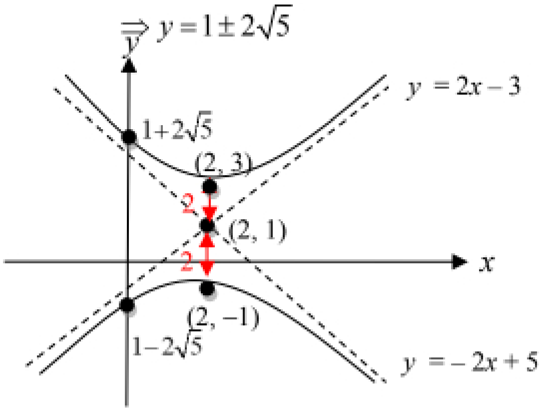
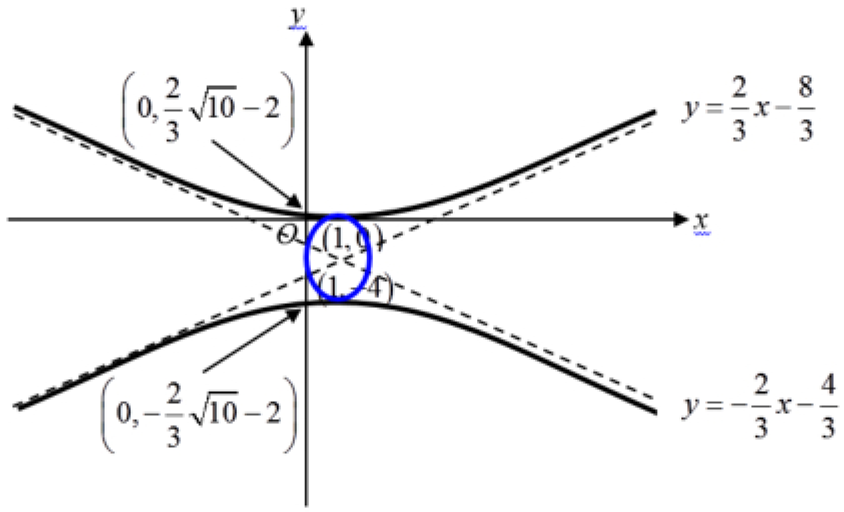
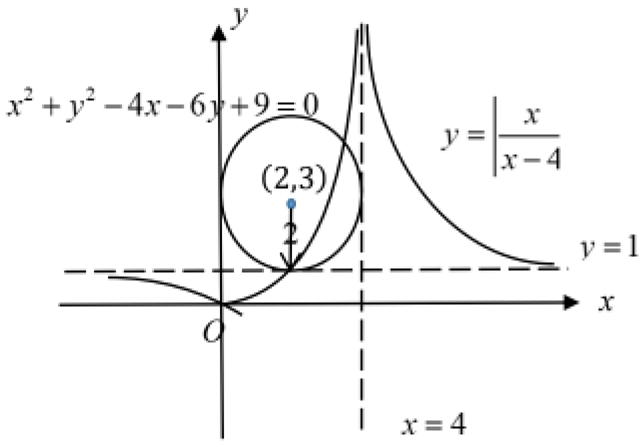
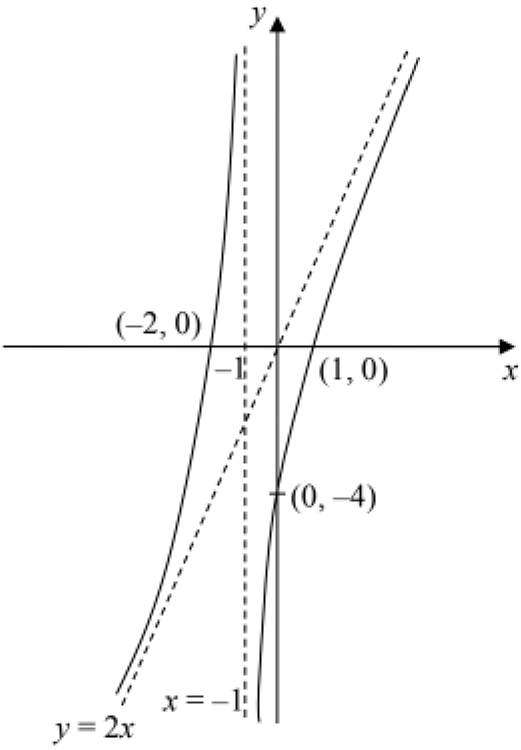
2022 C1 Block Test Revision Package Solutions
Chapter 2 Graphs and Transformations

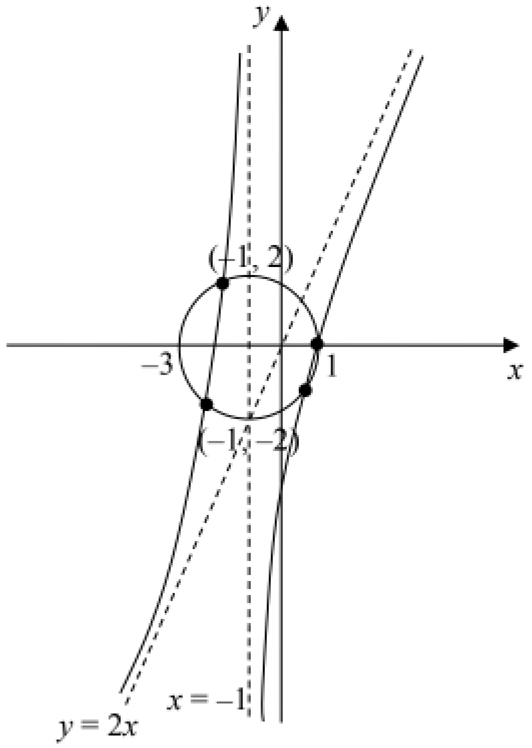
A Curves Sketching and Conic Sections

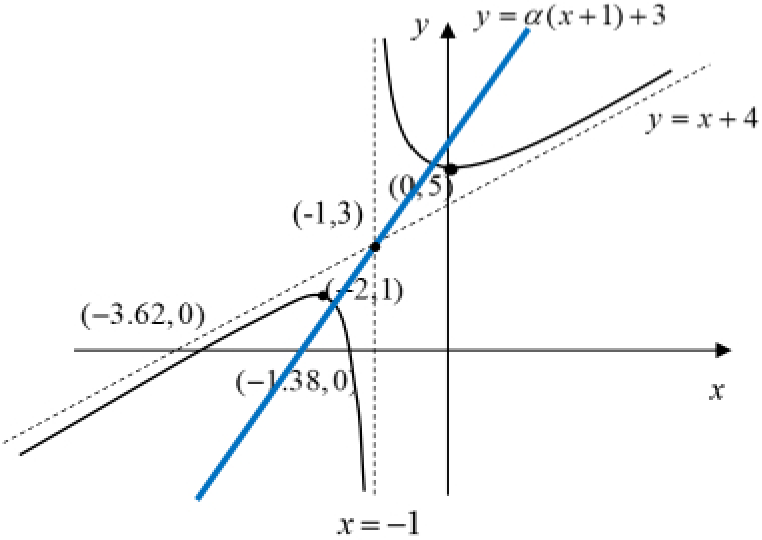
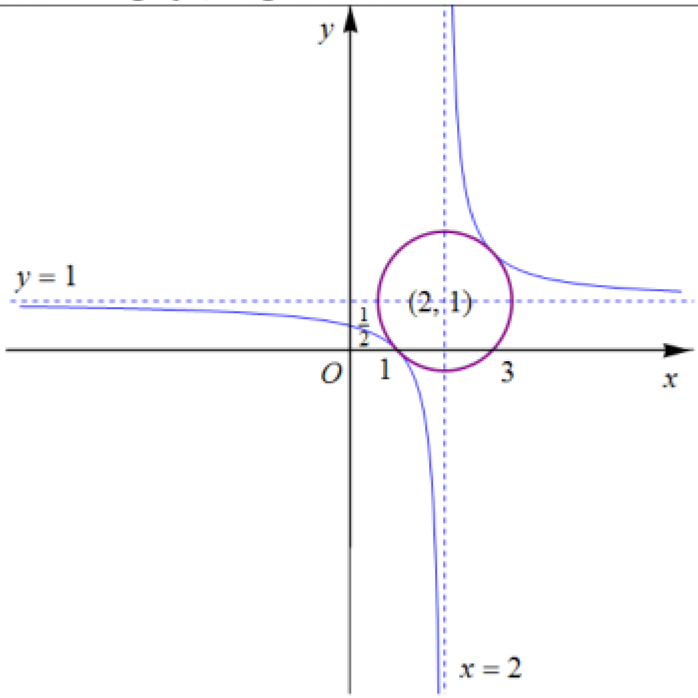
Qn	Solutions	Comments
1(i)	C is a hyperbola with centre at (2, 1). To find the asymptotes, as $x \to \pm \infty$ $\frac{(y-1)^2}{4}-(x-2)^2=0$ $\Rightarrow (y-1)^2=4(x-2)^2$ $\Rightarrow y-1=\pm 2(x-2)$ $\Rightarrow y=1\pm 2(x-2)$ $\Rightarrow y=1+2x-4$ or $y=1-2x+4$ $\Rightarrow y=2x-3$ or $y=-2x+5$ When $x=0$, $\frac{(y-1)^2}{4}-(0-2)^2=1$ $\Rightarrow (y-1)^2=20$ $\Rightarrow y=1\pm\sqrt{20}$ $\Rightarrow y=1\pm 2\sqrt{5}$ 	
1(ii)	Since $(x-2)^2+(y-1)^2=r^2$ is a circle with centre (2, 1) and radius r, hence to intersect the hyperbola in only two points, $r=2$.	
2(i)	$9y^2+36y-4x^2+8x-4=0$ $9\left(y^2+4y\right)-4\left(x^2-2x\right)-4=0$ $9\left(y^2+4y+4\right)-36-4\left(x^2-2x+1\right)+4-4=0$ $9\left(y+2\right)^2-4\left(x-1\right)^2=36$ $\frac{\left(y+2\right)^2}{2^2}-\frac{\left(x-1\right)^2}{3^2}=1$ (shown)	

