TEMASEK JUNIOR COLLEGE, SINGAPORE

Preliminary Examination 2015
Higher 2

MATHEMATICS 9740/02

Paper 2

16 September 2015

Additional Materials: Answer paper
List of Formulae (MF15)

3 hours

READ THESE INSTRUCTIONS FIRST

Write your Civics group and name on all the work that you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use a graphic calculator.

Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
At the end of the examination, fasten all your work securely together.

This document consists of 6 printed pages.

Section A: Pure Mathematics [40 marks]

1 Sketch, on an Argand diagram, the locus of the point representing the complex number z such that $\arg(z-\sqrt{3}+\mathrm{i})=\frac{5\pi}{6}$. [2]

Give a geometrical description of the locus of the point representing the complex number w such that $|w+\mathrm{i}|=k$, where k is real. [1]

(i) Given that the two loci intersect at exactly one point, show that $k = a$ or $k \geq b$ where a and b are real constants to be determined. [3]

(ii) In the case when k takes the value of a, find the complex number representing the point of intersection, in the form $x + \mathrm{i}y$, where x and y are exact. [3]

2 **(a)** Given that the sequence $5, 11, 17, \cdots, x$ is arithmetic, solve the equation

$$5+11+17+\cdots+x=2760.$$ [4]

(b) Mr Tan set aside \$80,000 for his two sons. On the first day of the year that his sons turned 7 and 17 years old, he deposited \$$x$ into the younger son's bank account and the remaining sum of money into the elder son's bank account. Mr Tan adds a further \$1000 into the younger son's account on the first day of each subsequent year. The bank pays a compound interest at a rate of 2% per annum on the last day of each year. Each son will withdraw the full sum of money from his account (after interest had been added) on the last day of the year that he turns 21 years old.

(i) Find the amount of money the elder son will withdraw in terms of x. [1]

(ii) Show that the younger son will withdraw $\$\left(1.02^{15}x+51000\left(1.02^{14}-1\right)\right)$. [3]

Find the value of x if Mr Tan wanted both sons to receive the same withdrawal amount, giving your answer to the nearest integer. [2]

3 **(a)** By considering a standard series expansion, find the general solution of the differential equation $x = 1 + \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right) + \frac{1}{2!}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2 + \frac{1}{3!}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^3 + \cdots + \frac{1}{r!}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^r + \cdots$. [4]

(b) An empty rectangular tank has vertical sides of depth H metres and a horizontal base of unit area. Water is pumped into the tank at a constant rate such that if no water flows out, the tank can be filled up in time T seconds. Water flows out at a rate which is proportional to the depth of water in the tank. At time t seconds, the depth of water in the tank is x metres.

When the depth of water is 1 metre, it remains at this constant value. Show that

$$\frac{\mathrm{d}x}{\mathrm{d}t} = k(1-x), \text{ where } k \text{ is a constant in terms of } H \text{ and } T. \quad [2]$$

Find x in terms of t, H and T. [4]

4 The curve C has equation $y = \frac{a + bx^2}{b + ax^2}$ where $x \in \mathbb{R}$ and, a and b are constants such that $0 < a < b$.

(i) **(a)** Using an algebraic method, find the range of values of y in terms of a and b. [3]

(b) Find the equation of the asymptote and the coordinates of the stationary point of C. [4]

(c) Sketch C, indicating the intercept(s) on the axes. [1]

(ii) Given that $b = 2a$, find $\int \frac{a + bx^2}{b + ax^2}\,\mathrm{d}x$. [3]

Section B: Statistics [60 marks]

5 A popular online retail store sells and provides home delivery of clothes and electrical appliances. The store manager wants to find out from the store customers their opinions on the quality of the products they have purchased.

