Name:	Class:	Class Register Number:

中正中学

CHUNG CHENG HIGH SCHOOL (MAIN)

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Parent's Signature

PRELIMINARY EXAMINATION 2023
SECONDARY 4

ADDITIONAL MATHEMATICS 4049/02

Paper 2 **Tuesday 29 August 2023**

Candidates answer on the Question Paper. **2 hours 15 minutes**

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

For Examiner's Use	
Question Number	Marks Obtained
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
Total Marks	

This document consists of **20** printed pages and **2** blank pages.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1 **(a)** The line $4x = 3y + 2$ intersects the curve $x^2 - xy + 5 = 0$ at the points A and B.
Find the midpoint of AB. [5]

(b) Find the least value of the integer h for which $hx^2 + 5x + h$ is positive for all real values of x. [3]

(c) Given that the line $y = 3x + p$ is tangent to the curve $y = x^2 + 5x + q$, where p and q are integers, prove that p and q are consecutive numbers. [4]

2 **(a)** By considering the general term in the binomial expansion of $\left(x^3 - \frac{2}{x}\right)^7$, explain why there are only odd powers of x in this expansion. [3]

(b) Find the term independent of x in the expansion of $\left(x^3 - \frac{2}{x}\right)^7 \left(\frac{5}{x} - 2x^2\right)$. [3]

3 The expression $7\sin\theta + 3\cos\theta$ is defined for $0° \leq \theta \leq 360°$.

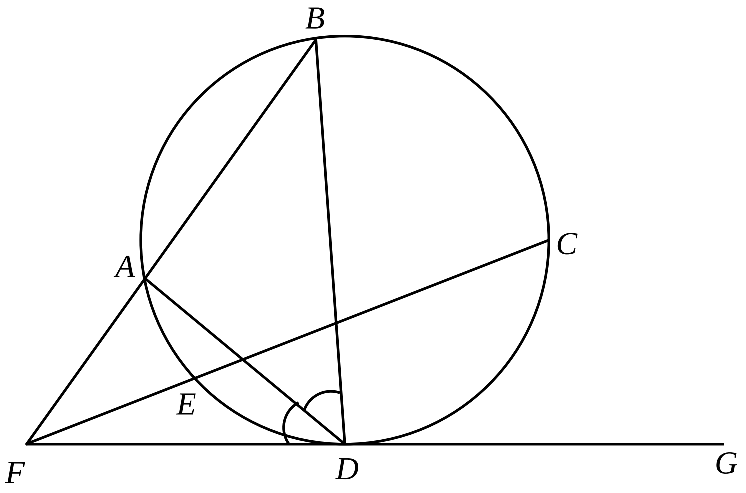
(a) Using $R\sin(\theta + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$, solve the equation

$$7\sin\theta = 5 - 3\cos\theta .$$

[5]

(b) State the largest and smallest values of $\left(7\sin\theta+3\cos\theta\right)^2-12$ and find the corresponding values of θ. [4]

4



The diagram shows a circle passing through the points A, B, C, D and E. The straight line FDG is tangent to the circle at D while FAB and FEC are secant lines.
Given that angle FDA = angle ADB,

(a) show that triangle ABD is an isosceles triangle, [2]

(b) prove that $AF \times BF = EF \times CF$. [3]

5 An object is heated in an oven until it reaches a temperature of X °C. It is then allowed to cool under room temperature. Its temperature, T °C, can be modelled by $T = 18 + 62e^{-kt}$, where t is the time in minutes since the object starts cooling.

(a) Find the value of X. [1]

When t = 10, the temperature of the object is 65 °C.

(b) Find the temperature of the object an hour later, giving your answer to one decimal place. [5]

(c) What does the model suggest about the room temperature? Explain your answer. [2]

6 The equation of a curve is $y = \ln\left(\frac{2x-1}{3x-1}\right)$, where $x > \frac{1}{2}$.

(a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$, expressing it as a single fraction. [3]

(b) Explain why the curve will be almost parallel to the x-axis as x becomes very large. [2]