2(ii)	Lines of symmetry are $x=1$ and $y=-2$.	
2 (iii)	 To find y-intercepts, let $x=0$, $\frac{(y+2)^2}{2^2}-\frac{(-1)^2}{3^2}=1$ $\frac{(y+2)^2}{2^2}=\frac{10}{9}$ $(y+2)^2=\frac{40}{9}$ $y+2=\pm\sqrt{\frac{40}{9}}$ $y=\pm\sqrt{\frac{40}{9}}-2$ $y=\pm\frac{2}{3}\sqrt{10}-2$ To find asymptotes, let $\frac{(y+2)^2}{2^2}-\frac{(x-1)^2}{3^2}=0$. $(y+2)^2=\frac{2^2(x-1)^2}{3^2}$ $y+2=\pm\frac{2}{3}(x-1)$ $y=\frac{2}{3}(x-1)-2$ or $y=-\frac{2}{3}(x-1)-2$ $\therefore y=\frac{2}{3}x-\frac{8}{3}$ or $y=-\frac{2}{3}x-\frac{4}{3}$	

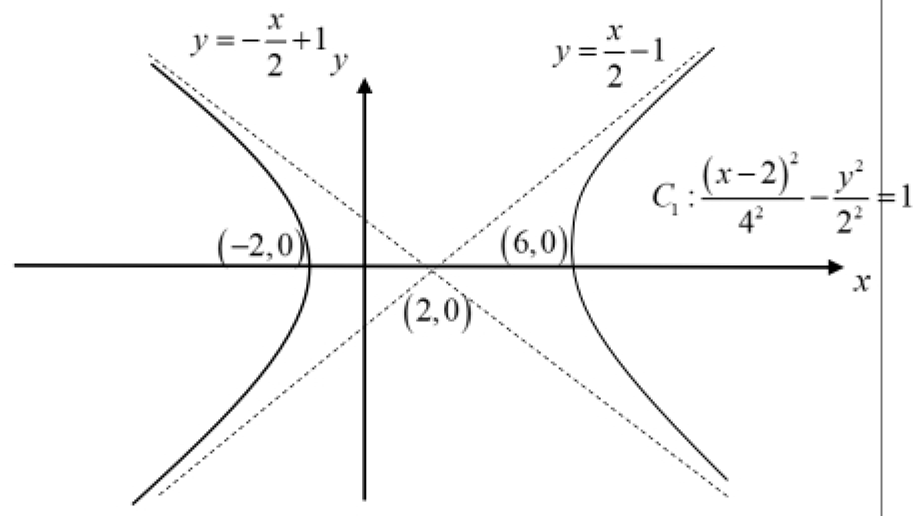
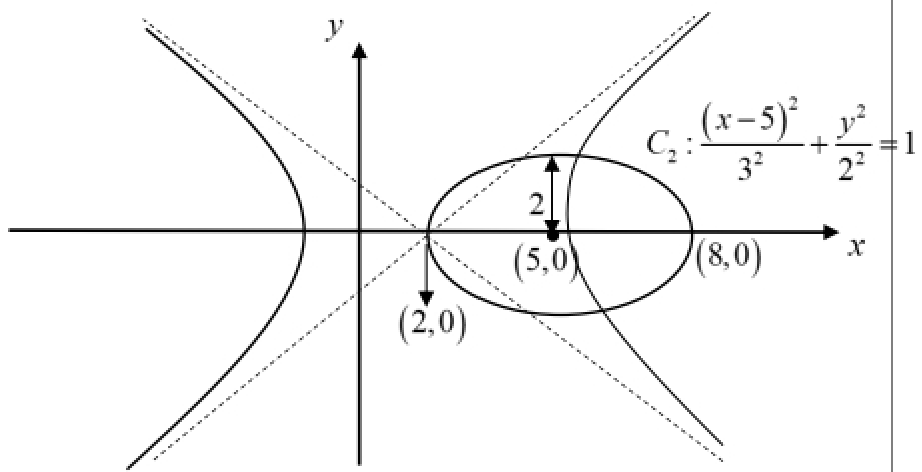
2.(iv)	$2n(x-1)^2+(y+2)^2=2n$ $(x-1)^2+\dfrac{(y+2)^2}{2n}=1$ $\dfrac{(x-1)^2}{1^2}+\dfrac{(y+2)^2}{\left(\sqrt{2n}\right)^2}=1$ For $2n(x-1)^2+(y+2)^2=2n$ and H intersect at least twice, $\sqrt{2n}\geq 2$ $n\geq 2$	
3(a) (i)	$y=\left\|\dfrac{x}{x-4}\right\|=\left\|1+\dfrac{4}{x-4}\right\|$ $x^2+y^2-4x-6y+9=0$ $(x-2)^2+(y-3)^2-4-9+9=0$ $(x-2)^2+(y-3)^2=4$ 	
3(a) (ii)	Points of intersection are (2, 1) and (3.28, 4.54).	
4(i)	$C: y=2x+\dfrac{k}{x+b}=\dfrac{2x^2+2bx+k}{x+b}$, for some constant k Comparing $ax^2+2x-4=2x^2+2bx+k$ $\Rightarrow \begin{cases} a=2 \\ 2=2b\Rightarrow b=1 \end{cases}$ <u>Alternatively</u>: $C: y=\dfrac{ax^2+2x-4}{x+b}$ $ax+(2-ab)$	

	$x+b\,\overline{)\,ax^2+2x-4}$ $\underline{ax^2+abx}$ $(2-ab)x-4$ $\underline{(2-ab)x+b(2-ab)}$ $-4-b(2-ab)$ $\therefore C: y = ax+(2-ab)+\dfrac{-4-b(2-ab)}{x+b}$ $\Rightarrow ax+2-ab=2x$ $\Rightarrow \begin{cases} a=2 \\ 2-ab=0 \Rightarrow b=1 \end{cases}$	
4(ii)	$C: y=\dfrac{2x^2+2x-4}{x+1}$ $\dfrac{dy}{dx}=\dfrac{(x+1)(4x+2)-(2x^2+2x-4)}{(x+1)^2}$ $=\dfrac{4x^2+6x+2-2x^2-2x+4}{(x+1)^2}$ $=\dfrac{2x^2+4x+6}{(x+1)^2}$ $=\dfrac{2[(x+1)^2+2]}{(x+1)^2}$ $>0 \quad \text{for } x \neq -1$ Therefore, the gradient of C is positive for all $x \in \mathbb{R},\ x \neq -1$.	
4(iii)		

