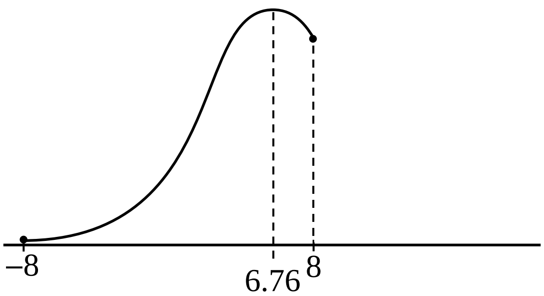
Question 6

No.	Solution
(i)	Let μ and σ denote the mean and standard deviation of X. $\text{P}(X<0)=0.06 \Rightarrow \text{P}\left(Z<\frac{0-\mu}{\sigma}\right)=0.06$ where $Z\sim N(0,1)$ $\Rightarrow \frac{0-\mu}{\sigma}=-1.55477$ $\Rightarrow \mu-1.55477\sigma=0$ --- (1) $\text{P}(X\ge 15)=0.029 \Rightarrow \text{P}\left(Z\ge\frac{15-\mu}{\sigma}\right)=0.029$ where $Z\sim N(0,1)$ $\Rightarrow \frac{15-\mu}{\sigma}=1.89570$ $\Rightarrow \mu+1.89570\sigma=15$ --- (2) Solving (1) and (2), $\mu=6.76$ and $\sigma=4.35$ (3 sf)
(ii)	

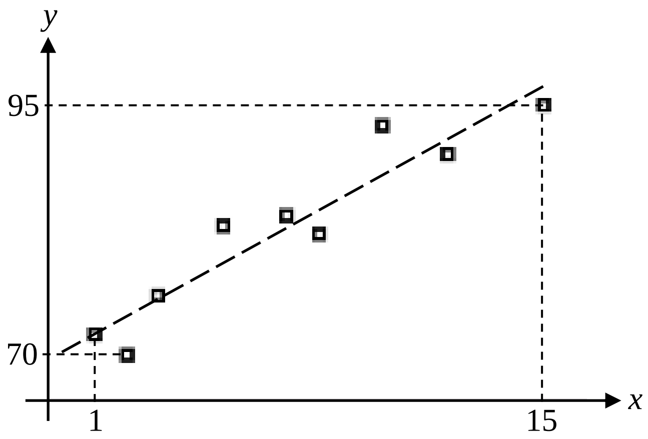
Question 7

No.	Solution
(i)	$\mathrm{P}(A \mid B) = \frac{1}{6} \Rightarrow \frac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)} = \frac{1}{6} \Rightarrow \mathrm{P}(B) = 6k$ $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$ $\Rightarrow \frac{9}{20} = \frac{1}{5} + 6k - k$ $\therefore k = \frac{1}{20}$
(ii)	$\mathrm{P}(A' \cap B) = \mathrm{P}(A \cup B) - \mathrm{P}(A) = \frac{9}{20} - \frac{1}{5} = \frac{1}{4}$
(iii)	$\mathrm{P}(A') \times \mathrm{P}(B) = \frac{4}{5} \times 6k = \frac{6}{25}$ Since $\mathrm{P}(A' \cap B) \neq \mathrm{P}(A') \times \mathrm{P}(B)$, the events A' and B are not independent.

Question 8

No.	Solution
(i)	Let X denote the number of defective lamps in a sample of 30 lamps $X \sim \mathrm{B}(30, 0.05)$ The expected number of defective lamps is $\mathrm{E}(X) = 30 \times 0.05 = 1.5$
(ii)	$\mathrm{P}(X \geq 1) = 1 - \mathrm{P}(X = 0) = 0.785$ (3 sf)
(iii)	Required probability $= \mathrm{P}(X \leq 2 \mid X \geq 1)$ $= \frac{\mathrm{P}(1 \leq X \leq 2)}{\mathrm{P}(X \geq 1)}$ $= \frac{\mathrm{P}(X = 1) + \mathrm{P}(X = 2)}{\mathrm{P}(X \geq 1)}$ $= \frac{0.59754}{0.78536} = 0.761$ (3 sf)

Question 9

No.	Solution
(i)	
(ii)	From GC, $r = 0.945$ (3 sf) r is close to 1. There is a strong positive linear correlation between the number of years, x, of experience of the prosecutors and the percentage, y, of their cases that ended in guilty pleas. As the number of years of experience of the prosecutors increases, the percentage of their cases that ended in guilty pleas increases.
(iii)	From GC, $y = 1.77x + 70.4$ (3 sf) The percentage of a prosecutor's cases that ended in guilty pleas is expected to increase by 1.77 % when the number of years of experience of the prosecutor increases by 1.
(iv)	From GC, when $x = 20$, $y = 106$ (3 sf) An estimate for the percentage of cases that ended in guilty pleas for a prosecutor with 20 years of experience is 106 %. The estimate is not reliable because (1) $x = 20$ is outside the data range of $1 \le x \le 15$ and (2) the estimated percentage of 106 does not make sense since it exceeds 100 %.

