2 Kinematics

Main concept(s)

1. Rectilinear Motion
2. Non-Linear Motion (including Projectile Motion)

Learning Outcome(s)

Candidates should be able to:

(a) show an understanding of and use the terms distance, displacement, speed, velocity and acceleration.

(b) use graphical methods to represent distance, displacement, speed, velocity and acceleration.

(c) identify and use the physical quantities from the gradients of displacement-time graphs and areas and gradients of velocity-time graphs, including cases of non-uniform acceleration.

(d) derive, from the definitions of velocity and acceleration, equations which represent uniformly accelerated motion in a straight line.

(e) solve problems using equations which represent uniformly accelerated motion in a straight line, including the motion of bodies falling in a uniform gravitational field without air resistance.

(f) describe qualitatively the motion of bodies falling in a uniform gravitational field with air resistance.

(g) describe and explain motion due to a uniform velocity in one direction and a uniform acceleration in a perpendicular direction.

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Introduction

Today, we use electronic stopwatches, video recorders and other sophisticated instruments to analyse motion, but it has not always been so. Galileo, who in the early 1600s was the first scientist to study motion experimentally, used his pulse to measure time! Galileo made a useful distinction between the *cause* of motion and the *description* of motion. “Kinematics” is the modern term for the mathematical description of motion without regard to causes. The term comes from the Greek word *kinema*, meaning “movement”. This chapter begins with visualising motion and developing the representations for describing the position, displacement, velocity and acceleration of moving objects in space as a function of time. We will begin our study of kinematics with motion in one dimension, followed by two dimensions.

Essential Questions

- How do we describe the motion of objects?
- How can the motion of objects be represented, quantified and predicted?
- How can we tell if an object is moving with a constant acceleration?
- How would an object experiencing constant acceleration in one direction and constant velocity in a perpendicular direction move?
- How would an object falling in a uniform gravitational field move?

2.1 RECTILINEAR MOTION

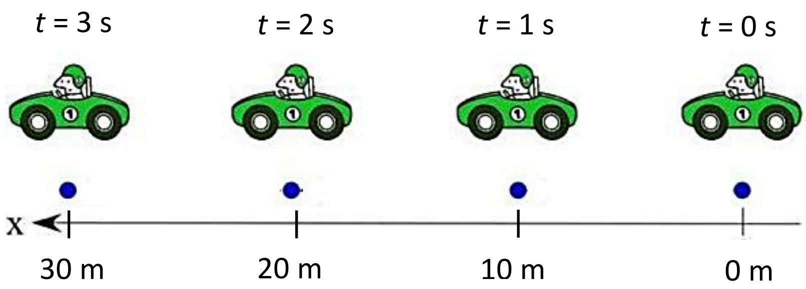
Rectilinear motion means motion in a straight line.

2.1.1 Displacement vs Distance

Displacement is the straight-line distance of an object or a point, in a specified direction, from some reference point. The specified direction is directed from the reference point to that object's position or that point. Displacement is a *vector* quantity.

Example 1

Consider a car moving to the left. The positions of the car at various instances in time *t* are represented as dots in a straight line. This straight line serves as a reference to a coordinate axis, just like the x-axis.



To make the discussion of *displacement* meaningful,

- **A *reference point* must be defined. For example,**
  - **If t = 0 s is taken as the reference point, then at time t = 2 s, the displacement *from* that point is 20 m.**
  - **If t = 1 s is taken as the reference point, then at time t = 2 s, the displacement *from* that point is 10 m.**
- ***Additionally, a positive direction* must be defined.**
  - **If the positive direction is defined to be leftwards and t = 1 s is taken as the reference point, then at time t = 0 s, the displacement *of the car from the reference point* is -10 m.**

The negative sign is a mathematical representation to say that it is in the opposite direction to the positive direction.

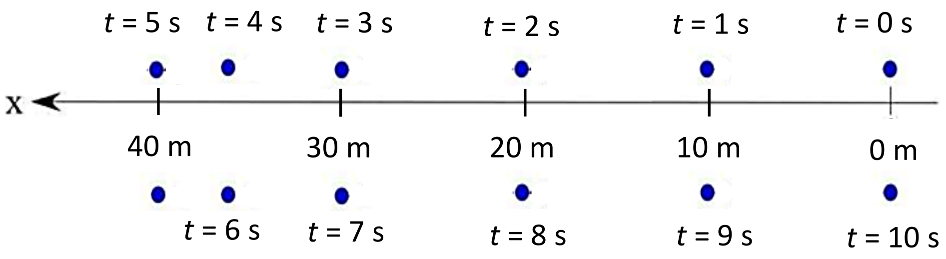
Note that **since displacement is a vector, both magnitude and direction are specified**.

Since distance is a scalar quantity while displacement is a vector quantity, the displacement of an object **may or may not** be equal in magnitude to the distance it travels.

Distance is the *total length of the actual path travelled by a moving object, irrespective of the direction of motion*. It is a *scalar* quantity.

Example 2
Suppose the car in Example 1 reversed direction after travelling 40 m and returned to its initial position at t = 10 s.

Motion diagram:



Distance moved in 10 s from t = 0 s is 80 m. But,
Displacement moved in 10 s from t = 0 s is 0 m.

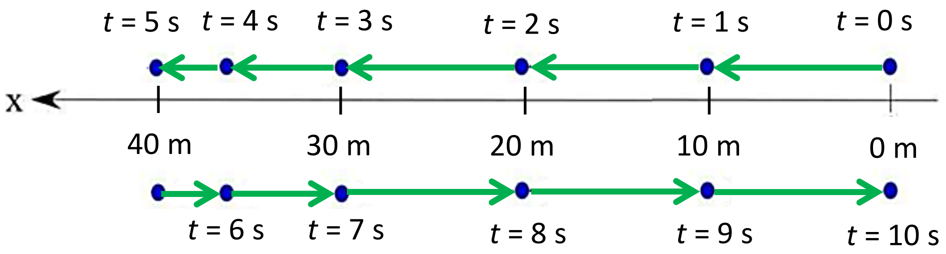
2.1.2 Speed and Velocity

Definition:

Speed is the rate of change of distance with respect to time.