4(iv)	$(x+1)^4+(2x^2+2x-4)^2=4(x+1)^2$ $(x+1)^2+\left(\dfrac{2x^2+2x-4}{x+1}\right)^2=4$ Hence, sketch the curve $(x+1)^2+y^2=2^2$ (circle). The two graphs intersect at 4 distinct points, therefore the equation $(x+1)^4+(2x^2+2x-4)^2=4(x+1)^2$ has 4 distinct real roots. 	
5(i)	$y=\dfrac{x^2+\lambda x+\lambda}{x+1}$ $xy+y=x^2+\lambda x+\lambda$ $x^2+(\lambda-y)x+(\lambda-y)=0$ For real values of x, $(\lambda-y)^2-4(1)(\lambda-y)\geq 0$ $(\lambda-y)(\lambda-4-y)\geq 0$ $y\leq\lambda-4$ or $y\geq\lambda$ Therefore, y cannot lie between the values of $\lambda-4$ and λ (shown).	Note the use of discriminant here. This is a commonly asked question.

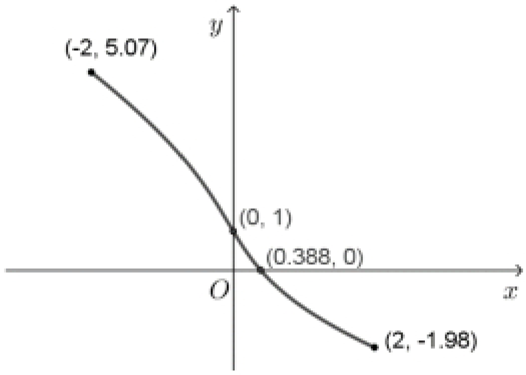
5(ii)		
5(iii)	$\dfrac{x^2+5x+5}{x+1}=\alpha(x+1)+3$ This is a straight line passing through (-1,3) with gradient α . From the graph, range is $\alpha>1$	
6(a) (i)	 Equations of asymptotes are $x=2$ and $y=1$. Axial intercepts are $(1,0)$, $(3,0)$, and $\left(0,\dfrac{1}{2}\right)$.	Can try to key into GC to see the relative positions of the two curves. $(x-2)^2+(y-1)^2=2$ is a circle with centre (2,1) and radius $\sqrt{2}$ which is the distance from (2,1) to (0,1)
6(a) (ii)	Substituting,	

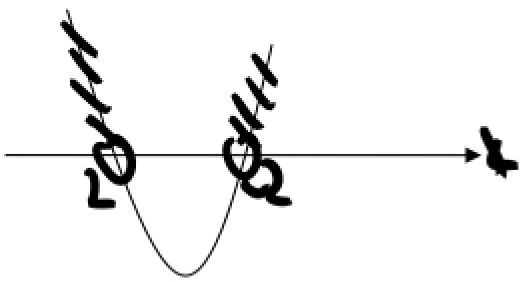
	$$(x-2)^2+(y-1)^2=2$$ $$(x-2)^2+\left(\left(\frac{x-1}{x-2}\right)-1\right)^2=2$$ $$(x-2)^2+\left(\frac{1}{x-2}\right)^2=2$$ Therefore, the number of roots of the equation is equal to the number of points of intersection of C_1 and C_2, which is 2.	
6(a) (iii)	C_3 is a circle with center $(2,1)$ radius $\sqrt{h}$. From the previous part, we see that any such circle with radius greater than $\sqrt{2}$ will intersect with C_1 at 4 points. Consequently, $\sqrt{h}>\sqrt{2}\Rightarrow h>2$.	
6(b)	From the vertical asymptote at $x=2$, $C=-2$. By long division, $$\frac{Ax^2+Bx+11}{x-2}=Ax+(B+2A)+\frac{11+2(B+2A)}{x-2}$$ By comparing $y=Ax+(B+2A)$ with the oblique asymptote $y=x+5$, we have $$A=1$$ $$B+2A=5\Rightarrow B=3$$ **Alternative** : Since $y=x+5$ is an oblique asymptote, $$\frac{Ax^2+Bx+11}{x-2}=x+5+\frac{K}{x-2}$$ $$Ax^2+Bx+11=\left(x+5+\frac{K}{x-2}\right)(x-2)$$ $$Ax^2+Bx+11=x^2+3x-10+K$$ By comparing coefficients, we get $A=1$ and $B=3$.	
7(i)	$x^2-4x-4y^2-12=0$ $(x-2)^2-4-4y^2-12=0$ $(x-2)^2-4y^2=16$ $\frac{(x-2)^2}{4^2}-\frac{y^2}{2^2}=1$	

		
7(ii)	$x=5+3\sin\theta \Rightarrow \sin\theta=\frac{x-5}{3}$ $y=2\cos\theta \Rightarrow \cos\theta=\frac{y}{2}$ using $\sin^2\theta+\cos^2\theta=1$ $\left(\frac{x-5}{3}\right)^2+\left(\frac{y}{2}\right)^2=1$ $\frac{(x-5)^2}{3^2}+\frac{y^2}{2^2}=1$	Use trigonometric identities to eliminate the parameter θ.
7(iii)		
7(iv)	$\frac{(x-2)^2}{4^2}-\frac{y^2}{2^2}=1$ ...(1) $\frac{(x-5)^2}{3^2}+\frac{y^2}{2^2}=1$ ...(2)	

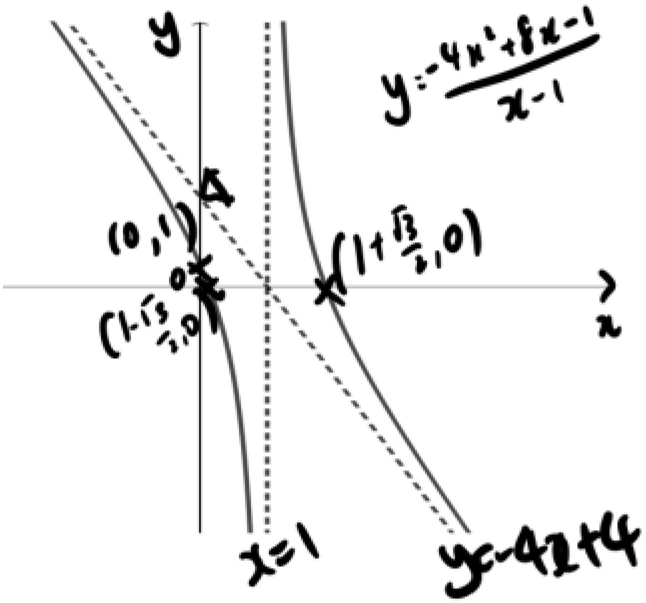
	$(1)+(2)$: $\frac{(x-2)^2}{4^2}+\frac{(x-5)^2}{3^2}=2$ (shown) Therefore the x-coordinate satisfies the equation. Using GC, $x=0.847$ (reject as $x>6$) or $x=6.99$ (3 s.f)	

