CHAPTER 4

INTRODUCTION

The logarithm of *Y* to the base *a* is the exponent to which a must be raised to yield *Y*. That is,

$$\log_a Y = x \quad \text{if and only if} \quad a^x = Y$$

Thus $\log_2 4 = 2$ since $2^2 = 4$ and $\log_2 8 = 3$ since $2^3 = 8$

Example 1 a) $3^2 = 9$; then $2 = \log_3 9$
2 is the logarithm of 9 to base 3.

b) $10^2 = 100$; so $2 = \log_{10} 100$
2 is the logarithm of 100 to base 10.

Example 2 If $\log_{10} N = 3$, find the value of N.

Example 3 Given that $\log_x 81 = 4$, find the value of x.

Example 4 Evaluate, $\log_8 2$ without using calculator.

4.1 LAWS OF LOGARITHMS

A. Product	$\log_a xy = \log_a x + \log_a y$
B. Quotient	$\log_a \frac{x}{y} = \log_a x - \log_a y$
C. Power	$\log_a x^n = n\log_a x$

4.2 SPECIAL LOGARITHMS

1. $\log_a a = 1$ e.g. $\log_2 2 = 1$, $\log_5 5 = 1$
2. $\log_a 1 = 0$ e.g. $\log_2 1 = 0$, $\log_9 1 = 0$
3. $\log_a \frac{1}{x} = -\log_a x$
4. Natural logarithm $\ln N = \log_e N$ (where e = 2.71828183)

Example 5 *Simplify* $\log_3 2 + \log_3 5 + \log_3 20 - \log_3 25$

Example 6 Simplify $3\log 3 + \log 10 - \log 3$ (assume same base)

Example 7 *Simplify* $\log_2 16 - \log_2 8 + \log_2 4$

Example 8 Write the following expression as a single logarithm.

$$2\log_{10} x + 3\log_{10}(x+2) - \log_{10}(x^2+5)$$

Example 9 Solve the equation $2^x = 5$

Example 10 *Given that* $\log_3 2 = 0.631, \log_3 5 = 1.465;$ *find the value of* $\log_3 1.2$

4.3 CHANGE OF BASE

a) Logarithms to *base 10* are called "common logarithms", and are denoted by lg. When the base is 10, this number is generally omitted.

i.e. *log N* denotes the logarithm of N to the base 10.

b) Logarithms to *base e* are called "natural logarithms", and are denoted by ln. e has approximately the value 2.718.

i.e. $\log_e N$ is written as *ln N*

c) Logarithms can be to any base; however common logarithms are exclusively used for calculations at this stage.

d) Where logarithms to other bases are encountered, they have to be changed to base 10 for a numerical answer.

To change from base a to base b :

$$\log_a N = \frac{\log_b N}{\log_b a}$$

Example 11 Find the value of (a) $\log_3 4$ (b) $\log_2 10$

Example 12 Find the value of x: $\log_2 x + \log_4 x = \frac{3}{2}$

Example 13 Find the positive value of x: $\log_2 x = \log_4 (x + 6)$

Example 14 Find the value of $x : \log_3 x - 4\log_x 3 + 3 = 0$

TUTORIAL 4

1. Write each of the following in logarithmic form. For example, $3^4 = 81$ can be written as $\log_3 81 = 4$.

   (a) $2^4 = 16$ *(b)* $125 = 5^3$ *(c)* $64 = 16^{\frac{3}{2}}$ *(d)* $81 = (\frac{1}{3})^{-4}$

2. Write each of the following in exponential form. For example, $\log_5 125 = 3$ can be written as $5^3 = 125$.

   (a) $\log_2 32 = 5$ (b) $2 = \log_5 25$ (c) $7 = \log_2 128$ (d) $-2 = \log_3 (1/9)$

   (e) $\log_e 1 = 0$ (f) $2 = \log_a X$ (g) $\ln 20.09 = 3$

3. Determine the value of each of the following logarithms.

   (a) $\log_2 64$ (b) $\log_{10} 10^7$ (c) $\log_{27} 3$ (d) $\log_5 125$ (e) $\log_{10} 10^{-6}$

4. Write each of the following as a single logarithm.

   (a) $3\log_a 2 + 2\log_a 3 - 2\log_a 6$ (b) $3\log_2 5 - 2\log_2 7$

   (c) $\frac{1}{2}\log_5 64 + \frac{1}{3}\log_5 27 - \log_5 (x^2 + 4)$

   (d) $3\log_2 (x+2) + \log_2 8x - 2\log_2 (x+8)$

   (e) $2\log_5 x - 3\log_5 (2x+1) + \log_5 (x-4)$

5. Evaluate, without using calculator:

(a) $3\log_{10}2 + 2\log_{10}5 - \log_{10}20$

(b) $\log_{10}\frac{41}{35} + \log_{10}70 - \log_{10}\frac{41}{2} + 2\log_{10}5$

(c) $\log_{10}\frac{14}{15} + \log_{10}\frac{21}{20} - \log_{10}\frac{49}{50}$

6. Solve the equations:

(a) $3^x = 2$ (b) $3^{4x} = 4$ (c) $2^x 2^{(x+1)} = 10$

(d) $(\frac{1}{2})^x = 6$ (e) $(\frac{2}{3})^x = \frac{1}{16}$

7. Given that $\log_2 3 = 1.585$, and $\log_2 5 = 2.322$, calculate the values of $\log_2 60$ and $\log_2 0.3$.

8. If $\log_7 2 = 0.356$ and $\log_7 3 = 0.565$, find the value of $\log_7\frac{8}{9} + 2\log_7\frac{9}{2}$

9. Solve each of the following equations.

(a) $\log_2 x + \log_2(x+2) = 3$ (b) $\log_3 x - \log_3(2x+3) = -2$

10. Find the values of x in

(a) $\log_2 x + \log_x 2 = 2$ (b) $\log_3 x - 2\log_x 3 = 1$

Challenging Questions

1 If $u = \log_4 x,$ find in term of u

(a) x

(b) $\log_4 2x$

(c) $\log_x 64$

2 (a) If $\log_8 x = p$, express $\log_2 x$ in terms of p.

Given that $\log_q(xy) = 3$ and $\log_q\left(x^2 y^3\right) = 4$.

Calculate the values of $\log_q x$ and $\log_q y$

3 (a) Calculate the value of $\log_3 8$. Giving your answer correct to 3 significant figures.

(b) Evaluate x if $\log_2(1+x) + \log_2(5-x) - \log_2(x-2) = 3$