Speed is a *scalar* quantity.
SI unit: metre per second, m s^{-1}

In Example 2,
The spacing between dots becomes smaller from t = 3 s to t = 5 s, indicating that the **change in** displacement **per unit time** decreases, hence speed decreases.
The spacing between dots becomes wider from t = 5 s to t = 7 s, indicating that the **change in** displacement **per unit time** increases, hence speed increases.



The lengths of the arrows in the motion diagram represents the speed. If the spacing is wider for adjacent dots, the higher the speed of the car.

From t = 0 s to t = 3 s, the lengths of the arrows are the same. In addition, they are also the same as the lengths of the arrow between t = 7 s to t = 10 s. Therefore, the speed of the car within these timings is the same. However, travelling to the left is different from travelling to the right, thus, a *vector representation* of how fast the car is moving in a particular direction is required.

Definition:

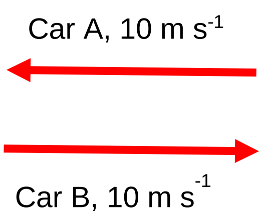
Velocity is the rate of change in displacement with respect to time.

Velocity is a *vector* quantity.
Magnitude of velocity equals speed.

Therefore, if the car in Example 2 is moving at 10 m s^{-1} between t = 0 s to t = 3 s, then, it is travelling at -10 m s^{-1} between t = 7 s to t = 10 s. [Note that positive direction was defined earlier as leftwards.]

Example 3
Car A is moving to the left at 10 m s^{-1} and car B is moving to the right at 10 m s^{-1}.

Car A and car B are said to have the *same speed* but *different velocity* because the direction of their velocities are different.

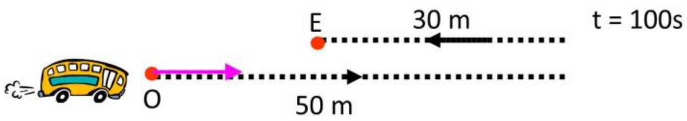


Note:

Velocities of the two cars are represented by arrows. The length of the arrows represent the magnitude. Since the two car are travelling at 10 m s^{-1}, the magnitude of the velocity vectors are the same.

Directions of velocity: car A point towards the left and car B point towards the right.

Example 4
A bus travels 50 m due east and makes a U-turn back to travel a further distance of 30 m. The total time taken is 100 s. Determine its
(i) average speed
(ii) average velocity

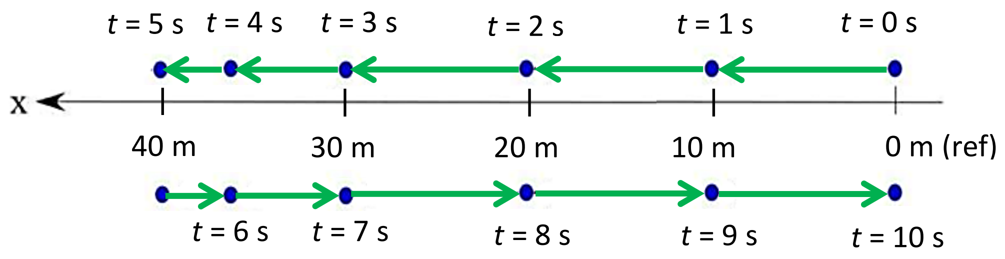


(i) Ave speed $= \dfrac{\text{total change in distance}}{\text{total time}} = \dfrac{80}{100} = 0.8 \text{ m s}^{-1}$

(ii) Ave vel $= \dfrac{\text{change in displacement}}{\text{total time}} = \dfrac{20}{100} = 0.2 \text{ m s}^{-1}$ due east

2.1.3 Motion Diagram with Velocity Vectors

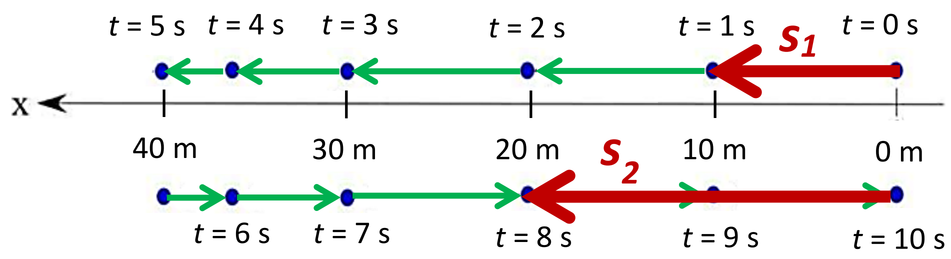
Back to **Example 2,**



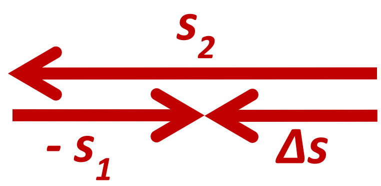
Define reference point to be the initial position, and, direction to the right as positive.

Average speed of the car between t = 1 s and t = 8 s is (50 m / 7 s) = 7.1 m s^{-1}.
Average velocity of the car in the same time interval is determined as follows:

Displacement at 1 s, s_1 = -10 m
Displacement at 8 s, s_2 = -20 m
Change in displacement, $\Delta s = s_2 - s_1$ = (-20) – (-10) = -10 m (i.e. 10 m to the left)
Average velocity = -10 m / 7 s = -1.4 m s^{-1} (i.e. 1.4 m s^{-1} to the left)



Using vector subtraction to determine Δs: $\quad \Delta s = s_2 - s_1$
$\Delta s = s_2 + (-s_1)$



Average velocity of the car between t = 0 s and t = 10 s is 0 m s^{-1} as the change in displacement is 0.
Instantaneous velocity of the car between t = 0 s and t = 10 s is varying in magnitude, and, direction could be to the left or right.