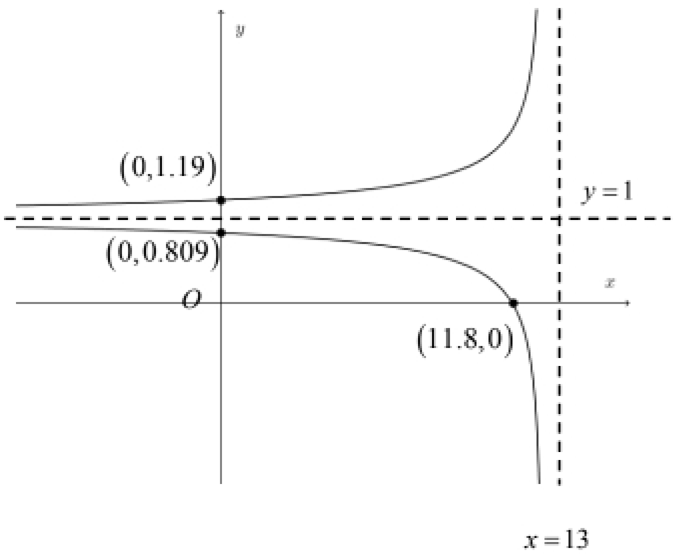
B Parametric Equations

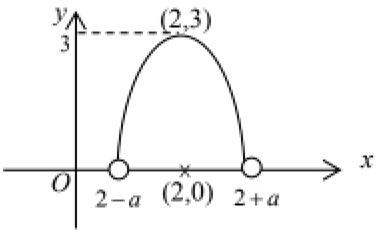
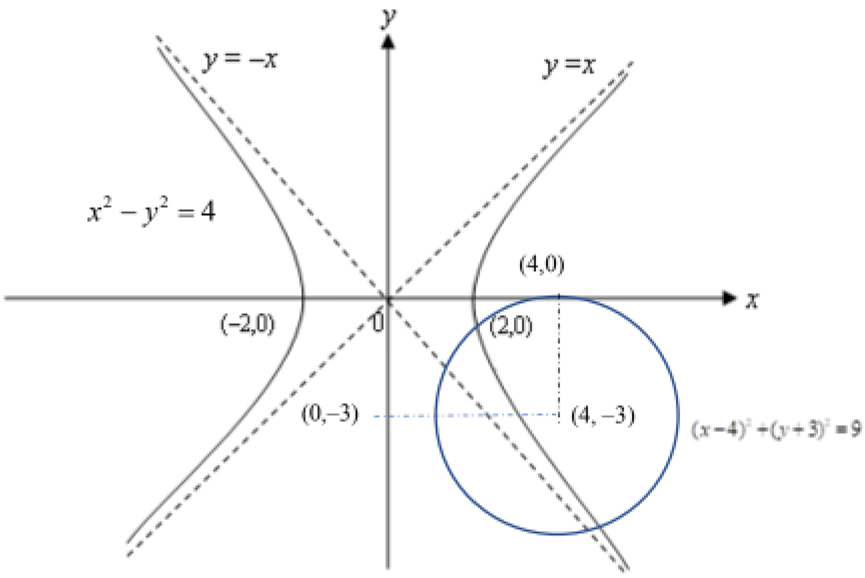
Qn	Solutions	Comments
8(i)	For x-intercept, let $y=0$. $-e^t+2e^{-t}=0$ $2e^{-t}=e^t$ $e^{2t}=2$ $t=0.5\ln 2$ $x=(0.5\ln 2)^3+(0.5\ln 2)$ ≈ 0.388 For y-intercept, let $x=0$. $t^3+t=0$ $t(t^2+1)=0$ $t^2+1=0$ (N.A.) or $t=0$ $y=-e^0+2e^0=1$ For end-points, At $t=-1$, $x=(-1)^3+(-1)=-2$ $y=-e^{-1}+2e^1=5.07$ At $t=1$ $x=1^3+1=2$ $y=-e^1+2e^{-1}=-1.98$ $y=-e^0+2e^0=1$ By GC, 	Need to adjust the Tmin and Tmax under WINDOW in GC after keying in parametric equations.
8(ii)	Substitute $x=t^3+t$, $y=-e^t+2e^{-t}$ into $y=x-1$, $-e^t+2e^{-t}=t^3+t-1$. Using GC, $t=0.48678$.	Use GC to find the point of intersection

	Substitute into x and y, the point of intersection is (0.602, –0.398)	between $y=-\mathrm{e}^{x}+2\mathrm{e}^{-x}$ and $y=x^3+x-1$.
9(i)	Given $y=\dfrac{-4x^2+8kx-5k^2+4}{x-k}$ $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{(x-k)(-8x+8k)-(-4x^2+8kx-5k^2+4)(1)}{(x-k)^2}$ $=\dfrac{-8x^2+8kx+8kx-8k^2+4x^2-8kx+5k^2-4}{(x-k)^2}$ $=\dfrac{-4x^2+8kx-3k^2-4}{(x-k)^2}$ For stationary points, $\dfrac{\mathrm{d}y}{\mathrm{d}x}=0$. $\dfrac{-4x^2+8kx-3k^2-4}{(x-k)^2}=0$ $\Rightarrow -4x^2+8kx-3k^2-4=0 --(1)$ Since there are two stationary points, there are two distinct real roots for the equation (1). Hence, discriminant >0 . $64k^2-4(-4)(-3k^2-4)>0$ $64k^2+16(-3k^2-4)>0$ $64k^2-48k^2-64>0$ $16k^2-64>0$ $k^2-4>0$ $(k-2)(k+2)>0$  Therefore, $k<-2$ or $k>2$.	
9(ii)	Working for long division:	

	$$\begin{array}{r} -4x+4k \\ x-k\overline{)-4x^2+8kx+(4-5k^2)} \\ -)\ \underline{-4x^2+4kx} \\ 4kx+\left(4-5k^2\right) \\ -)\ \underline{4kx-4k^2} \\ 4-k^2 \end{array}$$ $y=\dfrac{-4x^2+8kx-5k^2+4}{x-k}=-4x+4k-\dfrac{k^2-4}{x-k}$ Oblique asymptote: $y=-4x+4k$ Since the oblique asymptote cuts the y-axis at (0,4), $4k=4$ $k=1$	
9(iii)	Since $k=1$, there is no turning points for the curve C. $y=\dfrac{-4x^2+8x-1}{x-1}=-4x+4+\dfrac{3}{x-1}$ **Asymptotes**: Vertical: $x=1$ Oblique Asymptote: $y=-4x+4$ **Intercepts:** When $x=0$, $y=\dfrac{-1}{-1}=1 \quad \therefore \left(0\ ,\ 1\right)$ When $y=0$, $-4x^2+8x-1=0$ $4x^2-8x+1=0$ $x=\dfrac{8\pm\sqrt{64-4(4)(1)}}{2(4)}$ $=\dfrac{8\pm\sqrt{48}}{8}$ $=1\pm\dfrac{\sqrt{16\times3}}{8}$ $=1\pm\dfrac{4\sqrt{3}}{8}$ $=1\pm\dfrac{\sqrt{3}}{2}$ $\left(1+\dfrac{\sqrt{3}}{2}\ ,\ 0\right)$ or $\left(1-\dfrac{\sqrt{3}}{2}\ ,\ 0\right)$	

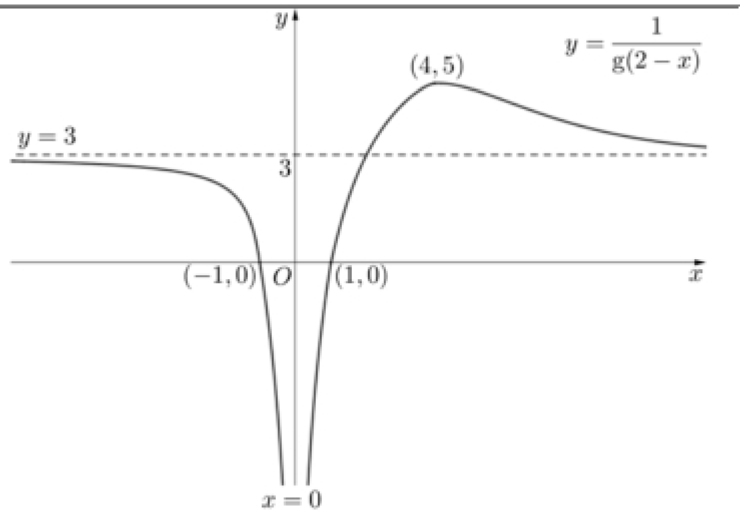
		
9(iv)	By using the trigonometric identity: $\tan^2 t + 1 = \sec^2 t$ $(x-1)^2 + 1 = \left(\frac{y}{b}\right)^2$ $\left(\frac{y}{b}\right)^2 - (x-1)^2 = 1$	
9(v)	The asymptotes of the hyperbola in (iv) are $y = \pm b(x-1)$ and centre is (1,0). The asymptotes of the hyperbola $y = \pm b(x-1)$ passes through the point (1 , 0) and will therefore pivot around the point. Hence, for the hyperbola to intersect the curve *C* at most twice, $b \geq 4$.	
10(i)	At $y = 0$, $\frac{t-3}{3} = 0 \Rightarrow t = 3$ $x = 16 - \sqrt{18} = 11.8$ At $x = 0$, $16 - \sqrt{t^2+9} = 0$ $\sqrt{t^2+9} = 16$ $t^2 + 9 = 256$ $t = \pm\sqrt{247}$ $\therefore y = \frac{\sqrt{247}-3}{\sqrt{247}}$ or $y = \frac{-\sqrt{247}-3}{-\sqrt{247}}$ $= 0.809$ $\quad$ $= 1.19$ Coordinates of the points are (11.8, 0), (0, 0.809), (0, 1.19).	

