_1 + r_2)]$$

$$i_3 = \frac{E_1r_1 + E_2r_2}{r_1r_2 + R\left(r_1 + r_2\right)}$$

p.d. across R , $V_R = i_3R = \dfrac{\left(E_1r_1 + E_2r_2\right)R}{r_1r_2 + R\left(r_1 + r_2\right)}$

If $r_1 = r_2 = r$, then $V_R = \dfrac{\left(E_1 + E_2\right)rR}{r^2 + 2rR} = \dfrac{\left(E_1 + E_2\right)R}{r + 2R}$

If $r << R$, then $V_R = \dfrac{\left(E_1 + E_2\right)}{2}$

Hence, if the internal resistances of the cells are negligible, the total e.m.f. of the cells when connected in parallel is the average of the e.m.f.s of the cells.

Tutorial 15 D.C. CIRCUITS

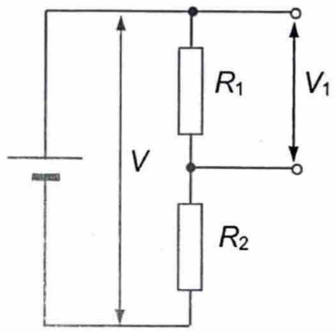
H2 PHYSICS 9749

Self - Check Questions

S1 Two resistors of resistances R_1 and R_2 are connected in parallel. Show that the equivalent resistance R of the combination is given by: $R = \dfrac{R_1 R_2}{R_1 + R_2}$

S2 Three identical resistors each has a resistance R. Draw diagrams to show how the resistors (one, two or three of them) can be combined to obtain different values of resistance. For each combination, obtain the value of the equivalent resistance in terms of R.

S3 For the circuit shown below, derive an expression for the potential difference V_1 in terms of R_1, R_2 and V.



S4 State how the resistance of
(a) a thermistor vary with temperature,
(b) a light-dependent resistor vary with intensity of light.

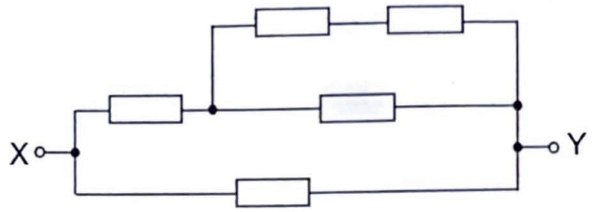
S5 Draw circuit diagrams to show how a thermistor or a light-dependent resistor is used in potential dividers to provide a p.d. which is dependent on temperature and illumination respectively.

S6 Draw a circuit diagram in each of the following cases to show how the potentiometer may be used to
(a) measure the e.m.f. of a cell,
(b) determine the internal resistance of a cell,
(c) compare the resistance of two resistors.

S7 State one advantage and one disadvantage of using a potentiometer to determine an unknown p.d.

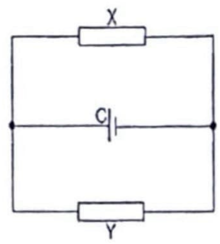
Self - Practice Questions

SP1 The figure shows a network of resistors each of resistance 10 Ω. What is the effective resistance between points X and Y?



SP2 Each of the resistors X and Y has resistance 6 Ω. The cell C has emf 12 V and internal resistance 3 Ω. What is the current in Y?

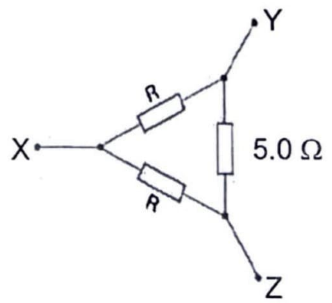
N88/I/16



SP3 The diagram shows a network of 3 resistors. Two of these marked R are identical. The other one has a resistance of 5.0 Ω.

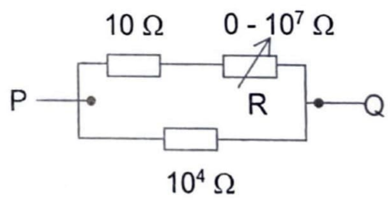
The resistance between Y and Z is found to be 2.5 Ω. What is the resistance between X and Y?

