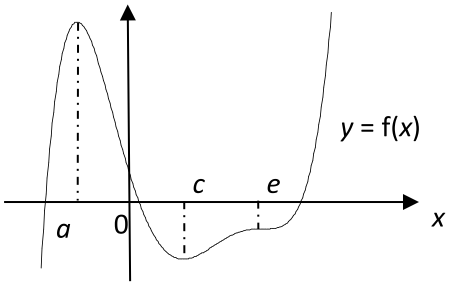
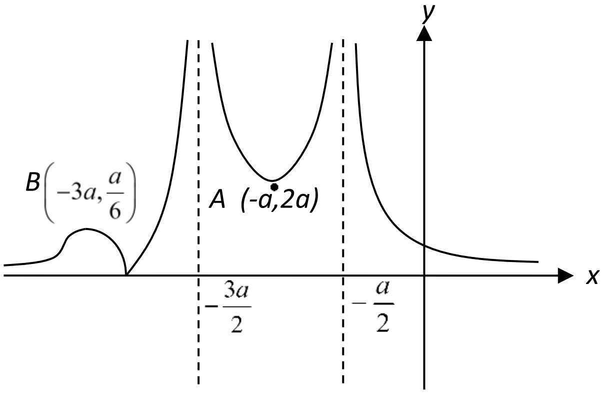
1.2 Graphs and Transformations 2 (Suggested Solutions)

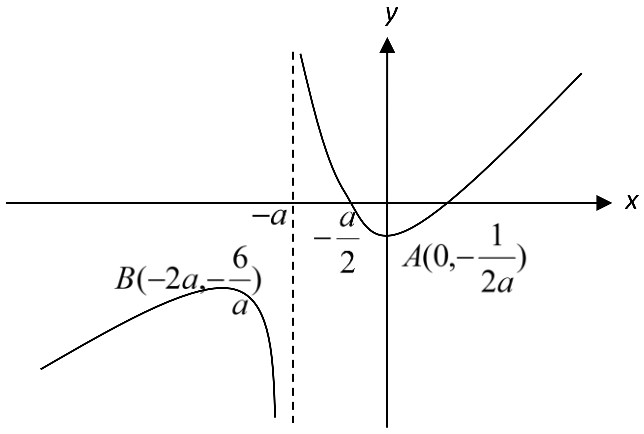
Transformation of Curves

1	(i) Range of values of x for which the graph of $\mathrm{f}(x)$ (a) is strictly increasing : $x < a$ or $x > b$ (b) concaves upwards: $x > h$ (ii) Maximum point at $x = a$ Minimum point at $x = b$
2	(i) Maximum point at $x = a$ Minimum point at $x = c$ Stationary point of inflexion at $x = e$ (ii) 