10(ii)	$\frac{dx}{dt} = -\frac{1}{2}\cdot\frac{1}{\sqrt{t^2+9}}(2t)$ $\quad y = 1-\frac{3}{t}$ $= -\frac{t}{\sqrt{t^2+9}}$ $\quad \frac{dy}{dt} = \frac{3}{t^2}$ $\frac{dy}{dx} = \frac{3}{t^2}\times\frac{\sqrt{t^2+9}}{-t}$ $= -\frac{3\sqrt{t^2+9}}{t^3}$ Since $t^2+9>0$ for all real values of t, $3\sqrt{t^2+9} \neq 0$. $\therefore \frac{dy}{dx} \neq 0$ $\therefore C$ has no stationary point.	
10 (iii)	y is undefined when $t = 0$. $\Rightarrow x = 16-\sqrt{0^2+9} = 13$ $\therefore x = 13$ is the vertical asymptote.	
10 (iv)	As $x \to -\infty$, $16-\sqrt{t^2+9} \to -\infty$ $t^2 \to \infty$ $t \to \pm\infty$ As $t \to \pm\infty$, $y = 1-\frac{3}{t} \to 1$	
10(v)		Use the information from (i) to (iv) to sketch the graph.
11(i)	For $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$,	

	$-1<\sin\theta<1$ $\Rightarrow 2-a<a\sin\theta+2<2+a$ $\Rightarrow 2-a<x<2+a$ $0<\cos\theta\le 1$ $\Rightarrow 0<3\cos\theta\le 3$ $\Rightarrow 0<y\le 3$	
11(ii)	For $0<a<2$, Curve C is a half-ellipse with centre (2,0) and x-intercepts $(2\pm a,0)$. 	May substitute a value of $0<a<2$ into the equations and use GC to check the shape of the curve but the labelling of intercepts are in terms of a. Use range in (i) to help sketch the graph.
12(i)	For C_1: Circle with centre (4, -3) and radius 3 x-intercept: (4, 0) For C_2: Hyperbola with centre (0, 0) Asymptotes: $y=x$ and $y=-x$ x–intercepts: (–2, 0) and (2, 0) 	

12(ii)	C_2: $x^2 - y^2 = 4$ --- (1) Sub $x = 3\sin\theta + 4$, $y = 3\cos\theta - 3$, into (1) , $(3\sin\theta + 4)^2 - (3\cos\theta - 3)^2 = 4$ $(9\sin^2\theta + 24\sin\theta + 16) - (9\cos^2\theta - 18\cos\theta + 9) = 4$ $9(\sin^2\theta - \cos^2\theta) + 24\sin\theta + 18\cos\theta + 3 = 0$ $3(\sin^2\theta - \cos^2\theta) + 8\sin\theta + 6\cos\theta + 1 = 0$ (shown)	
12(iii)	Solving using G.C: $\theta = 2.5083 \Rightarrow (5.78, -5.42)$ $\theta = 5.6014 \Rightarrow (2.11, -0.671)$	

C Graph Transformations

Qn	Solutions	Comments
13(a)	$\frac{x^2}{6^2}+\frac{(y+3)^2}{2^2}=1$ $\Rightarrow x^2+\frac{(y+3)^2}{\left(\frac{2}{6}\right)^2}=6^2$ $\Rightarrow x^2+\left(\frac{y+3}{\frac{1}{3}}\right)^2=6^2$ Scale parallel to the y-axis by a factor of 3. Translate in the positive y-direction by 9 units. OR Translate in the positive y-direction by 3 units. Scale parallel to the y-axis by a factor of 3.	
13(b) (i)		Perform translation by 2 units in negative x direction, then reflection about y axis.

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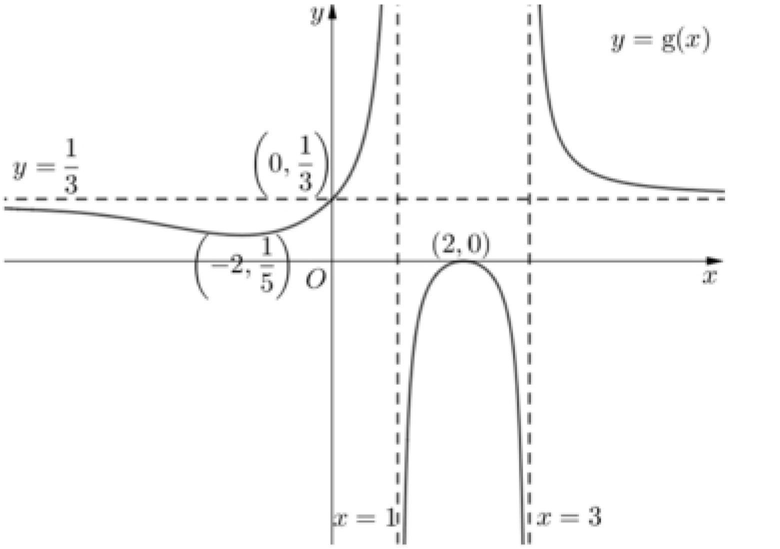
(b)(ii)</td>
<td>
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<td>Do reciprocal to given graph.</td>
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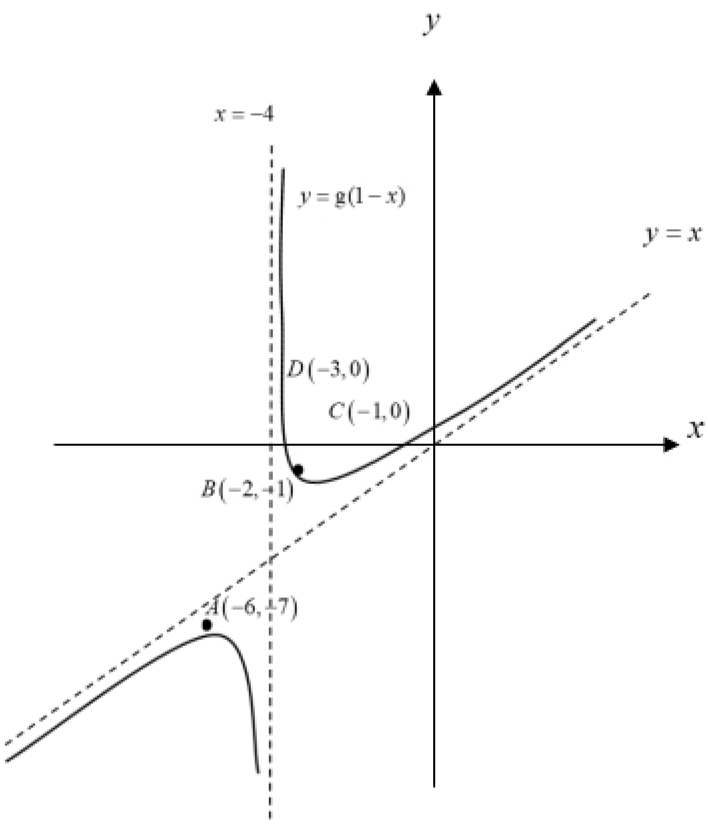
<td>14(i)</td>
<td>
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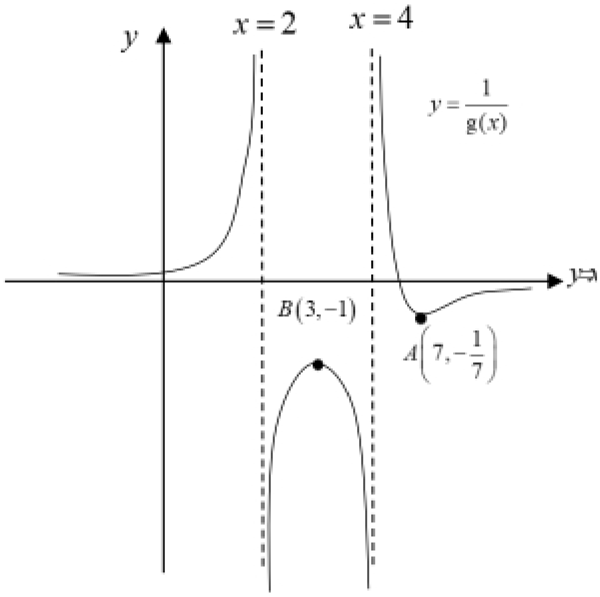
<tr><td>14(ii)</td><td>
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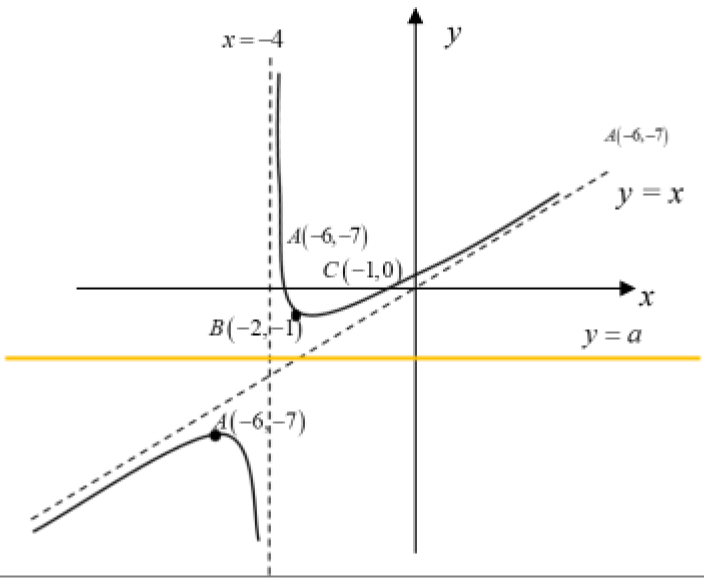
<tr><td>14</td><td>The inequality $g(1-x) > a$, where a is a constant, has the solution set $\{x \in \mathbb{R} : x > -4\}$. Therefore, $\{a \in \mathbb{R} : -7 \le a < -1\}$.