J91/I/14



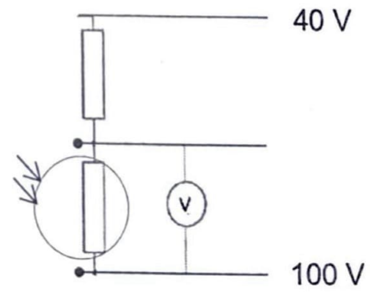
SP4 The variable resistor R can be adjusted over its full range from zero to 10^7 Ω. What are the approximate limits for the resistance between P and Q?

N88/I/15



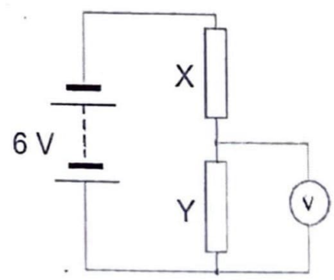
SP5 The LDR and a 500 Ω resistor form a potential divider between voltage lines held at +40V and +100 V.

The resistor of the LDR ranges from 100 to 1000 Ω. What is the potential difference across it in complete darkness?

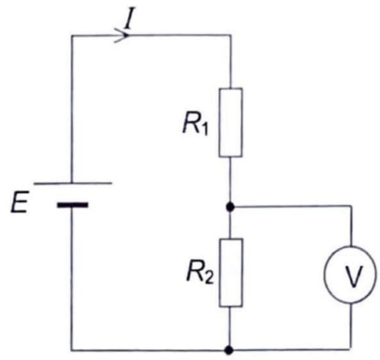


SP6 Resistors X and Y, each of resistance *R*, are connected to a 6 V battery of negligible internal resistance. A voltmeter, also of resistance *R*, is connected across Y. What is the reading of the voltmeter?

J90/I/15; N96/I/14



SP7 A battery of negligible internal resistance is connected to two resistors and a high resistance voltmeter. E = 9.0 V, R_1 = 120 Ω and R_2 = 60 Ω.

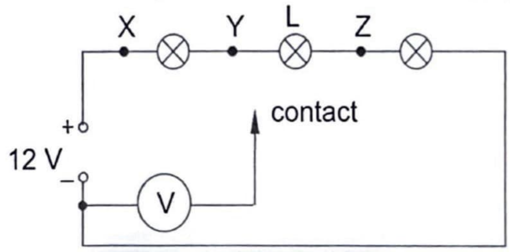


(a) **(i)** Calculate the current I flowing in the circuit.

(ii) Determine the reading on the voltmeter.

(b) The circuit is now modified by replacing R_1 with a thermistor. Explain whether the reading on the voltmeter increases or decreases when the temperature increases.

SP8 The diagram below shows three lamps in series with a 12 V supply.

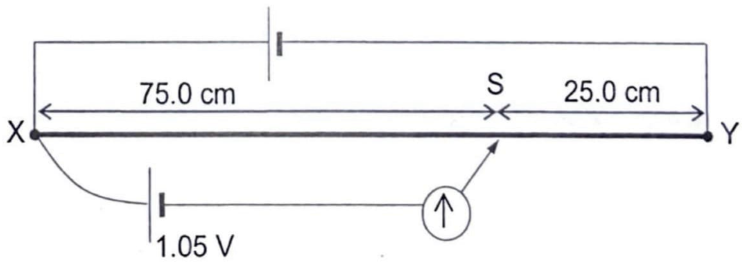


To test the circuit, the contact is connected in turn to points X, Y and Z. The lamps **do not light up** because lamp L has a broken filament.

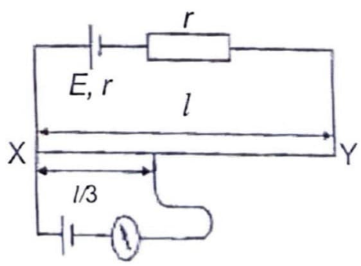
State the voltmeter reading when the contact is connected to

(a) X **(b)** Y **(c)** Z N05/I/23

SP9 A standard cell of e.m.f. 1.05 V is used to determine the potential difference across the wire XY as shown in the diagram below. It is found that there is no current in the galvanometer when the sliding contact is at S, 75.0 cm from X and 25.0 cm from Y. What is the potential difference across X and Y?



