STATISTICS AND PROBABILITY

4052 | ELEMENTARY MATHEMATICS

S2: PROBABILITY

CONTENT

- probability as a measure of chance
- probability of single events (including listing all the possible outcomes in a simple chance situation to calculate the probability)
- probability of simple combined events (including using possibility diagrams and tree diagrams, where appropriate)
- addition and multiplication of probabilities (mutually exclusive events and independent events)

PROBABILITY AS A MEASURE OF CHANCE

- Probability is a measure of how likely an event is to occur. The probability of an event is a number between 0 and 1, where:
  - **0** means the event will not occur.
  - **1** means the event will definitely occur.

Formula for the probability of single event A:

$$P(A) = \frac{Number\ of\ favourable\ outcomes}{Total\ number\ of\ favourable\ outcomes.}$$

EXAMPLE: SIX-SIDED DIE

If you roll a six-sided die, the probability of rolling a 3 is $P(3) = \frac{1}{6}$. One favourable outcome and six possible outcomes.

PROBABILITY OF SINGLE EVENTS

LISTING ALL THE POSSIBLE OUTCOMES (SIMPLE CHANCE SITUATIONS)

EXAMPLE: TOSSING A COIN

Possible outcomes = {Heads, Tails}

Total outcomes = 2

Probability of Heads = $\frac{1}{2}$

Probability of Tails = $\frac{1}{2}$

EXAMPLE: ROLLING A DIE AND GETTING AN EVEN NUMBER

Possible outcomes = {1, 2, 3, 4, 5, 6}

Total outcomes = 6

Probability of rolling an even number = $\frac{3}{6} = \frac{1}{2}$

EXAMPLE: SELECTING A DAY OF THE WEEK THAT FALLS ON A WEEKEND

Possible outcomes = {Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday}

Total outcomes = 7

Probability of selecting a weekend day = $\frac{2}{7}$

PROBABILITY OF SIMPLE COMBINED EVENTS

POSSIBILITY DIAGRAMS

- **Possibility Diagrams** are useful when you want to show all the possible outcomes of a situation, particularly when events are sequential.

EXAMPLE: TOSSING A COIN AND ROLLING A DIE

Two fair six-sided dice are rolled.
Find the probability that the sum of the numbers showing on the two dice is an odd number greater than 5, giving your answer as a fraction in simplest form.

2nd dice

1st dice	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

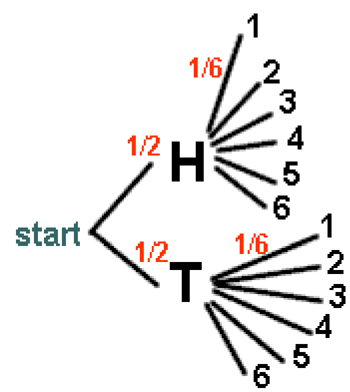
P(odd numbers greater than 5) = $\frac{12}{36} = \frac{1}{3}$

TREE DIAGRAMS

- **Tree Diagrams** are helpful for understanding the probabilities of different sequences of events. Each branch of the tree represents a possible outcome, and the probabilities are multiplied along the branches.

EXAMPLE: TOSSING A COIN AND ROLLING A DIE

1st event: rolling a die, {1, 2, 3, 4, 5, 6}
2nd event: tossing a coin, {Heads, Tails}



P(2 and Heads) = $\frac{1}{6} \times \frac{1}{2} = \frac{1}{12}$

ADDITION AND MULTIPLICATION OF PROBABILITIES

ADDITION RULE (FOR MUTUALLY EXCLUSIVE EVENTS)

- **Mutually exclusive events** are events that **cannot happen at the same time**. If A and B are mutually exclusive, then:

$$P(A\ or\ B) = P(A) + P(B)$$

EXAMPLE

A fair die is rolled. The probability of getting a $2[P(A)]$ is $\frac{1}{6}$, and the probability of getting a $5[P(B)]$ is $\frac{1}{6}$. Since getting a 2 and getting a 5 is mutually exclusive:

$P(A\ or\ B) = \frac{1}{6} + \frac{1}{6} = \frac{2}{6}$

$P(A\ or\ B) = \frac{1}{3}$

MULTIPLICATION RULE (INDEPENDENT EVENTS)

- **Independent events** are events where the occurrence of one event does **not affect the probability of the other**. If A and B are independent, then:

$$P(A\ and\ B) = P(A) \times P(B)$$

EXAMPLE

You flip a fair coin and roll a fair die. The probability of getting heads $[P(A)]$ is $\frac{1}{2}$, and the probability of rolling a $6[P(B)]$ is $\frac{1}{6}$.

$P(A\ and\ B) = \frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$

GENERAL ADDITION RULE (FOR NON-MUTUALLY EXCLUSIVE EVENTS)

- If two events **can** happen at the same time, we use:

$$P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)$$

EXAMPLE

A card is drawn from a deck. Let A be the event of drawing a heart $\left[P(A) = \frac{13}{52}\right]$ and B be the event of drawing a face card $\left[P(B) = \frac{12}{52}\right]$. Since 3 of these face cards are also hearts,

$P(A\ and\ B) = \frac{3}{52}$

$P(A\ or\ B) = \frac{13}{52} + \frac{12}{52} - \frac{3}{52} = \frac{22}{52}$

$P(A\ or\ B) = \frac{11}{26}$