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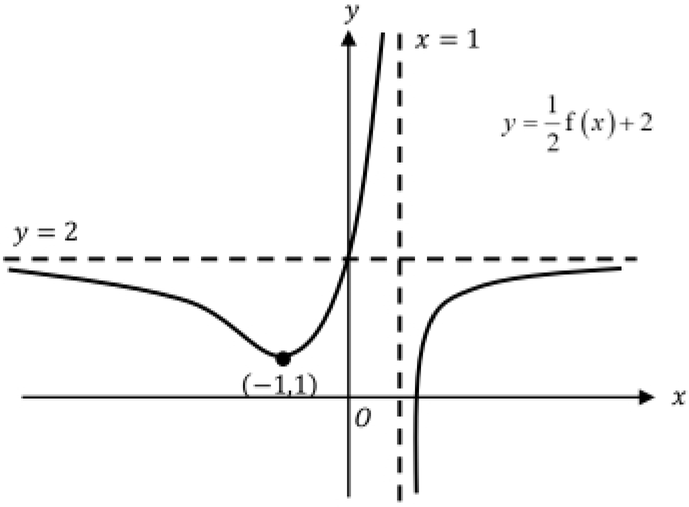
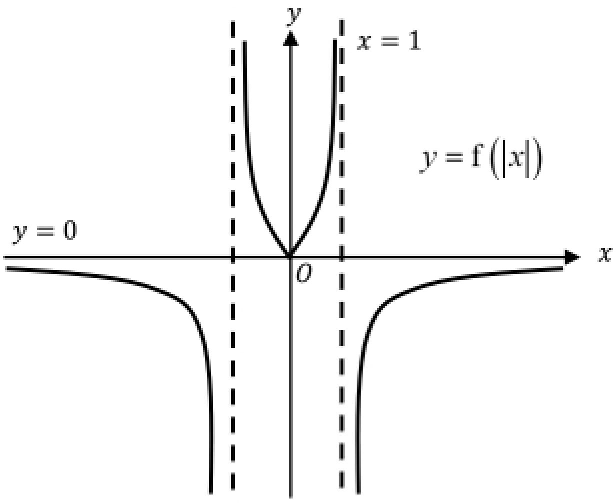
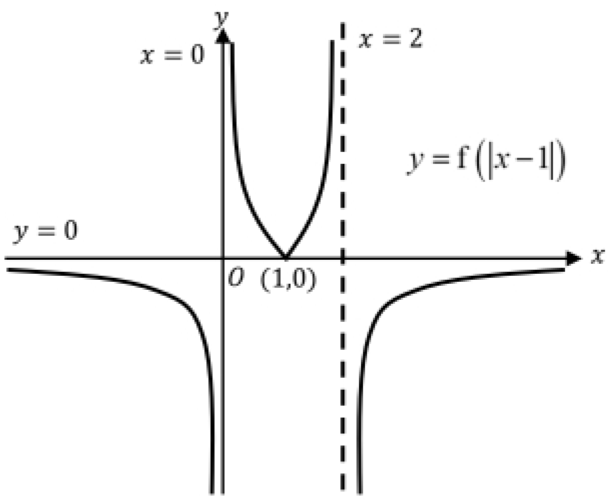
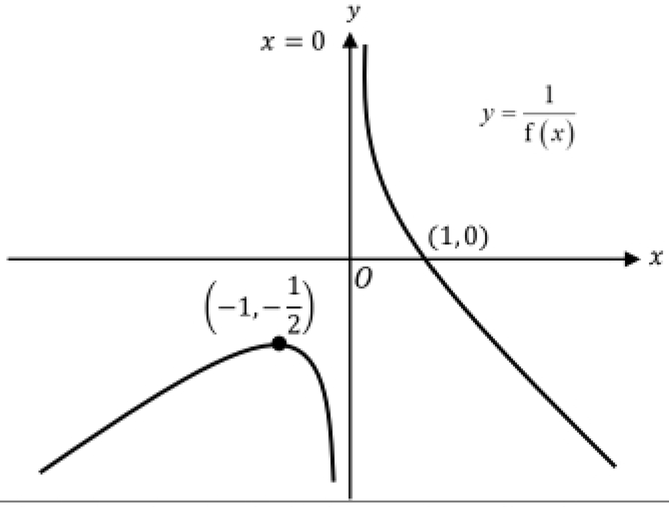
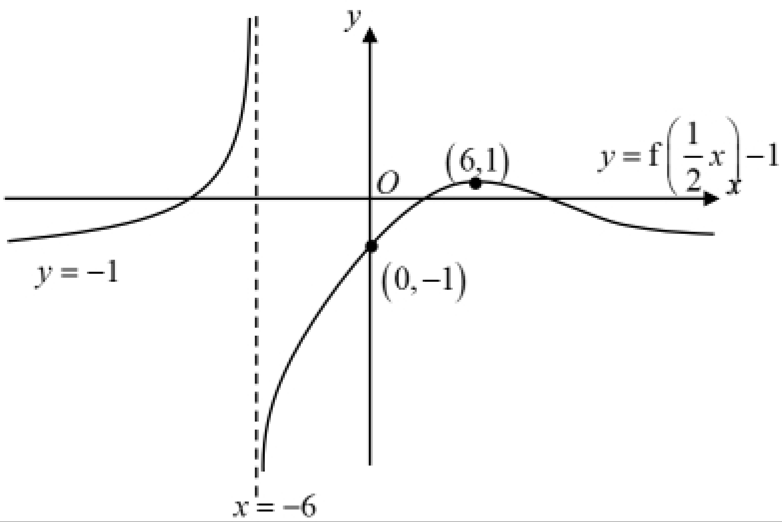
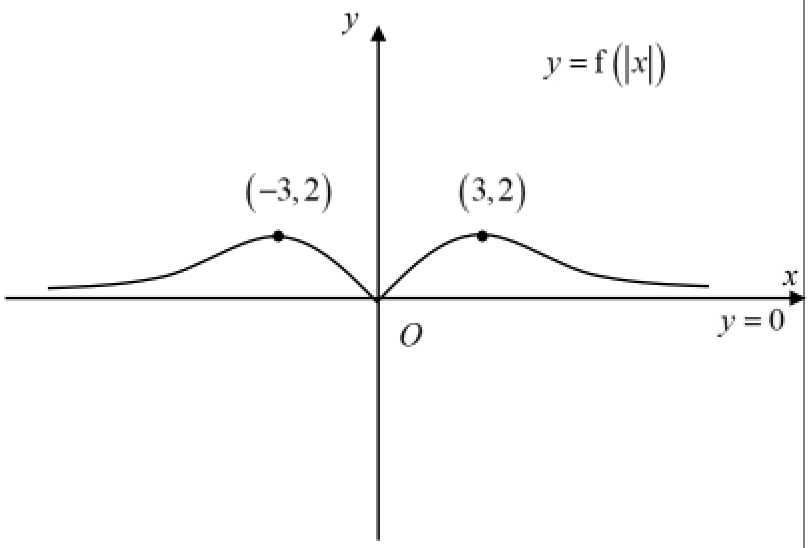
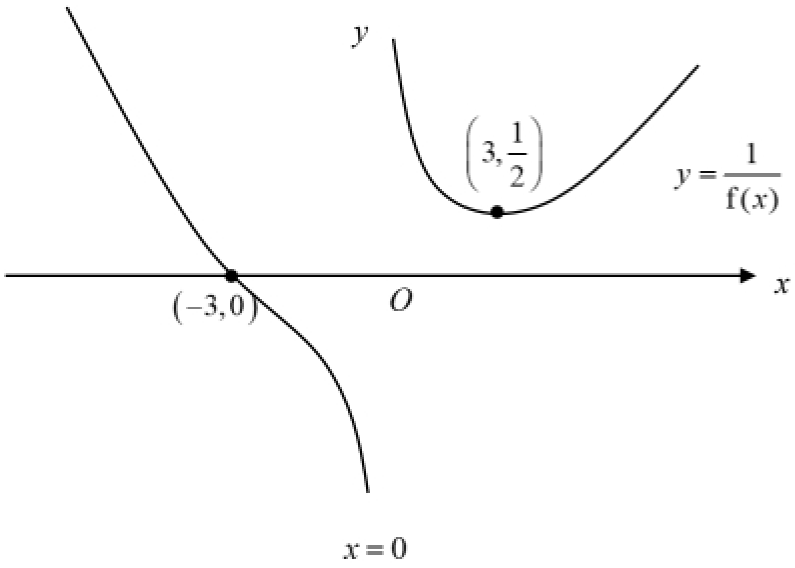
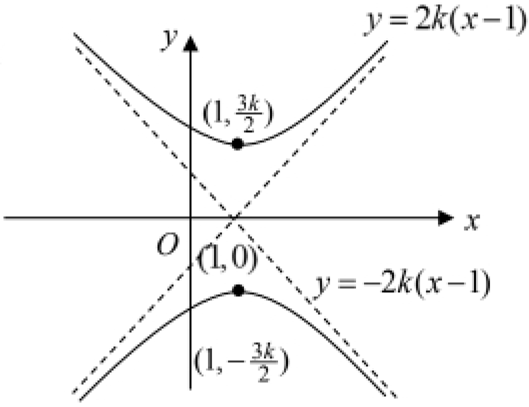
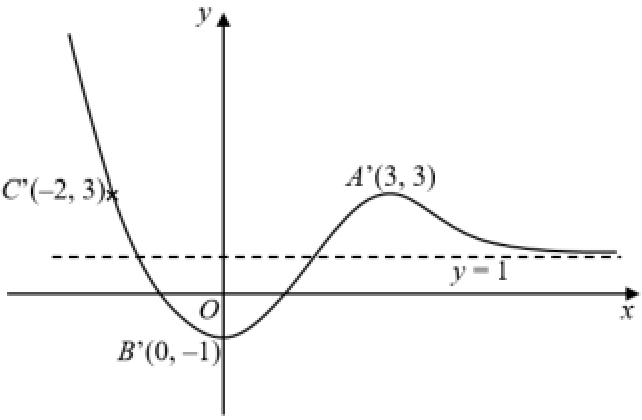
15(a) (i)		Perform the transformations : $y = \text{f}(x) \to \frac{y}{\frac{1}{2}} = \text{f}(x)$ $\to y - 2 = \frac{1}{2}\text{f}(x)$
15(a) (ii)		Perform the transformations : $y = \text{f}(x) \to y = \text{f}(\lvert x \rvert)$ $\to y = \text{f}(\lvert x-1 \rvert)$