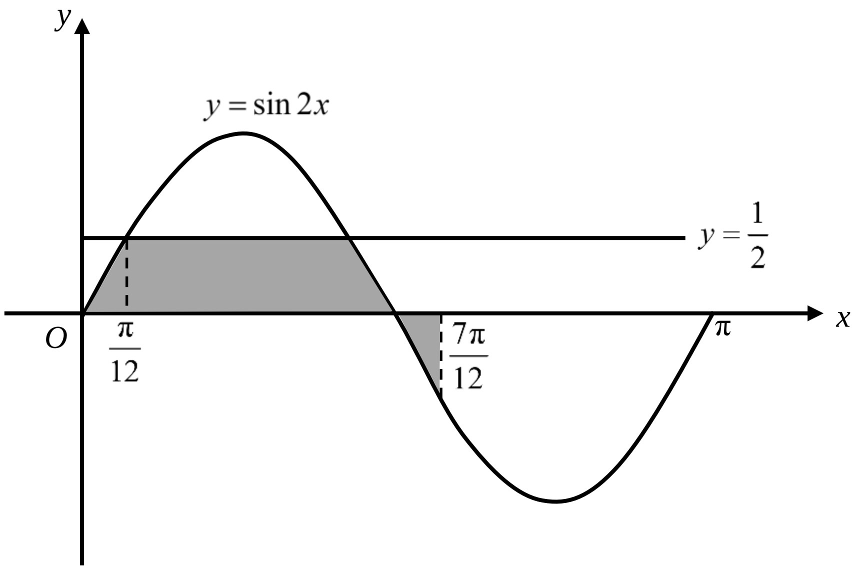
(c) Find the value of x at the instant when the rate of change of x is twice the rate of change of y. [3]

7 **(a)** Show that $\tan A + \cot A = 2\operatorname{cosec} 2A$. [3]

(b) Hence, solve the equation $\dfrac{1}{\tan A + \cot A} = \dfrac{1}{4}$ for $0 \le A \le 2\pi$. [3]

(c) The diagram shows, for $0 \leq x \leq \pi$, the curve $y = \sin 2x$ and the line $y = \dfrac{1}{2}$.

Showing all your working, find the area of the shaded region. [5]



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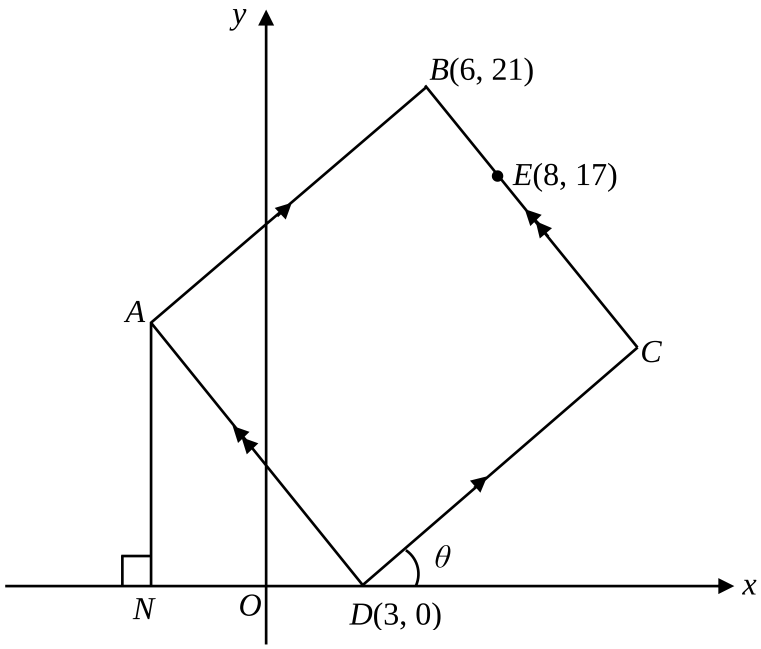


The diagram shows a parallelogram with vertices A, $B(6, 21)$, C and $D(3, 0)$. The point $E(8, 17)$ lies on BC. The line CD makes an angle θ with the positive x-axis such that $\tan\theta = 1$. A line is drawn, parallel to the y-axis, from A to meet the x-axis at N.