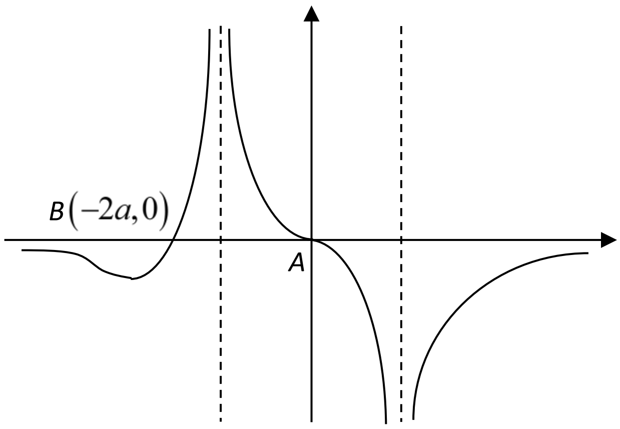
3(i) $y = \left|\mathrm{f}(x+a)\right|$



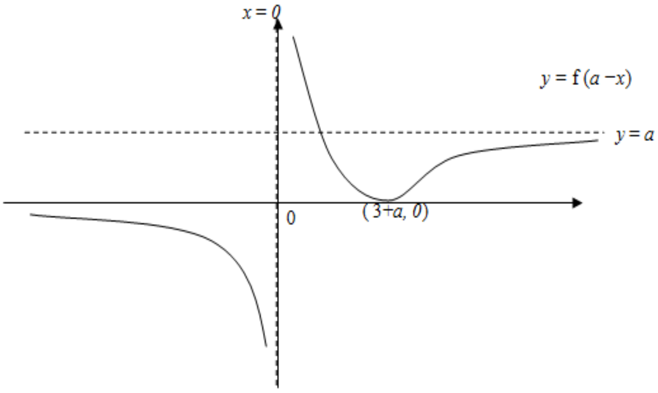
3(ii) $y = \frac{1}{\mathrm{f}(x)}$



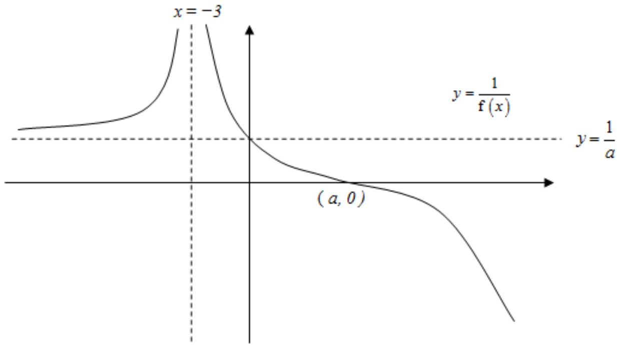
3(iii) $y = \mathrm{f}'(x)$



4(i) $y = f(a-x)$



4(ii) $y = \dfrac{1}{\mathrm{f}(x)}$



5ai) **Method 1:** $x + (2y + 2k)^2 = 1$

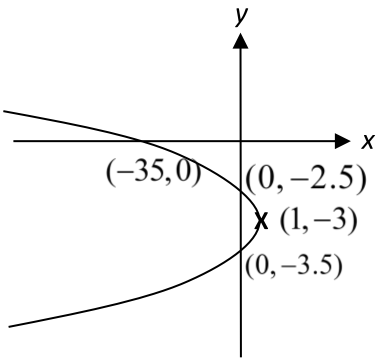
(1): Translation by <u>$2k$</u> units in the negative y-direction;
(2): Scale parallel to the y-axis by <u>factor 1/2</u>.

OR

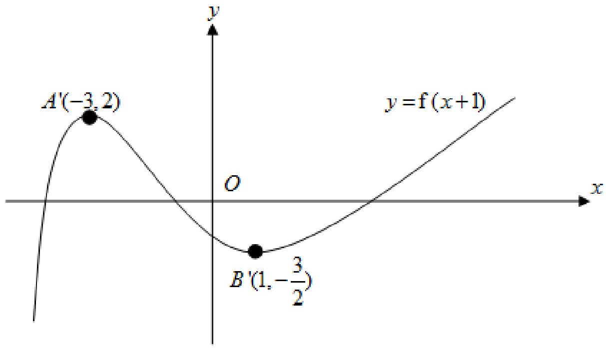
<u>Method 2</u>: $x + [2(y+k)]^2 = 1$

(1): Scale parallel to the y-axis by <u>factor ½</u>
(2): Translation by <u>k</u> units in the negative y-direction.

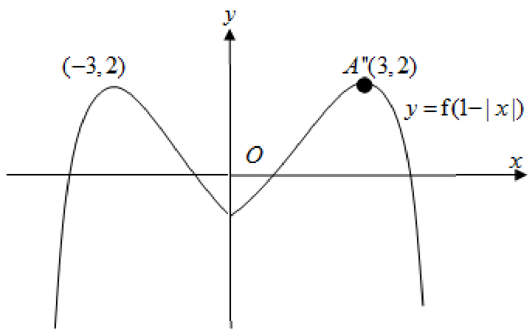
5aii $x + 4(y+3)^2 = 1$



6(ai) $y = f(x+1)$



6(aii) $y = \mathrm{f}(1-|x|)$



6(bi)

Equation of ellipse: $\dfrac{(x-p)^2}{p^2}+\dfrac{(y-p)^2}{(2p)^2}=1$

6(bii)

A : Scale the resulting graph parallel to the y-axis by a scale factor of 2.