SP10 A potentiometer has a wire XY of length l and resistance R. It is powered by a battery of emf E and internal resistance r in series with a resistor of resistance r. With a cell in the branch circuit, the null point is found to be $l/3$ from X. What is the emf of the cell in terms of E, R and r?

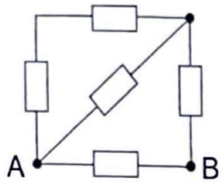


J79/I/18

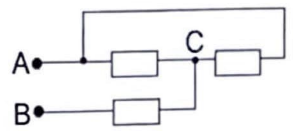
Discussion Questions

D1 Given that each resistor has a resistance of R, obtain a value in terms of R for the total resistance between the points A and B in the following circuits.

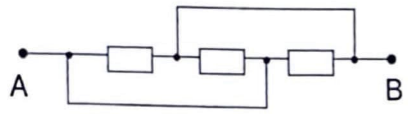
(a)



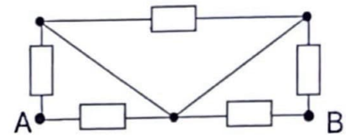
(b)



(c)



(d)



[4]

D2 A set of 12 lamps are arranged in series to a 30 V supply as shown in Fig. 2. The total power of the lamps is 9.0 W.

Fig. 2

(a) For a single lamp, calculate

(i) the current, [1]

(ii) the potential difference, [1]

(iii) the resistance. [1]

(b) The lamps do not light up when the set is plugged in, so a voltmeter is used to test the circuit. For each of the following observations, identify the fault.

(i) The potential difference between A and M is zero. [1]

(ii) The potential difference is zero across every lamp except EF, across which the potential difference is 30 V. [1]

(iii) The potential difference between A and M is 30 V, but the potential difference is zero across every single lamp. [1]

N90/III/4 (Modified)

D3 In the circuit shown in Fig. 3, the 3.0 V cell has negligible internal resistance. Determine the potential difference between X and Y.

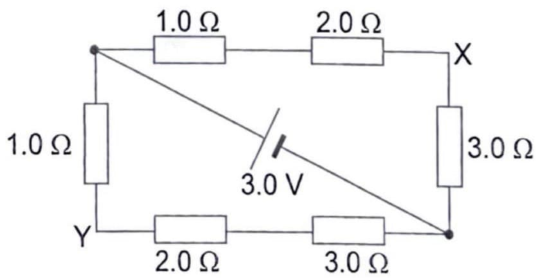
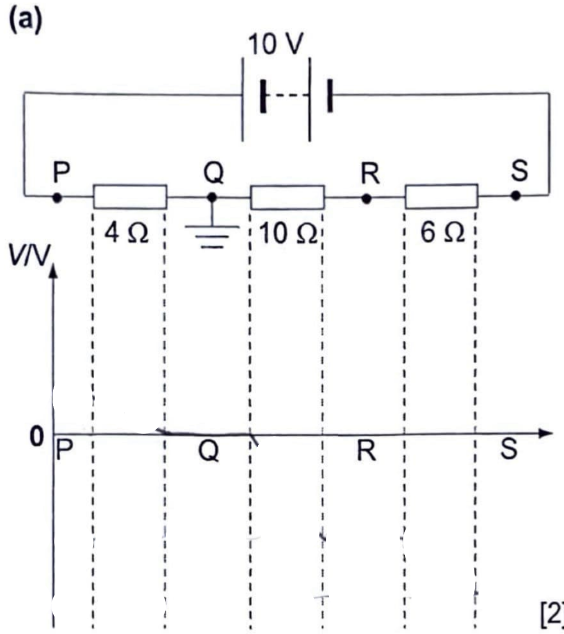


Fig. 3

[3]

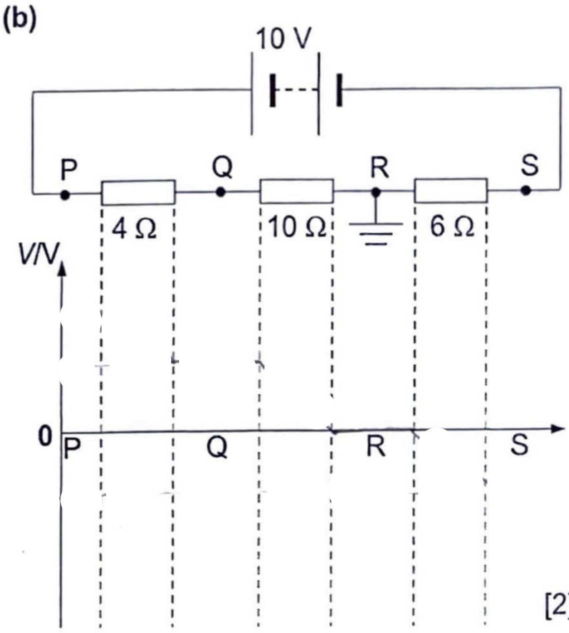
D4 Draw the variation in potential along the network of resistors for the following circuits:

(a)



[2]

(b)



[2]

D5 Fig. 5 shows a potential divider arrangement using a fixed resistor of resistance 4.0 kΩ and a variable resistance of maximum resistance 20 kΩ, with a slide contact connected to terminal S. The e.m.f of the battery is 12 V and it has negligible internal resistance.

It is possible to obtain different continuously-variable voltage ranges by selecting, as the input, particular pairs of terminals from S, X, Y and Z.

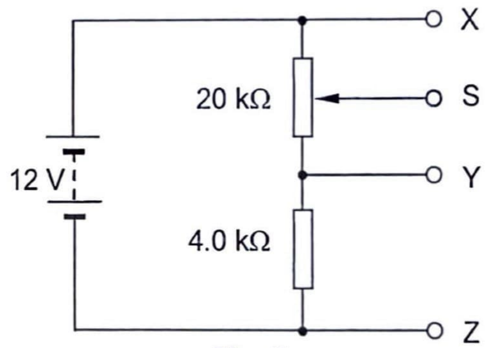


Fig. 5

(a) Determine the voltage ranges obtainable between the terminals

(i) S and X, [2]

(ii) S and Z. [2]

In each case, state the polarity of terminal S relative to the other terminal.

(b) The slide contact S is set at the mid-point of the 20 kΩ resistance track. A voltmeter of resistance 10 kΩ is then connected between S and Y. Calculate the reading on the voltmeter. [2]

D6 The resistance of a light dependent resistor (LDR) decreases when light shines on it. A typical LDR, with resistance varying from 2.0 MΩ when placed in the dark to 100 Ω when in bright light, is used in a light sensing circuit (shown in Fig. 6) to turn on a nightlight when dusk falls.

(a) State whether the nightlight should be connected parallel to the LDR or the fixed resistor *R*. [1]

(b) A 3.0 V power supply and a 2.0 MΩ nightlight are connected. If the nightlight is switched on when the voltage across it is above 2.5 V and is switched off when the voltage across it is less than 0.50 V, determine the range for the resistance of *R*. [4]

(c) Assuming that the p.d. across the nightlight is 2.5 V at night, calculate the power delivered to the nightlight. [2]

(d) Hence, comment on the practicality of this design of a nightlight.

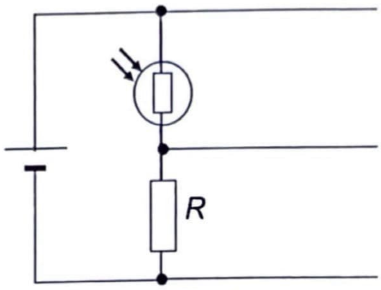


Fig. 6

D7 Two similar cells, each of e.m.f. 1.5 V and internal resistance 0.25 Ω, are connected in series with a fixed resistor of resistance 2000 Ω and a thermistor, as shown in Fig. 7.

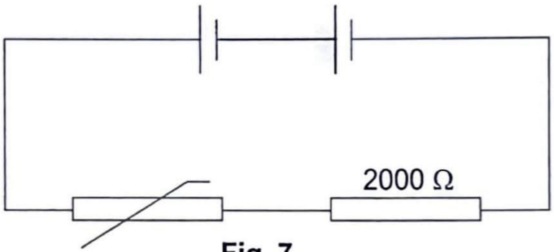


Fig. 7

The thermistor has resistance 4000 Ω at 0 °C and 1800 Ω at 20 °C.

(a) Explain why, in this circuit, the internal resistance of the cells may be considered to be negligible. [1]

(b) Determine the potential difference across the thermistor at

(i) 0 °C, [1]

(ii) 20 °C. [1]

(c) In one particular application of the circuit of Fig. 7, it is desired that the potential difference across the fixed resistor should range from 1.2 V at 0 °C to 2.4 V at 20 °C.