(a) Show that the coordinates of A are $(-3, 12)$. [5]

(b) Hence, find the area of parallelogram *ABCD*. [2]

(c) A point *F* with *y*-coordinate of 5 lies on the line *CD*.
Explain why *AEFN* is a parallelogram. [5]

9 **(a)** Differentiate $x \ln x$ with respect to x. [2]

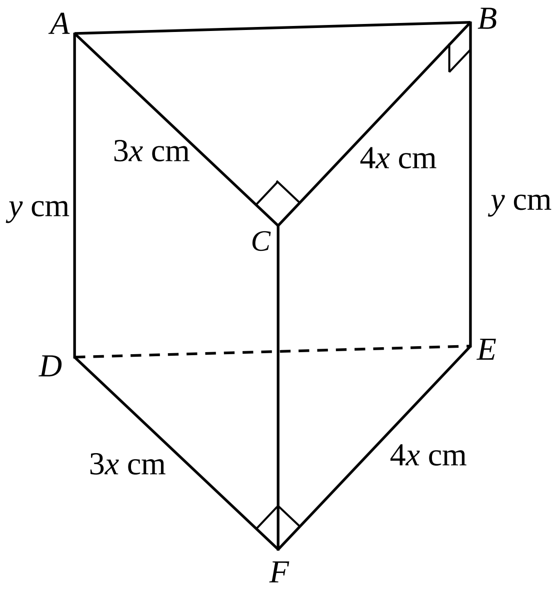
(b) A curve $y = \mathrm{f}\left(x\right)$ is such that $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = 24x^2 + \frac{16}{x}$, where $x > 0$. The line $y = 24x - 40$ is parallel to the tangent of the curve at $P(1, -16)$.

By using the result found in part **(a)**, find the equation of the curve. [6]

Continuation of working space for question **9(b)**.

(c) Explain why the condition $x > 0$ is necessary. [1]

10



The diagram shows a solid prism with right-angled triangular ends that are perpendicular to the parallel sides *AD*, *BE* and *CF*, which are each y cm in length. The right-angled triangular ends have sides *AC* and *DF*, which are $3x$ cm, and sides *BC* and *EF*, which are $4x$ cm.

Given that the volume of the prism is 1200 cm^3,

(a) find an expression for y in terms of x, [2]

(b) show that the total surface area of the prism, S cm^2, is given by $S = 12x^2 + \frac{2400}{x}$. [3]

(c) Given that x can vary, find the value of x for which the total surface area of the prism is a stationary value. [3]

(d) Explain why this value of x gives the smallest surface area possible. [2]

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Answer Key

Qns	Ans
1(a)	$\left(1,\ \frac{2}{3}\right)$
1(b)	$h = 3$
1(c)	$q = 1 + p$, therefore consecutive numbers
2(a)	Power of $x = 21 - 4r$. Since **$4r$ is an even number** for all non-negative integer values of r and 21 is **an odd number**, then $21 - 4r$ is always an **odd number.**
2(b)	-3360
3(a)	$\theta = 17.8°$ or $115.8°$ (1 d.p.)
3(b)	Largest value $= 46$ when $\theta = 66.8°$ or $246.8°$ (1 d.p.) Smallest value $= -12$ when $\theta = 156.8°$ or $336.8°$ (1 d.p.)
4(a)	Show qns
4(b)	Show qns
5(a)	80
5(b)	For $t = 60$, $T = 29.8°C$ For $t = 70$, $T = 26.9°C$
5(c)	Room temp. = 18°, $62e^{-kt}$ approaches zero as t becomes larger.
6(a)	$\frac{1}{(2x-1)(3x+1)}$
6(b)	Since $(2x-1)(3x+1)$ becomes a very large number as x becomes very large, $\frac{dy}{dx}$ approaches 0, the curve will almost horizontal, thus it will be almost parallel to the x-axis.
6(c)	$x = 1$
7(a)	Show qns
7(b)	$A = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}$ or $\frac{17\pi}{12}$
7(c)	0.725 units^2
8(a)	Show qns
8(b)	162 units^2
8(c)	Since gradients of lines AE and FN are the same, the lines AE and FN are parallel. Since F lies directly below E, the line EF is a vertical line. Thus, the lines EF and AN are parallel. Since there are 2 pairs of parallel lines (or equivalent), $AEFN$ is a parallelogram.
9(a)	$1 + \ln x$
9(b)	$y = 2x^4 + 16x\ln x - 18$
9(c)	$\ln x$ is defined for $x > 0$ or $\ln x$ is undefined for $x < 0$.
10(a)	$y = \frac{200}{x^2}$
10(b)	
10(c)	$x = 4.64$
10(d)	$\frac{d^2S}{dx^2} = 72 > 0$ $\because$ minimum