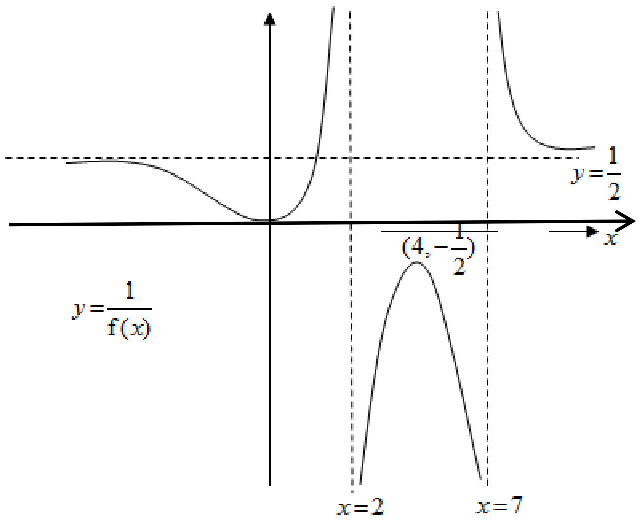
B : Translate the resulting graph in the negative y-direction by p units (positive y-direction by $-p$ units)

Alternatively,

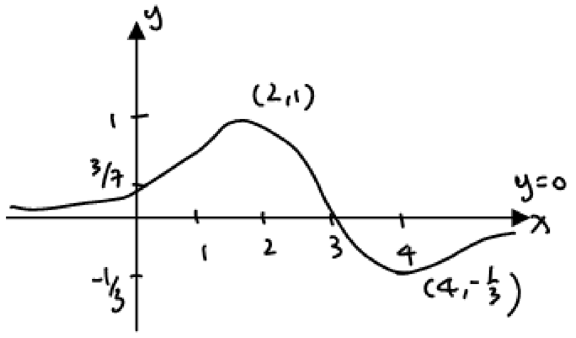
A' : Translate the resulting graph in the negative y-direction by $\dfrac{p}{2}$ units (positive y-direction by $\dfrac{-p}{2}$ units)

B': Scale the resulting graph parallel to the y-axis by a scale factor of 2.