Determine whether, by substituting a different fixed resistor in the circuit of Fig. 7, it is possible to achieve this range of potential differences. [3]

N09/III/7 (part)

D8 In the circuit shown in Fig. 8, cell A has a constant e.m.f. of 2.0 V and negligible internal resistance. Wire XY is 100 cm long with a resistance of 8.0 Ω. Cell B has an e.m.f. of 1.2 V and an internal resistance of 0.50 Ω.

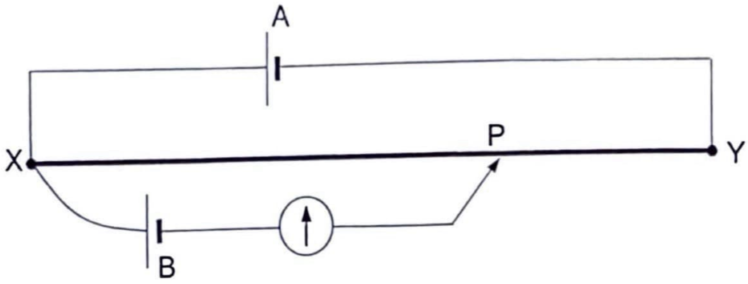


Fig. 8

Calculate the length XP required to produce zero current in the galvanometer

(a) in the circuit as shown in Fig. 8, [1]

(b) when a 2.0 Ω resistor is placed in series with A, [2]

(c) when this resistor is placed in series with B, [2]

(d) when this resistor is placed in parallel with B. [2]

D9 The potentiometer in Fig. 9 below is used to measure the e.m.f. E_2 and internal resistance r of a battery C. Slide wire AB is 50.0 cm long, with a resistance of 10.0 Ω. The driver battery has an e.m.f. E_1 = 2.00 V and negligible internal resistance. R_1 and R_2 have resistances 15.0 Ω and 5.0 Ω respectively. With the switches S_1 and S_2 both open, the galvanometer has zero deflection when AJ is 31.25 cm. With the two switches closed, the balance length AJ is 5.00 cm.

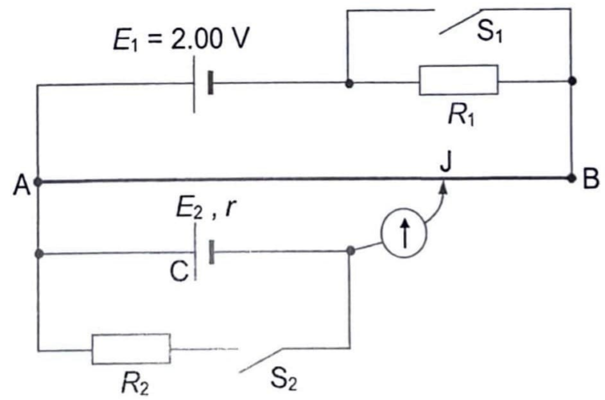


Fig. 9

Calculate

(a) the e.m.f. E_2 and internal resistance r of battery C, [3]

(b) the balance length AJ when S_1 is open and S_2 is closed, [2]

(c) the balance length AJ when S_2 is open and S_1 is closed. [2]

D 10 The movable contact J in the circuit of Fig. 10 below is moved along the 100 cm potentiometer wire AB to find the point C at which the galvanometer registers zero current.

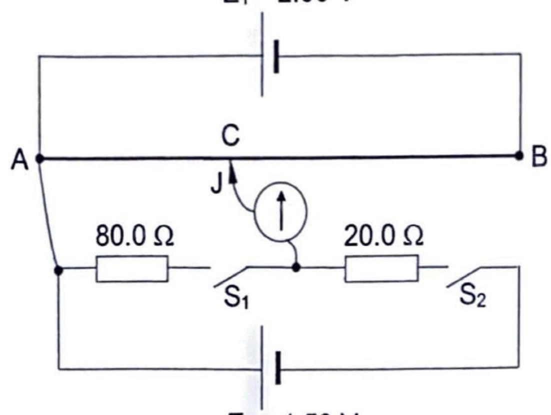


Fig. 10

(a) Both cells have negligible internal resistance. Calculate the length AC.