2.1.4 Acceleration

Definition:

Acceleration is the rate of change of velocity with respect to time.

Acceleration is a *vector* quantity.
SI unit: metre per second per second, m s^{-2}

Average acceleration is defined as the *ratio* of the *change in* velocity to the time interval.

$$\textbf{Average acceleration, } \langle a \rangle = \frac{\text{change in velocity}}{\text{time taken}} = \frac{\Delta v}{\Delta t}$$

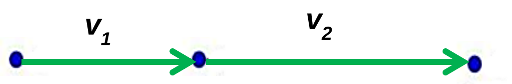
Instantaneous acceleration is the acceleration at a particular instant in time.

$$\textbf{Instantaneous acceleration , } a = \frac{dv}{dt}$$

where: *v* is the velocity
t is the time taken

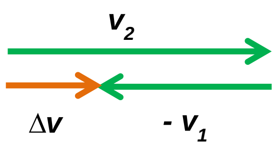
In the motion diagram representation,

Motion diagram with only velocity vectors drawn:



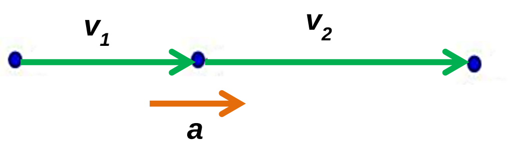
Use vector subtraction to determine Δv:

$$\Delta v = v_2 - v_1$$
$$\Delta v = v_2 + (-v_1)$$



Direction of *Δv* is also the direction of the average acceleration vector *a*.

Return to the original motion diagram. Draw the acceleration vector as an arrow beside the dot and this completes the motion diagram:



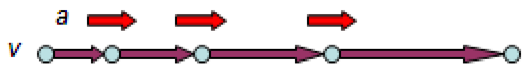
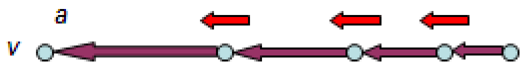
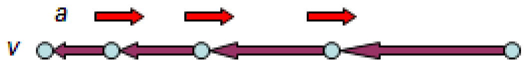
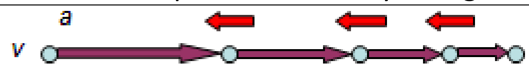
A complete motion diagram consists of:

- Dots, representing the positions of the object at equal time intervals.
- Velocity vectors, joining every two adjacent dots.
- Acceleration vectors, drawn beside the dots.

<u>How to tell when an object is slowing down or speeding up?</u>

➔ Look at the direction (or signs) of BOTH velocity & acceleration.

Taking rightwards as positive (+) and leftwards as negative (-)

Case (a): Speeding up to the right	 Both acceleration and velocity are positive.
Case (b): Speeding up to the left	 Both acceleration and velocity are negative.
Case (c): Slowing down to the left	 Acceleration is positive. Velocity is negative.
Case (d): Slowing down to the right	 Acceleration is negative. Velocity is positive.

Whenever the acceleration and velocity are in the same direction (their signs are the same), the speed of the object <u>increases</u>.

Whenever the acceleration and velocity are in **opposite** directions (their signs are opposite), the speed of an object <u>decreases</u> and it is said to be "decelerating" or "retarding".

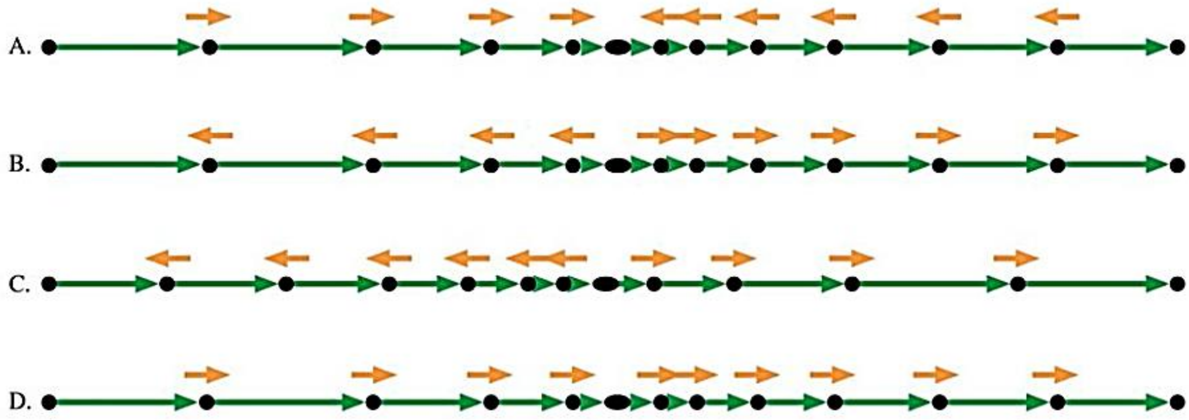
Quiz: Does a "negative acceleration" necessarily mean that the object is decelerating?
(Hint: Observe the cases above)

After the analysis in the table above, the following statements are concluded:

- *Negative acceleration does not necessarily indicate that the object is decelerating.*
- *"deceleration" and "negative acceleration" are different.*
    - ***The word "decelerating" is a textual representation to mean that an object is slowing down.***
    - ***The mathematical <u>sign</u> of acceleration tells you which way the acceleration vector points.*** *Negative acceleration refers to acceleration vector pointing in the negative direction.* ***If the term of direction of velocity is not known, direction of acceleration alone does not tell you if an object is speeding up or slowing down.***

Quiz

A cyclist riding at 7 m s^{-1} sees a stop sign and comes to a complete stop in 4 s. He then, in 6 s, returns to a speed of 5 m s^{-1}. Which of the following shows his motion diagram?



Solution:

Answer is either __________ based on the __________ directions of the acceleration and velocity vectors. Since the cyclist is ______________ in the first part of his motion, the acceleration and velocity vectors should be in __________ directions. Since he ___________ after the stop, the acceleration and velocity vectors should be in _________ direction.

Answer is _________ because the final velocity is _________ than the initial velocity, so the last arrow should be __________ than the first arrow.