15(a) (iii)	 	
15(b)	Undo C: Translate 2 units in the negative x-direction. i.e. replace x with $x+2$ $y=\frac{(x+2)^2-2}{(x+2)+1}=\frac{x^2+4x+2}{x+3}$ Undo B: Scale parallel to the x-axis by a scale factor of $\frac{1}{3}$. i.e. replace x with $3x$ $y=\frac{(3x)^2+4(3x)+2}{3x+3}=\frac{9x^2+12x+2}{3x+3}$ Undo A: Translated 4 units in the positive y-direction. i.e. replace y with $y-4$ $y-4=\frac{9x^2+12x+2}{3x+3}$ $y=\frac{9x^2+12x+2}{3x+3}+4$ $y=\frac{9x^2+12x+2+12x+12}{3x+3}$ $y=\frac{9x^2+24x+14}{3x+3}$	
16(i)	 	

16(ii)		
16(iii)		
17(i)	$C_1:\ 4y^2 = 9 + 16x^2$ After translation by 1 unit in the positive x-direction, $4y^2 = 9 + 16(x-1)^2$ After scaling parallel to the y-axis by factor k, $4\left(\frac{y}{k}\right)^2 = 9 + 16(x-1)^2$	
17(ii)		

	$\frac{4}{9}\left(\frac{y}{k}\right)^2-\frac{16}{9}(x-1)^2=1$ $\frac{y^2}{\left(\frac{3k}{2}\right)^2}-\frac{(x-1)^2}{\left(\frac{3}{4}\right)^2}=1$  Oblique asymptote: $y=\pm\frac{3k/2}{3/4}(x-1)=\pm 2k(x-1)$	
18.(i)	$y=\mathrm{f}(-x)+3$  	

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<td>18.(ii)</td>
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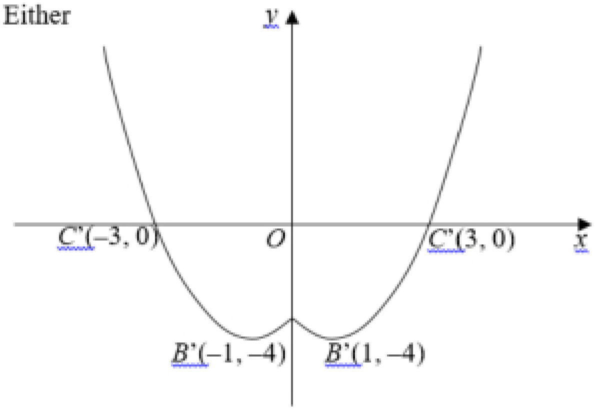
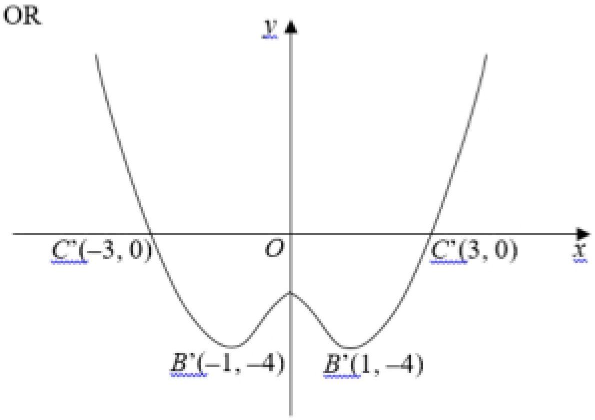
$y=\mathrm{f}\left(|x|-1\right)$