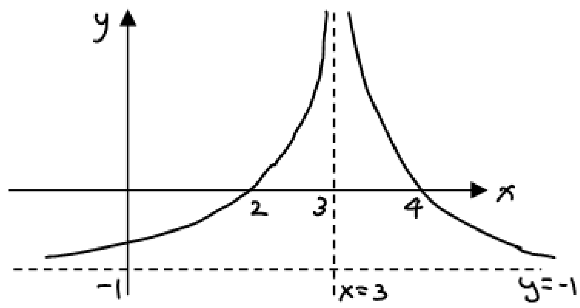
7(a) $y=\dfrac{1}{\mathrm{f}(x)}$



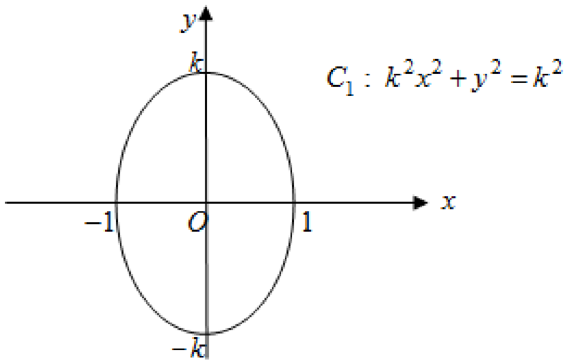
8(i) $y=\dfrac{1}{\mathrm{f}(x)}$



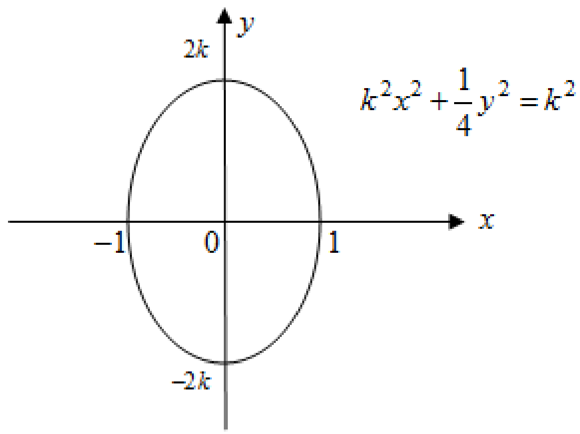
8(ii) $y=\dfrac{\mathrm{d}}{\mathrm{d}x}\left[\mathrm{f}(x)\right]$



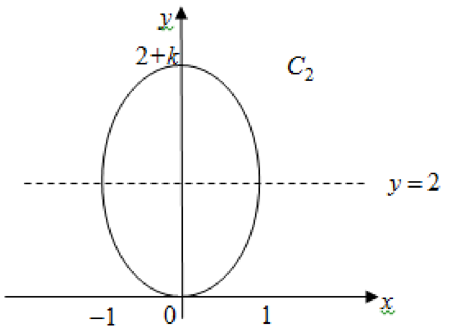
9(i)



9(ii)



9(iii)



$k = 2$

9(iv) Translation parallel to the y-axis by 2 units

9(v) $4x^2 + (y-2)^2 = 4$

10a

Method 1: $\dfrac{(2y-1)^2}{4} - \dfrac{x^2}{9} = 1$

(1): Translate $\underline{1}$ unit in the positive y-direction;

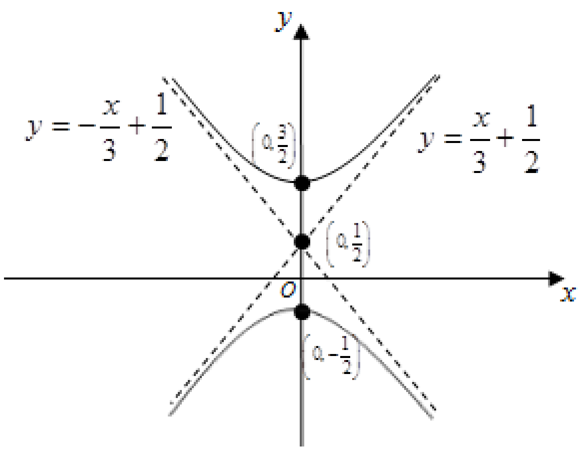
(2): Scale parallel to the y-axis by <u>factor 1/2</u>.

Method 2: $\dfrac{\left[2\left(y-\frac{1}{2}\right)\right]^2}{4} - \dfrac{x^2}{9} = 1$

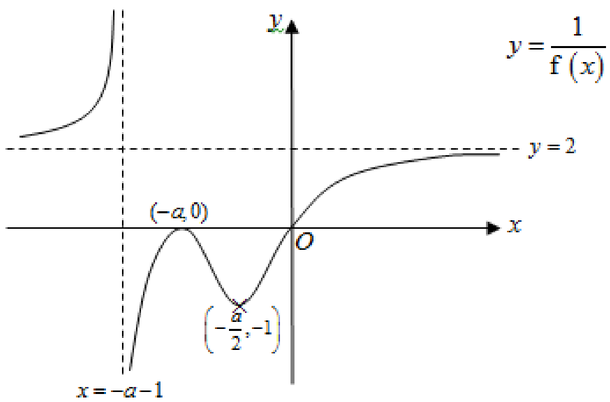
(1): Scale parallel to the y-axis by <u>factor 1/2</u>;

(2): Translate <u>1/2</u> unit in the positive y-direction.