Quiz

Let us define eastward as positive and westward as negative. Are the following statements *True or False?*

a) If a car is traveling eastward, its acceleration must be eastward.
b) If a car is slowing down, its acceleration may be positive.

Solution:

a) False. Velocity of car must be eastward but acceleration can be either eastward or westward. If acceleration is directed eastward, car is speeding up. If westward, car is slowing down.
b) True. A car slowing down indicates that its acceleration and velocity vectors have opposite directions. If the car is travelling westward and is slowing down, its acceleration must be directed eastward, that is, acceleration is positive.

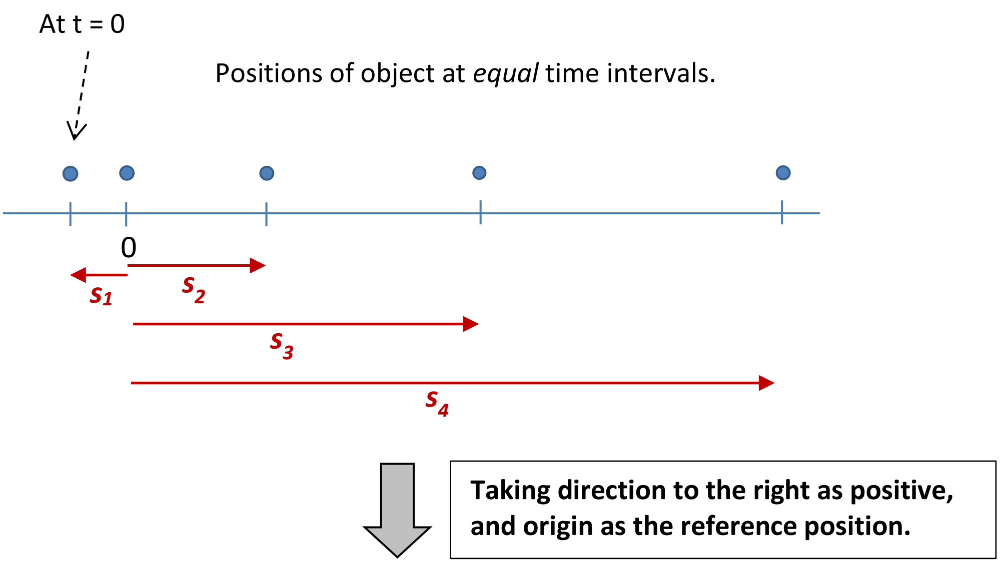
2.1.5 Graphical Representation of motion

A *motion diagram* tells us only the positions of an object at a few discrete instants of time, and only allows us to determine the average values of velocity.

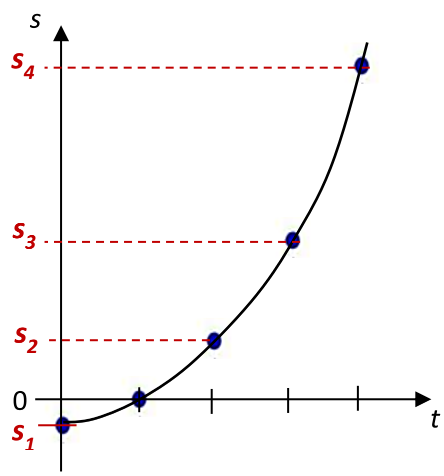
A *graph* would allow us to obtain the position of the object at all instants in time and the instantaneous values of velocity and acceleration.

(A) Displacement – Time graph (*s-t* graph)

Motion Diagram:



Graph:



In the example above, at t = 0, object was to the left of the origin it has a negative displacement initially. Velocity is increasing with time as observed from both the motion diagram and s-t graph.

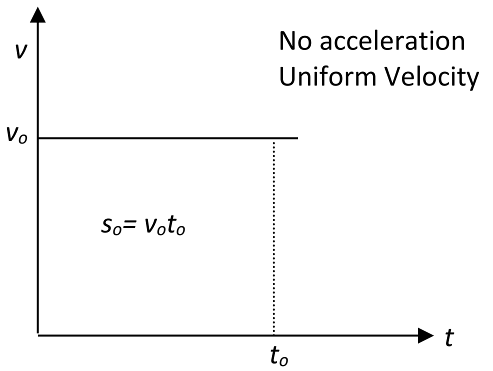
From the definition of velocity, $\boldsymbol{v} = \frac{ds}{dt}$

Thus, **gradient of a displacement-time graph at a time *t* is the velocity of the object at that time**.

(B) Velocity – Time graph (*v-t* graph)

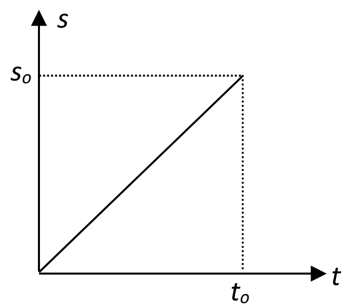
From the definition of acceleration, $a = \frac{dv}{dt}$
Thus, **acceleration is determined from the gradient of a velocity-time graph**.

From the definition of velocity, $v = \frac{ds}{dt}$
Integrating, $\int v\,dt = \int ds = s_f - s_i = \Delta s$, ***change in displacement***
Thus, ***change in*** **displacement is determined from the area under the velocity-time graph.**