(i) with switches S_1 and S_2 both closed, [2]

(ii) with switch S_1 open and S_2 closed. [2]

(b) The 1.50 V cell develops an internal resistance of a few ohms. Identify and explain without calculations, any effect on the two balance lengths determined in **(a)**. [2]

Challenging Questions

C1 Fig. 11.1 shows an infinitely long network made up of resistors each of resistance r. The resistance of the network, measured between the terminals C and D, is R.

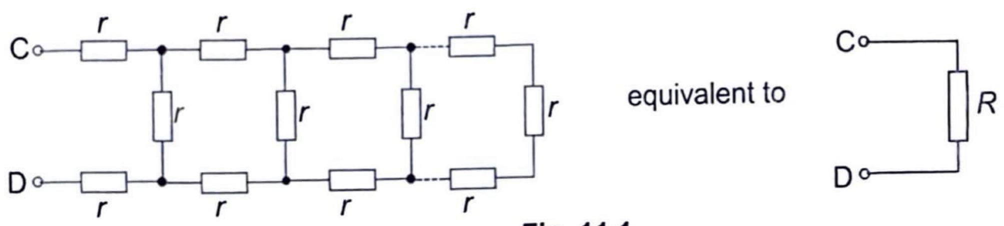


Fig. 11.1

An extra segment of three resistors is then added to the left-hand end of the network, as shown in Fig. 11.2. The combination of the extra segment and the infinite network is equivalent to the circuit shown in Fig. 11.3.

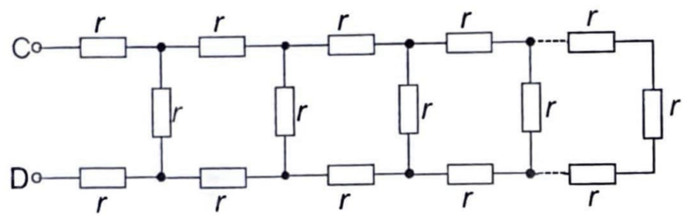


Fig. 11.2

equivalent to

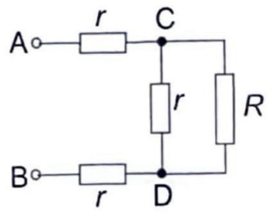


Fig. 11.3

(a) Obtain an expression, in terms of r and R, for the resistance of the circuit shown in Fig. 11.3 when measured between the terminals A and B.

(b) Because the network is infinitely long, the addition of the extra segment makes no difference to the overall resistance. Use this fact to show that

$$R = \left(1+\sqrt{3}\right)r$$

N97/0/10 (part)

C2 The circuit in Fig. 12 is used to measure the e.m.f. E. The driver cell has an e.m.f. of 2.00 V and negligible internal resistance. AB is a uniform wire of length 100.0 cm and resistance 2.00 Ω.

With S_1 closed and S_2 open, the balance length is 90.0 cm.

With S_2 closed and S_1 open, the balance length is 45.0 cm.

Determine the value of

(a) the e.m.f. E.

(b) the resistance R.

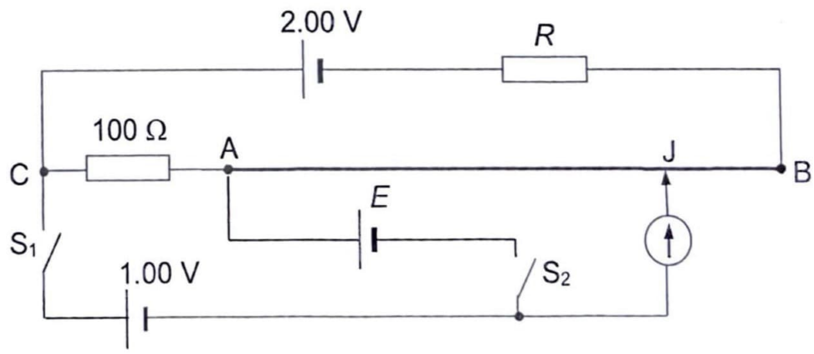


Fig. 12

Numerical answers for discussion and challenging questions:

D1 **(a)** $0.625R$ **(b)** $1.5R$ **(c)** $0.333R$ **(d)** R

D2 **(a)** **(i)** 0.30 A **(ii)** 2.5 V **(iii)** 8.3 Ω

D3 1.0 V

D5 **(a)** **(i)** 0 V to 10 V **(ii)** 2 V to 12 V
(b) 3.2 V

D6 **(b)** Range of R: between 0.50 kΩ and 200 kΩ. **(c)** 3.13 μW

D7 **(b)** **(i)** 2.0 V **(ii)** 1.4 V

D8 **(a)** 60.0 cm **(b)** 75.0 cm **(c)** 60.0 cm **(d)** 48.0 cm

D9 **(a)** $E_2 = 0.50\text{ V}, r = 7.5\ \Omega$ **(b)** 12.5 cm **(c)** 12.5 cm

D 10 **(a)** **(i)** 60.0 cm **(ii)** 75.0 cm

C1 **(a)** $R_{AB} = \dfrac{3rR + 2r^2}{R + r}$

C2 **(a)** 8.8 mV **(b)** 102 Ω

Suggested solutions

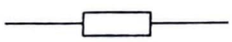
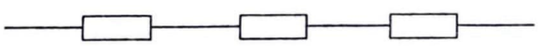
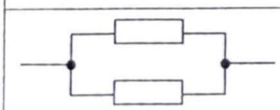
Self - Check Questions

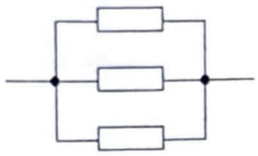
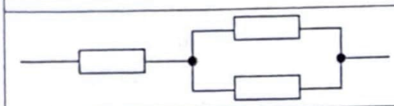
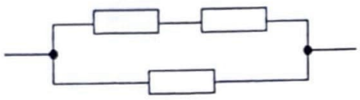
S1

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_1 + R_2}{R_1 R_2}$$

$$\Rightarrow R = \frac{R_1 R_2}{R_1 + R_2}$$

S2

Configuration	Equivalent resistance
	R
	$2R$
	$3R$
	$\frac{1}{2}R$

	$\frac{1}{3}R$
	$\frac{3}{2}R$
	$\frac{2}{3}R$

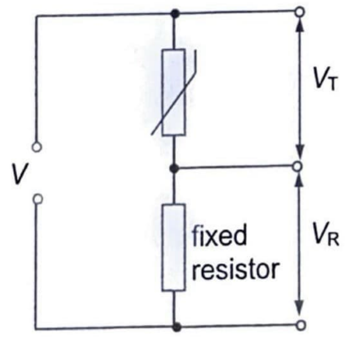
S3 Current through the resistors is given by $I = \dfrac{V}{R_1 + R_2}$

$$V_1 = IR_1 = \left(\frac{V}{R_1 + R_2}\right)R_1$$

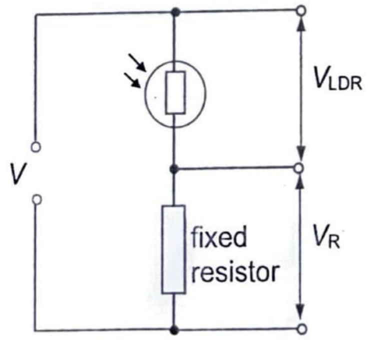
$$\therefore V_1 = \left(\frac{R_1}{R_1 + R_2}\right)V$$

S4 **(a)** The resistance of a thermistor decreases with increasing temperature.
(b) The resistance of a light-dependent resistor decreases with increasing brightness.

S5



As temperature increases,
resistance of thermistor decreases and hence,
V_T decreases whilst V_R increases.



As level of illumination increases,
resistance of LDR decreases and hence,
V_{LDR} decreases whilst V_R increases.

S6 Refer to lecture notes pg 21-23.

S7 Advantage: Since the potentiometer draws no current from the e.m.f. being measured at balance point, it gives a more accurate measurement than a voltmeter.

Disadvantage: Measurement is slow and hence it cannot be used to measure p.d. that varies rapidly.