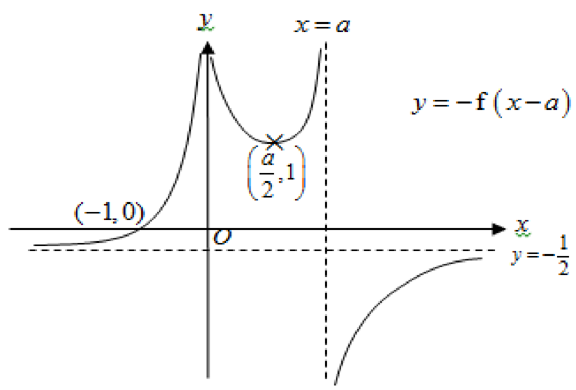
10(aii)



10(bi)



10(bii)

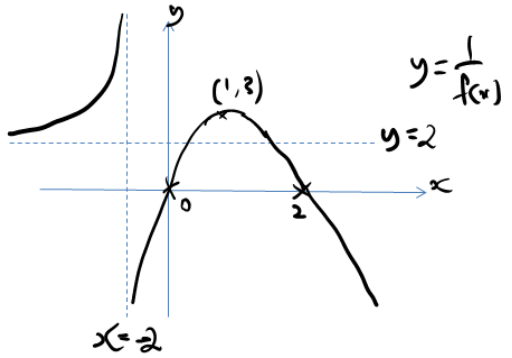


11a

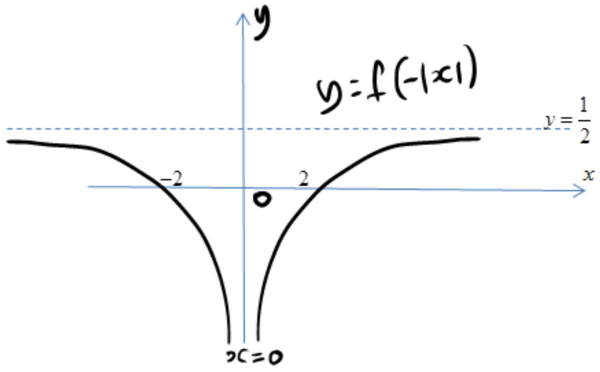
(1) Translate -3 units in the direction of x-axis, followed by

(2) Scale parallel to the x-axis with a scale factor of $\dfrac{1}{2}$.

11(bii) $y = \dfrac{1}{f(x)}$



11(biii) $y = f\left(-|x|\right)$



12

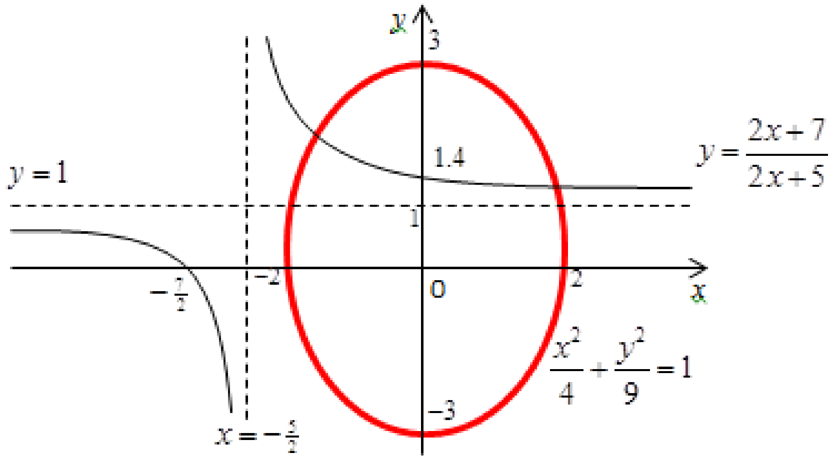
$$y = \frac{2x+7}{2x+5} = 1 + \frac{2}{2x+5}$$

$$\frac{1}{x} \xrightarrow{\text{Step 1}} \frac{1}{x+5} \xrightarrow{\text{Step 2}} \frac{1}{2x+5}$$

$$\xrightarrow{\text{Step 3}} \frac{2}{2x+5} \xrightarrow{\text{Step 4}} 1 + \frac{2}{2x+5}$$