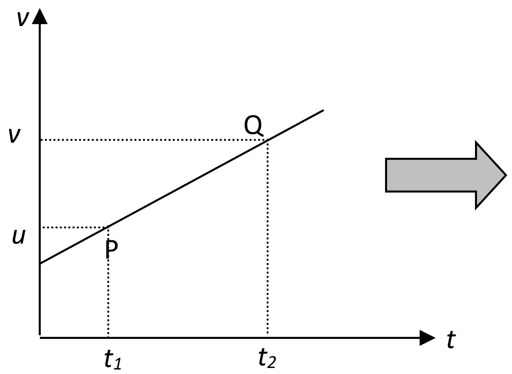


v-t graph with zero gradient

Corresponding displacement-time graph:



s-t graph with constant and non-zero gradient



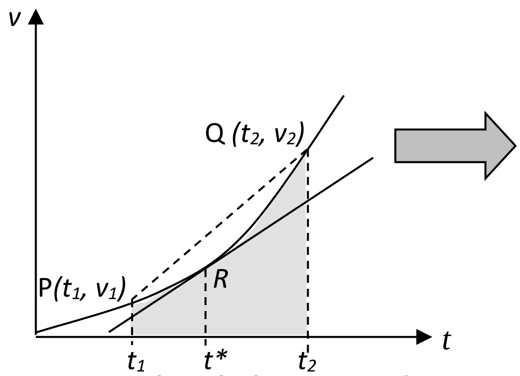
v-t graph with constant and non-zero gradient

Uniform Acceleration
(constant and non-zero gradient)

Instantaneous acceleration at any t = *Ave* acceleration

= gradient of PQ = $\frac{v - u}{t_2 - t_1}$

Non-Uniform Acceleration

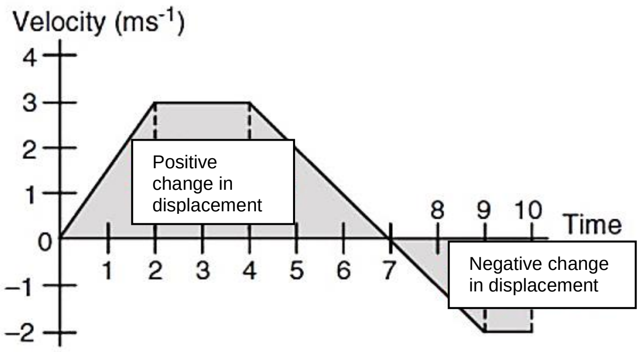


v-t graph with changing gradient

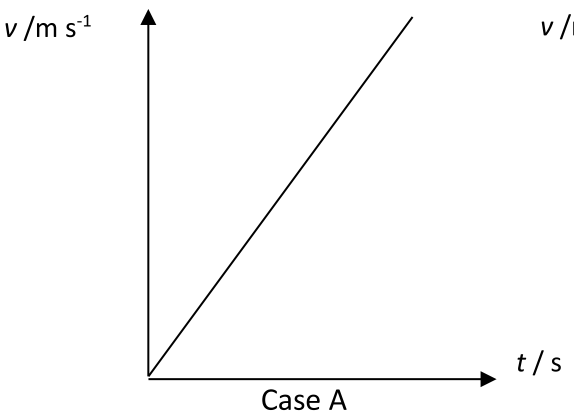
Instantaneous acceleration at t*
= gradient of tangent to curve at R = $\frac{dv}{dt}$

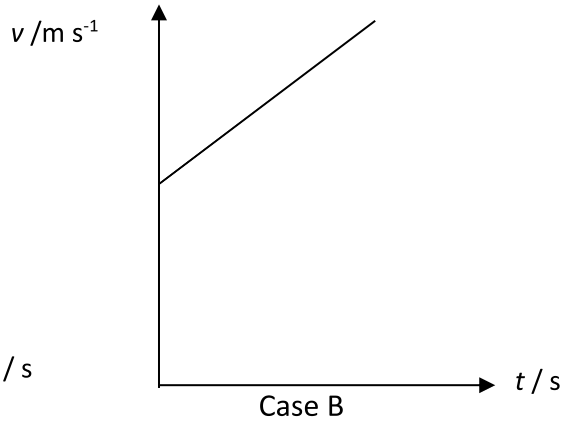
Average acceleration over time interval t_1 to t_2
= gradient of the line segment PQ
= $\frac{v_2 - v_1}{t_2 - t_1}$

Change in displacement from t_1 to t_2
= Area under the graph from t_1 to t_2



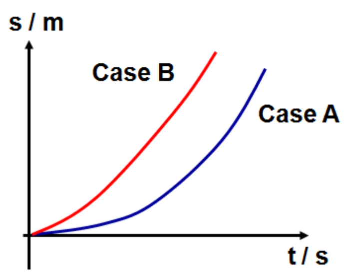
Example 5

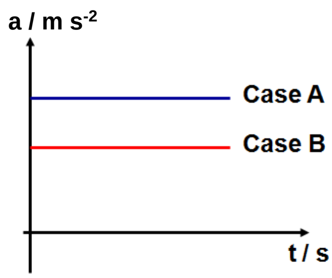




Sketch the displacement-time graphs and acceleration-time graphs for the motions.

Solution:



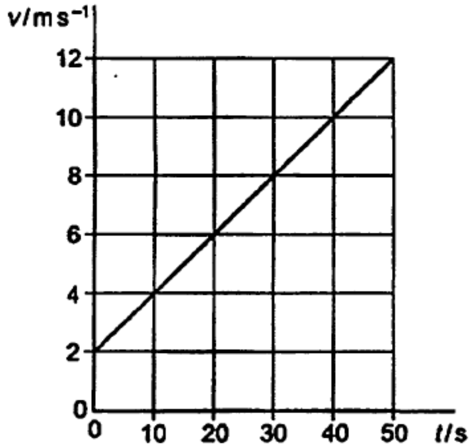


Notes:

- *Gradient at t = 0 is zero for case A, because velocity at t = 0 is zero.*
- *Object starts off with non-zero velocity for Case B.*
- *Compare magnitude of v for Case A and B. At each instant in time, velocity of Case B is larger in magnitude. Thus, gradient of s-t graph for Case B is steeper compared to Case A.*
- *Gradient of v-t graph gives the value of acceleration.*
- *Gradients of v-t graphs for both cases are non-zero and constant, so the value of acceleration is non-zero and constant.*
- *Gradient of v-t graph in case A is larger. Hence, acceleration of case A is larger.*

Example 6 (N04/I/3 modified)

A train travelling at 2.0 m s^{-1} passes through a station. The variation with time *t* of the speed *v* of the train after leaving the station is shown below.