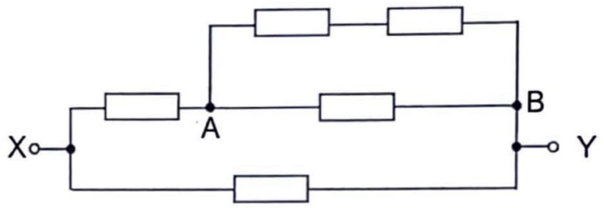
Self - Practice Questions

SP1 $\frac{1}{R_{AB}} = \frac{1}{10} + \frac{1}{20}$

$R_{AB} = 6.667\ \Omega$

$\frac{1}{R_{XY}} = \frac{1}{10+6.667} + \frac{1}{10}$

$R_{XY} = 6.25\ \Omega$

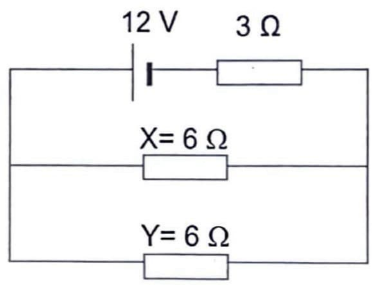


SP2 $\frac{1}{R_{XY}} = \frac{1}{6} + \frac{1}{6} = \frac{2}{6} \Rightarrow R_{XY} = 3\ \Omega$

$R_{eq} = 3 + 3 = 6\ \Omega$

curent through C, $I = \frac{V}{R_{eq}} = \frac{12}{6} = 2\ \text{A}$

hence $I_Y = 1\ \text{A}$



Equivalent circuit

SP3 Consider YZ:

$\frac{1}{2.5} = \frac{1}{2R} + \frac{1}{5.0}$

$R = 2.5\ \Omega$

Consider XY:

$\frac{1}{R_{XY}} = \frac{1}{2.5} + \frac{1}{2.5+5.0}$

$R_{XY} = 1.9\ \Omega$

SP4 When $R = 0$:

$\frac{1}{R_{PQ}} = \frac{1}{10^4} + \frac{1}{10+0}$

$R_{PQ} \approx 10\ \Omega$

When $R = 10^7$:

$\frac{1}{R_{PQ}} = \frac{1}{10^4} + \frac{1}{10+10^7}$

$R_{PQ} \approx 10^4\ \Omega$

$\therefore$ limits are between 10 and $10^4\ \Omega$

SP5 In complete darkness, resistance of LDR is at the highest at 1000 Ω.

$$V = \frac{1000}{1000+500} \times (100-40) = 40 \text{ V}$$

SP6

$$\frac{1}{R_{VY}} = \frac{1}{R} + \frac{1}{R}$$

$$R_{VY} = 0.5R$$

$$R_{eq} = 0.5R + R = 1.5\ R$$

voltmeter reading $V = \frac{R_{VY}}{R_{eq}} \times 6 = \frac{0.5R}{1.5R} \times 6 = 2.0 \text{ V}$

SP7 (a) (i) $I = \frac{E}{R_1 + R_2} = \frac{9.0}{120+60} = 0.050 \text{ A}$

(ii) $V = IR = (0.050)(60)$

$V = 3.0 \text{ V}$

(b) As the temperature increases, the resistance of the thermistor decreases whereas the resistance of R_2 remains the same. Based on the potential divider principle, i.e. ratio of resistance equals ratio of voltage, there will be a smaller p.d. across the thermistor and more across the R_2 resistor. Therefore, the voltmeter reading increases when temperature increases.

SP8 Broken filament at the centre lamp L means there is <u>no current</u> in the circuit so there is <u>no p.d. across the other two lamps</u>.

When contact is connected to X, voltmeter reads 12 V as the voltmeter measures the p.d. across the supply.

When contact is connected to Y, voltmeter reads 12 V because the p.d. across the lamp between X and Y is 0, so the voltmeter still measures the p.d. across the supply.

When the contact is connected to Z, voltmeter reads 0 V because, with the break at L, the voltmeter now measures the p.d. across right lamp, which is 0.

SP9 V_{XS} = 1.05 V

$$\frac{V_{XS}}{V_{XY}} = \frac{L_{XS}}{L_{XY}} \Rightarrow V_{XY} = \frac{(75.0+25.0)}{75.0} \times 1.05 = 1.40 \text{ V}$$

SP10

$$V_{XY} = \frac{R}{R+r+r} E$$

$$\text{e.m.f. of cell} = \frac{l/3}{l} \times V_{XY} = \frac{1}{3} \times \frac{R}{R+r+r} E = \frac{R}{3(R+2r)} E$$