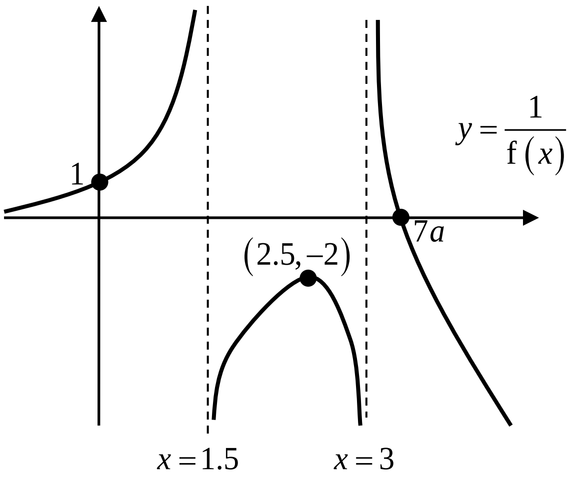
Steps:
1. Translate 5 units in the negative x-direction.
2. Scale parallel to the x-axis by factor $\frac{1}{2}$.
3. Scale parallel to the y-axis by factor 2.
4. Translate 1 unit in the positive y-direction.

12 cont'd

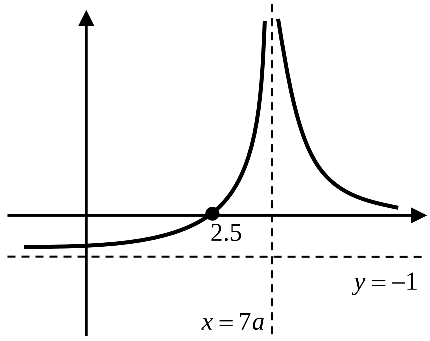


two real roots

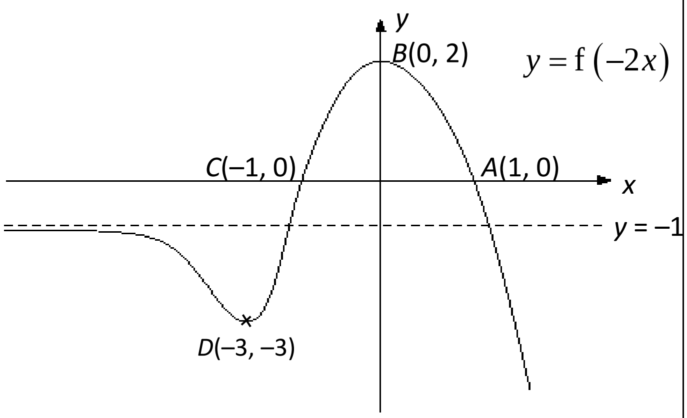
13(i) $y=\dfrac{1}{\mathrm{f}(x)}$



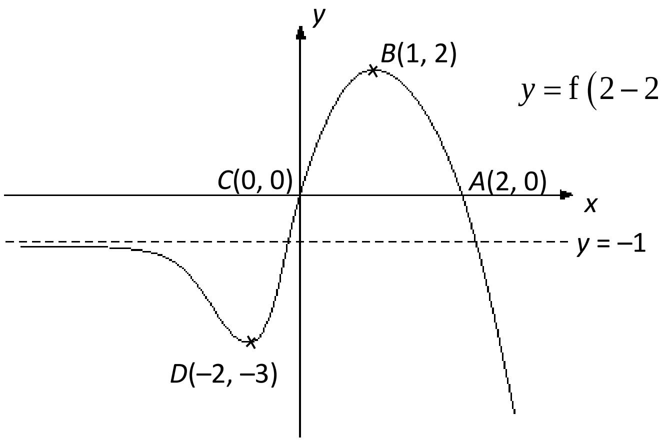
13(ii) $y = f\,'(x)$



14(i) $y = \mathrm{f}(-2x)$



14(ii) $y = \mathrm{f}(2-2x)$



14(iii) $\boldsymbol{y}=\dfrac{1}{2}\mathrm{f}\left(2\boldsymbol{x}\right)$