Question 10

No.	Solution
(i)	Let S (in grams) denote the mass of fries in a small packet. $S \sim \mathrm{N}(80, 2^2)$ $\mathrm{P}(S < 77) = 0.066807$ Required probability $= [\mathrm{P}(S < 77)]^2 = 0.00446$ (3 sf)
(ii)	Let L (in grams) denote the mass of fries in a large packet. $L \sim \mathrm{N}(150, 3^2)$ Let $X = L_1 + L_2 \sim \mathrm{N}(150 \times 2, 3^2 \times 2)$, i.e. $\mathrm{N}(300, 18)$ Let $Y = S_1 + \cdots + S_4 \sim \mathrm{N}(80 \times 4, 2^2 \times 4)$, i.e. $\mathrm{N}(320, 16)$ $Y - X \sim \mathrm{N}(320 - 300, 18 + 16)$, i.e. $\mathrm{N}(20, 34)$ $\mathrm{P}(Y - X \geq 15) = 0.80441 = 0.804$ (3 sf)
(iii)	$1.72S \sim \mathrm{N}(80 \times 1.72, 2^2 \times 1.72^2)$, i.e. $\mathrm{N}(137.6, 11.8336)$ $0.86L \sim \mathrm{N}(150 \times 0.86, 3^2 \times 0.86^2)$, i.e. $\mathrm{N}(129, 6.6564)$ $1.72S - 0.86L \sim \mathrm{N}(137.6 - 129, 11.8336 + 6.6564)$, i.e. $\mathrm{N}(8.6, 18.49)$ $\mathrm{P}(1.72S - 0.86L > k) > 0.7$ From GC, $k < 6.35$ $\therefore$ The largest integer value of k is 6. Alternatively, when $k = 5$, probability $= 0.79876 > 0.7$, when $k = 6$, probability $= 0.72729 > 0.7$, when $k = 7$, probability $= 0.64509 < 0.7$. $\therefore$ The largest integer value of k is 6.

Question 11

No.	Solution
(i)	Number of ways = ${}^{5}P_{4} \times 8! = 4838400$
(ii)	Required probability = $\dfrac{{}^{4}P_{3} \times 9!}{12!} = \dfrac{1}{55}$
(iii)	Required probability = $\dfrac{3! \times 2 \times 9!}{12!} = \dfrac{1}{110}$
(iv)	Number of ways for the team to consist of 2 pupils who play sports $= {}^{3}C_{2} \times {}^{9}C_{3} = 252$ Number of ways for the team to consist of 3 pupils who play sports $= {}^{3}C_{3} \times {}^{9}C_{2} = 36$ $\therefore$ Required probability = $\dfrac{252 + 36}{{}^{12}C_{5}} = \dfrac{4}{11}$
(v)	Let A denote the event that the team consists of at least 2 pupils who play sports, and let B denote the event that the team consists of all boys. $\mathrm{P}(A) = \dfrac{4}{11}$ $\mathrm{P}(B) = \dfrac{{}^{7}C_{5}}{{}^{12}C_{5}} = \dfrac{7}{264}$ $\mathrm{P}(A \cap B) = \dfrac{{}^{5}C_{3} \times {}^{2}C_{2}}{{}^{12}C_{5}} = \dfrac{5}{396}$ $\therefore$ Required probability $= \mathrm{P}(A \cup B)$ $= \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$ $= \dfrac{299}{792}$

Question 12

No.	Solution
(i)	Unbiased estimate of the population mean, $\bar{x} = \frac{3290}{60} = \frac{329}{6}$ or 54.8 (3 sf) Unbiased estimate of the population variance, $s^2 = \frac{1}{59}\left(201100 - \frac{(3290)^2}{60}\right) = 350.82 = 351$ (3 sf)
(ii)	A 1-tail test should be carried out as the investigation is to find out if the minimum recommended time is met, and thus we are only concerned if the mean time spent is less than 60 minutes.
(iii)	Let μ denote the population mean time spent. Test $H_0 : \mu = 60$ against $H_1 : \mu < 60$ at the 5 % significance level Under H_0, since n is large, by Central Limit Theorem, $\bar{X} \sim N\left(60, \frac{350.82}{60}\right)$ approximately Using a 1-tail test, $\bar{x} = \frac{3290}{60}$ gives p-value = 0.0163 (3 sf) Since p-value = 0.0163 < 0.05, we reject H_0 and conclude that there is sufficient evidence at the 5 % significance level that the minimum recommended time of 60 minutes is not met.
(iv)	Test $H_0 : \mu = 60$ against $H_1 : \mu \neq 60$ at the 5 % significance level Under H_0, since n is large, by Central Limit Theorem, $\bar{X} \sim N\left(60, \frac{15^2}{60}\right)$ approximately $Z = \frac{\bar{X} - 60}{\sqrt{\frac{15^2}{60}}} \sim N(0,1)$

	$z_{cal} = \dfrac{k-60}{\sqrt{\dfrac{15^2}{60}}}$ Using a two tailed test, reject H_0 when $z_{cal} \le -1.96$ or $z_{cal} \ge -1.96$ Since H_0 is rejected, $z_{cal} \le -1.96$ or $z_{cal} \ge -1.96$ $\dfrac{k-60}{\sqrt{\dfrac{15^2}{60}}} \le -1.96 \quad or \quad \dfrac{k-60}{\sqrt{\dfrac{15^2}{60}}} \ge -1.96$ Hence $k \le 56.2$ or $k \ge 63.8$ (3 sf)