Determine the speed of the train when it is 150 m from the station.

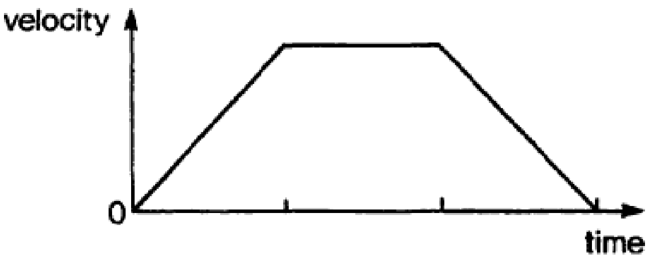
Solution:

Area per small square represents 20 m.
150 m is equivalent to 7.5 small squares.

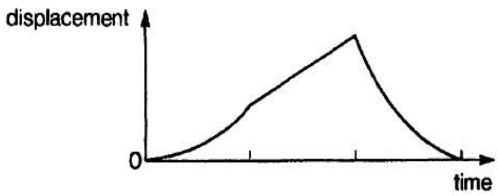
By counting number of squares under the graph starting from t = 0 s, when the train is 150 m from the station,
t = 30 s, v = 8 m s^{-1}

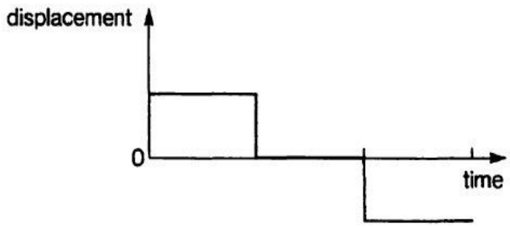
Quiz (N95/I/5)

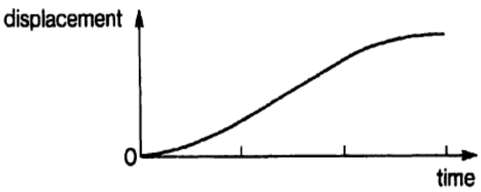
The graph of velocity against time for a moving object is shown.

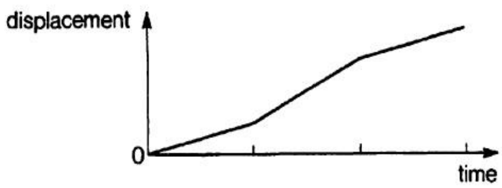


Which of the following is the corresponding graph of displacement against time?

A


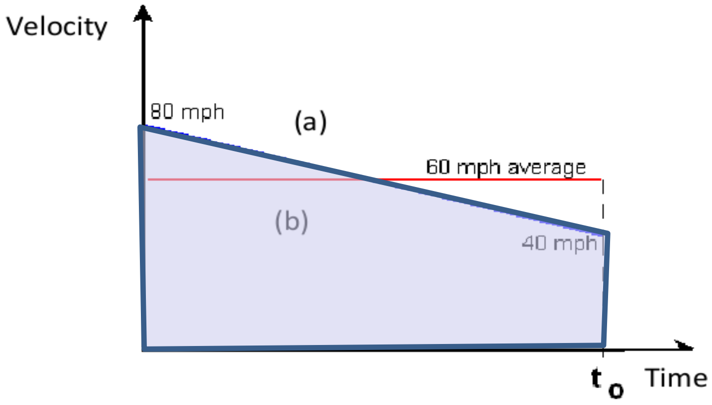
B


C


D


( )

The difference between "instantaneous" versus "average" velocity:



***v-t* graph of a car on a road**

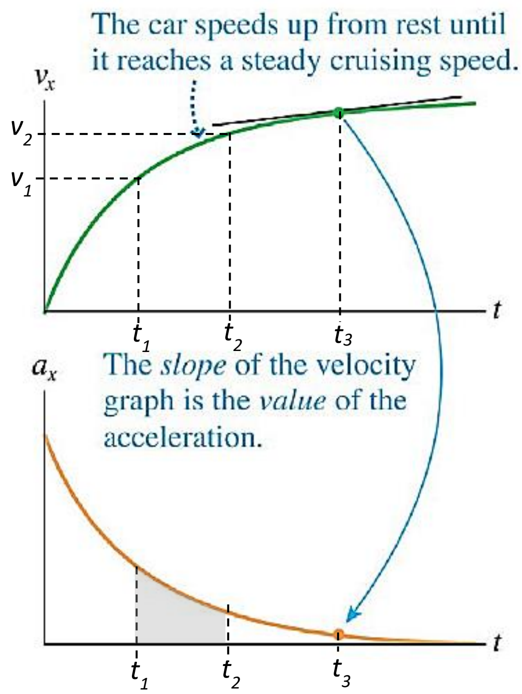
Instantaneous velocity	**Average velocity**
Mathematically, it is a differential quantity, given by the gradient of the s-t graph at time t. $v = \left.\frac{ds}{dt}\right\|_{\text{at time t}}$ On a v-t graph, it is the y-coordinate at time t.	Mathematically, it is a ratio quantity, given by the total change in displacement divided by the total time = area bounded by the line and the x-axis divided by total time (t_o)

(C) Acceleration – Time graph (*a-t* graph)

From the definition of acceleration, $\boldsymbol{a = \frac{dv}{dt}}$

Integrating, $\int a\,dt = \int dv = v_f - v_i = \Delta v$, ***change in velocity***

Thus, ***change in*** **velocity is determined from the area under the acceleration-time graph.**



This is a realistic graph for a car leaving a stop sign.

The gradient is not constant but becomes less and less positive. Thus, acceleration is not constant but becomes less and less positive in value.

This is the acceleration-time graph for the car with positive and decreasing values of acceleration.