(i) Describe how systematic sampling can be carried out with 5% of the customers. [2]

(ii) Explain briefly why the sample obtained in part **(i)** may not be a representative sample. [1]

[Turn Over

6 Find the number of ways to arrange the twelve letters of the word PERSEVERANCE

(i) in a row, [1]

(ii) in a circular way such that the vowels are placed adjacent to one another. [2]

Four letters are selected at random from the twelve letters of the word PERSEVERANCE to form a code word. The code word can contain at most two identical letters. Find the number of possible code words. [3]

7 The manufacturer of Mola snack biscuits claims that the mean biscuit content in a packet is 15 g. A random sample of 80 packets is taken and the weight, x g, of the content in each packet is measured. The results are summarised by

$$\sum(x-15)=53.6 \text{ and } \sum(x-15)^2=795.7.$$

(i) Calculate unbiased estimates of the population mean and variance. [2]

(ii) Test whether the mean weight differs from 15 g at 4% level of significance. [4]

(iii) Another large random sample of n packets is taken. Assume now that the population standard deviation is 3 g. Find the range of values of the sample mean, in terms of n, for which the claim that the mean weight differs from 15 g is supported at 4% level of significance. [3]

8 In a school, there are 28 and 18 teachers in the Science and Mathematics Departments respectively. Of the 28 Science teachers, there are 10 single men, 4 single women, 2 married couples, 8 married men and 2 married women. Of the 18 Mathematics teachers, there are 3 single men, 3 single women, 1 married couple, 5 married men and 5 married women. No Science teacher is married to a Mathematics teacher. Two teachers are chosen at random from the two departments to attend a focus group discussion.

Find the probability that

(i) they are married to each other, [2]

(ii) they are a man and a woman in the same department, [2]

(iii) they are married to each other given that a man and woman are chosen, [2]

(iv) they are from different departments given that a married man and a single woman are chosen. [3]

9 When a large number of mangoes are harvested, 37% of them are unripe. The mangoes are packed randomly in boxes of six.

(i) Show that the probability that there is no unripe mango in a box is 0.062524, correct to 5 significant figures. [1]

(ii) Find the probability that there are exactly 2 unripe mangoes in 2 boxes. [2]

A supermarket orders 50 boxes of mangoes daily.

(iii) By using a suitable approximation, find the probability that there are at least 9 boxes with no unripe mango in a particular day. [4]

(iv) When the supermarket places a daily order for 10 weeks, estimate the probability that the mean number of boxes with no unripe mango in a day is between 3 and 9. [3]

10 The table gives the research and development (R&D) expenditures, x, and earnings, y, in suitable units for 11 pharmaceutical companies in a particular year.

R&D expenditure, x	8.5	12	6.5	4.5	2	0.5	1.5	6	9	7.5	2.5
Earnings, y	83	147	69	50	43	35	40	64	97	53	45

(i) Draw the scatter diagram for these values, labelling the axes clearly. [2]

(ii) Give a possible reason why one of the data points does not seem to follow the trend. [1]

(iii) Using your answer in part **(i)**, explain whether $y = a + bx$ or $y = c + dx^2$ is a better model for the data. [1]

(iv) Explain how you would verify your choice of model in part **(iii)** by calculating the product moment correlation coefficients. [2]

Hence estimate the earnings of a company with a R&D expenditure of 10 units, showing your calculations clearly. [3]

(v) For the model $y = c + dx^2$, give an interpretation, in context, of the value of d. [1]

[Turn Over

11 A power station has one generator. It has been observed that the generator has on average 3 breakdowns in 2 years. Assume that the number of breakdowns can be modelled using a Poisson distribution.

(i) Given that the probability that no breakdown occurs in a period of n months is 0.8, find n, correct to 1 decimal place. [3]

(ii) Using a suitable approximation, find the probability that the number of breakdowns in 10 years is less than 10. [4]

For each breakdown, the time taken for the repair work is normally distributed with mean 2 hours and standard deviation 0.7 hour.

(iii) Find the least integer m such that the probability that the total time taken to repair 3 randomly chosen breakdowns is less than m hours is at least 0.85. [3]

The government imposes a fine of \$10,000 for the first breakdown of the generator. The fine is increased to \$20,000 for each subsequent breakdown within a year of the first breakdown. The power station faces an additional fine of \$50,000 if the generator is not repaired within 3 hours of a breakdown.

(iv) Find the probability that the power station pays less than \$50,000 in fines in a particular year. [3]

END OF PAPER