Either



OR



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Perform the transformations :

$y=\mathrm{f}(x)\rightarrow y=\mathrm{f}(x-1)$

$\rightarrow y=\mathrm{f}\left(|x|-1\right)$

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<td>18. (iii)</td>
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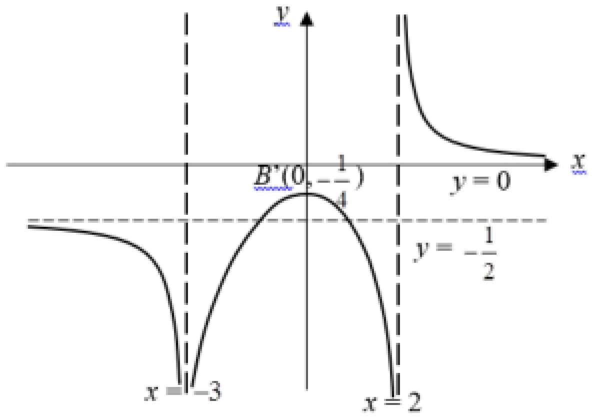
$y=\dfrac{1}{\mathrm{f}(x)}$



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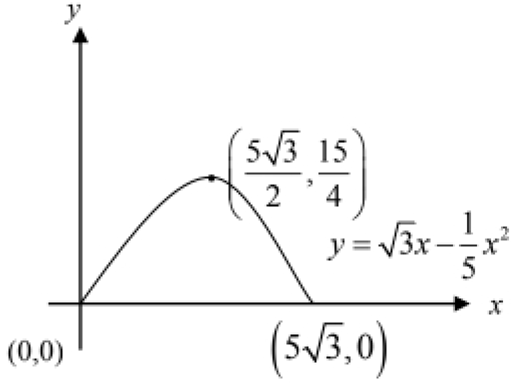
D Real Life Applications

Qn	Solutions	Comments
19	The y-intercept is $(0,\ 1.5351-0.2553)=(0,1.2798)$. $\therefore\ b=1.2798$. The asymptote is $y=\frac{b}{a}x$. $\frac{b}{a}=1.5096$ $\frac{1.2798}{a}=1.5096$ $a=0.848\ (3\text{ s.f})$	
20	Let $FG=h_1$ and let $BC=h_2$. Form equation of the ellipse: $\frac{x^2}{a^2}+\frac{y^2}{2000^2}=1$ --- (1) Since the areas of $ABCD$ and $EFGH$ are equal: $1000h_2=1435h_1$ --------- (2) Substitute the point $G\left(\frac{1435}{2},h_1\right)$ into (1): $\frac{717.5^2}{a^2}+\frac{h_1^{\ 2}}{2000^2}=1$ --------- (3) Substitute the point $C\left(\frac{1000}{2},h_2\right)$ into (1): $\frac{500^2}{a^2}+\frac{h_2^{\ 2}}{2000^2}=1$ --------- (4) Substitute (2) into (4): $\frac{500^2}{a^2}+\frac{(1.435h_1)^2}{2000^2}=1$ --- (5) From (3) and (5): $a^2=764806.25\Rightarrow a=874.532\Rightarrow$ $MN=2a=1749.06\approx 1749$ mm	
21(a)(i)	When Toy Rocket A hits the ground, $y=0$. $(10\sin\alpha)t-5t^2=0$ $t\left[(10\sin\alpha)-5t\right]=0$ $t=0$ or $(10\sin\alpha)-5t=0$ $t=2\sin\alpha$ Time taken is $2\sin\alpha$ s.	
21(a)(ii)	To find the range after $(2\sin\alpha)$s: $x=(10\cos\alpha)2\sin\alpha$ $=20\cos\alpha\sin\alpha$	

	Range is $20\cos\alpha\sin\alpha$ or $10\sin 2\alpha$.	
	Range $r = 20\sin\alpha\cos\alpha = 10\sin 2\alpha$. $\alpha = \frac{\pi}{4}$. **Justification** For r to be maximum, $\sin 2\alpha = 1$ and $2\alpha = \frac{\pi}{2}$ (since $0 < \alpha < \frac{\pi}{2}$). Hence, $\alpha = \frac{\pi}{4}$. **Or** $\frac{dr}{d\alpha} = \frac{d}{d\alpha}10\sin 2\alpha = 20\cos 2\alpha$ $20\cos 2\alpha = 0$ $\Rightarrow \cos 2\alpha = 0$ $\Rightarrow \alpha = \frac{\pi}{4} \quad \left(\text{since } 0 < \alpha < \frac{\pi}{2}\right)$ To test for nature of stationary point: $\frac{d^2r}{d\alpha^2} = \frac{d}{d\alpha}20\cos 2\alpha = -40\sin 2\alpha$ At $\alpha = \frac{\pi}{4}$, $\frac{d^2r}{d\alpha^2} = -40\sin\left[2\left(\frac{\pi}{4}\right)\right] < 0$ $\Rightarrow$ Stationary point is maximum. Hence, Toy Rocket A should be launched at $\frac{\pi}{4}$.	
21(b)(i)	Given $\alpha = \frac{\pi}{3}$, $x = 10(0.5)t \quad y = 10\left(\frac{\sqrt{3}}{2}\right)t - 5t^2$ $\quad = 5t \qquad\qquad = 5\sqrt{3}t - 5t^2$ $t = \frac{x}{5}$ Sub $t = \frac{x}{5}$ into $y = 5\sqrt{3}t - 5t^2$: $y = \frac{5\sqrt{3}}{5}x - 5\left(\frac{x}{5}\right)^2$ $\quad = \sqrt{3}x - \frac{1}{5}x^2$ (Shown)	
21(b)(ii)	Since the path is a parabola, maximum point occurs at $x = \left(0 + 20\cos\frac{\pi}{3}\sin\frac{\pi}{3}\right) \div 2 = 10\left(\frac{1}{2}\right)\frac{\sqrt{3}}{2} = \frac{5\sqrt{3}}{2}$. Maximum value of y	Can also find the turning point of the quadratic function by

	$y=\sqrt{3}\left(\frac{5\sqrt{3}}{2}\right)-\frac{1}{5}\left(\frac{5\sqrt{3}}{2}\right)^2$ $=\frac{15}{2}-\frac{75}{20}=3\frac{3}{4}$  	completing the square.
21(c)	The graph traced by Toy Rocket A is scaled by a scale factor of 5 parallel to the y-axis and then scaled by a scale factor of $\frac{1}{2}$ parallel to the x-axis. (Or vice versa)	
22(a)	(i) Scaling parallel to y-axis with scale factor 1/5; Scaling parallel to x-axis with scale factor 1/5	
	(ii) Equation of stencil circle: $(x-h)^2+(y-k)^2=r^2$ <u>Method 1</u> Replace x by $5x$ & replace y by $5y$: $\left(5x-h\right)^2+\left(5y-k\right)^2=r^2$ $\left(x-\frac{h}{5}\right)^2+\left(y-\frac{k}{5}\right)^2=\left(\frac{r}{5}\right)^2$ <u>Method 2</u> The centre is transformed to the point $\left(\frac{h}{5},\frac{k}{5}\right)$ The point $(h,k+r)$ is transformed to $\left(\frac{h}{5},\frac{k+r}{5}\right)$ Sso the new radius is $\frac{r}{5}$ So engraved shape is a circle with equation $\left(x-\frac{h}{5}\right)^2+\left(y-\frac{k}{5}\right)^2=\left(\frac{r}{5}\right)^2$	
	(iii) $A=\pi\left(\frac{r}{5}\right)^2=\frac{\pi r^2}{25}$	