Change in velocity from t_1 to t_2

= $v_2 - v_1$ from v_x - t graph

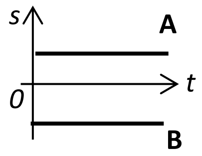
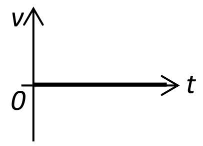
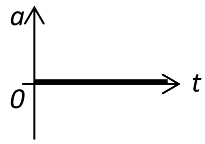
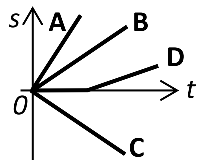
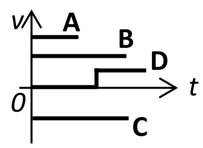
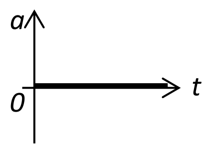
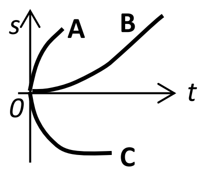
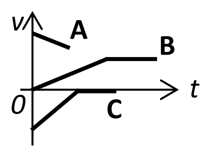
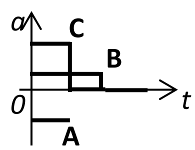
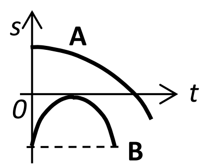
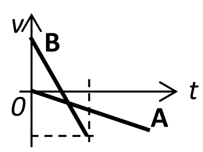
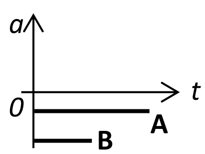
= Shaded area under the a_x – t graph from t_1 to t_2

Average acceleration between t_1 and t_2

$= \frac{v_2 - v_1}{t_2 - t_1}$ (on v_x-t graph)

$= \frac{\text{Area under } a_x\text{-t graph during the time interval } t_1 \text{ to } t_2}{t_2 - t_1}$ (on a_x-t graph)

Motion Graphs Summary

Displacement-Time	Velocity-Time	Acceleration-Time	Description
			Both objects A and B are at rest. A is at a positive displacement from the origin; B is at a negative displacement.
			All are at the origin at t = 0 with no acceleration. A, B and D are all moving in the positive direction, away from the origin, with constant speed; however D started moving later. A is moving at the greatest speed, followed by B and then D. C is moving at the same speed as B, but moving in the opposite direction away from the origin, hence represented as below the horizontal axis.
			All are at the origin at t = 0. A has the largest initial speed, while B has the smallest initial speed of zero. A is moving in the positive direction away from the origin, with decreasing speed. Its speed is decreasing at a constant rate. B moves off from rest in the positive direction away from the origin, with increasing speed. Its speed is increasing at a constant rate. Its acceleration is smaller in magnitude compared to A. It eventually travels at a constant speed. C is moving in the negative direction, away from the origin, with decreasing speed. Its speed is decreasing at a constant rate. C has the largest magnitude of acceleration. It eventually comes to rest.
			At t = 0, A is at a positive displacement from the origin. Its initial velocity is zero. It is moving towards the origin with increasing speed. Its speed is increasing at a constant rate. It eventually moves past the origin and has negative displacement. At t = 0, B is at a negative displacement from the origin. Its initial velocity is positive. It is moving towards the origin with decreasing speed. It is instantaneously at rest when it reaches the origin, thereafter reverses direction and moves away from the origin with increasing speed. It eventually arrives back at its initial position. Its speed is changing at a constant rate throughout, thus the durations for the to and fro journey are equal.

MINI-TEST 1

A car is travelling along a straight road at a constant velocity initially for 1 minute. After that, the driver steps on the accelerator, causing the car's speed to increase uniformly for 30 s. It then travels at constant velocity again for another minute before the driver slows down uniformly for 30 s to a momentary stop and then makes a U-turn back with the same acceleration for 30 s. Sketch the displacement-time, velocity-time and acceleration-time graphs to illustrate the motion of the car.

My solution:

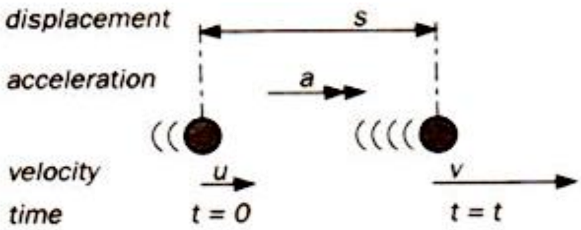
2.1.6 Mathematical Representation of motion (Kinematic Equations)

Using equations allows for description of relationship between variables more precisely and allows for manipulations that allows us to make calculations and predictions. Equations thus build more meaning on top of the graphical representations that have been discussed thus far.

2.1.6.1 Derivation of Kinematic Equations for UNIFORMLY ACCELERATED MOTIONS in a STRAIGHT LINE

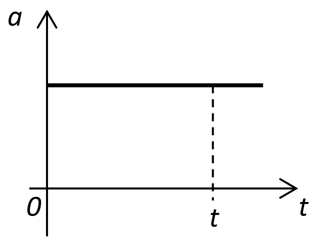
***** Knowledge of the following derivation is expected in the A-level syllabus.***

Consider an object moving in a **straight line** with **uniform acceleration** *(a)* from an initial velocity (*u*) to a certain velocity (*v*) in time (*t*). During this time interval, the object undergoes a change in displacement (*s*).



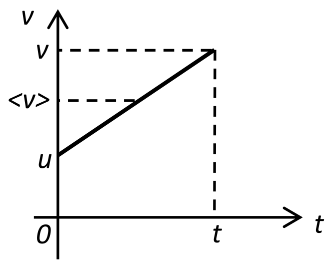
Uniformly accelerated motion

Consider the velocity-time graph of an object moving with uniform acceleration. The *v-t* graph is a straight line with non-zero gradient equal to *a* and vertical-intercept equal to *u*.



By definition of acceleration,

$$a = \frac{dv}{dt} = \text{gradient of } v - t \text{ graph} = \frac{v - u}{t}$$



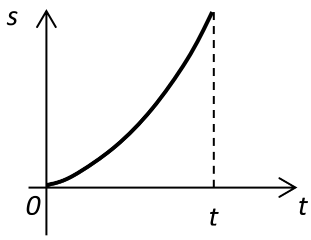
Rearranging, we obtain a linear equation for the velocity v at any time t:

$$\boxed{v = u + at}$$

This is the ***1st equation of motion***.

Average velocity, <*v*>, is midway between u and v

$$\langle v \rangle = \frac{u + v}{2}$$



By definition of average velocity,

<*v*> = change in displacement / time taken

change in displacement = <*v*> x time

$$\therefore s = \langle v \rangle \times t$$

$$s = \frac{(u + v)t}{2}$$

Alternatively, we arrive at the same result by considering area under the *v-t* graph:

change in displacement = $\int v \, dt$

= area under the *v-t* graph

$$s = \frac{1}{2}(u + v)t$$

Substituting ($v = u + at$) into this equation gives:

$$\boxed{s = ut + \frac{1}{2}at^2}$$

This is the ***2nd equation of motion***.

Eliminate t from ($v = u + at$) and $s = \frac{(u + v)}{2}t$:

$$s = \frac{u + v}{2} \cdot \frac{v - u}{a}$$

$$= \frac{v^2 - u^2}{2a}$$

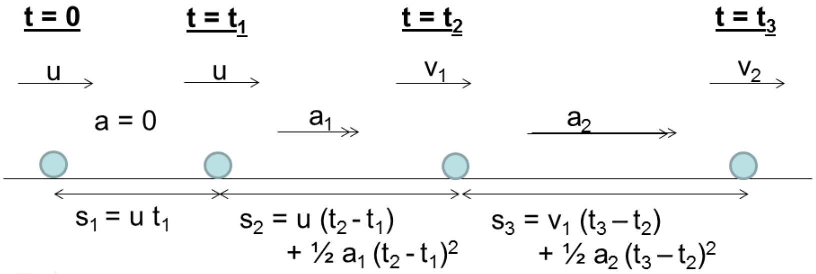
$$\boxed{v^2 = u^2 + 2as}$$

This is the ***3rd equation of motion***.

Notes:

- **Conditions for the 3 kinematic equations to be applicable:**
  - **(1) When acceleration is constant.**
  - **(2) When motion is in a straight line.**
- *t* in the equations above refers to the time duration for a particular fixed value of acceleration. For example, if an object is moving at constant velocity initially for a duration of t_1, and subsequently accelerates uniformly at *a*, then the time t_1 *is excluded in t*.
- *s* refers to the change in displacement from the object's position in time *t* as it accelerate uniformly at *a*.

Example:



2.1.6.2 Solving Rectilinear Kinematics Problems Quantitatively

1. Identify the state of motion: uniform motion or constant acceleration.
2. Draw a *pictorial representation* to represent the situation being studied.
3. Identify a direction to be positive (+). The opposite direction would be negative (-). Identify a convenient coordinate origin (reference point for displacement). Follow these identification throughout the course of your calculation.
4. List down all known quantities (with appropriate + and - signs).
5. List the quantity to be determine.
6. Use a *graphical representation* if it is appropriate for the problem.
7. Based on the lists in Steps 4 & 5, select appropriate equations of motion to find the unknown quantity.

   Uniform motion : $s = ut$

   Uniform acceleration : $v = u + at$ OR $s = ut + \frac{1}{2}at^2$ OR $v^2 = u^2 + 2as$
8. Evaluate the final answer: Does the answer make physical sense?

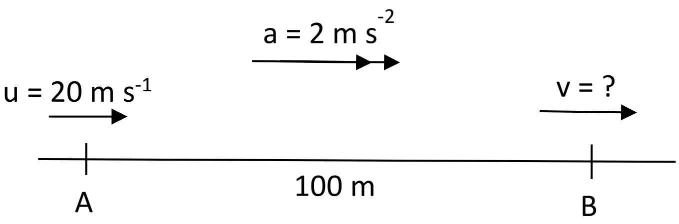
Example 7

A train travels on a straight track passing signal A at 20 m s^{-1}. It accelerates uniformly at 2 m s^{-2} and reaches signal B 100 m further than A. At B, the velocity of the train is

(A) 10 m s^{-1} (B) 20 m s^{-1} (C) 28 m s^{-1} (D) 56 m s^{-1}

Question analysis:

1. **Identify the motion:** It is a motion with constant acceleration of 2 m s^{-2}. This acceleration must be in the same direction as velocity as the train accelerates.
2. **Draw a pictorial representation with important quantities:**



3. Let **the direction of A to B to be positive.**
4. Since time duration of motion is unknown and the final velocity is to be determined, use $\boldsymbol{v^2 = u^2 + 2as}$. *u*, *a* and *s* are available from the question.

Solution:

Let the direction of A to B be positive.

Use $v^2 = u^2 + 2as$ [*Write the original equation*]

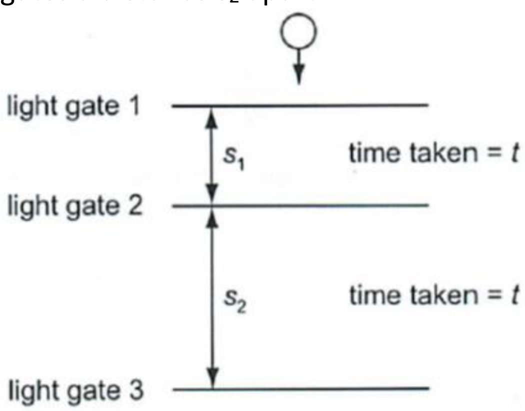
$v^2 = 20^2 + 2(2)(100)$ [*Substitute quantities in the context, including sign*]

$v = +28$ m s^{-1}

(Reject -28 m s^{-1} since the train is speeding up from A to B and so it is impossible for the train to reverse direction of velocity.)

Example 8: N10/1/5

An object falls freely with constant acceleration a from above three light gates. It is released from rest just above the first light gate. It is found that it takes a time t to fall between the first two light gates a distance of s_1 apart. It then takes an additional time, also t, to fall between the second and third light gates a distance s_2 apart.

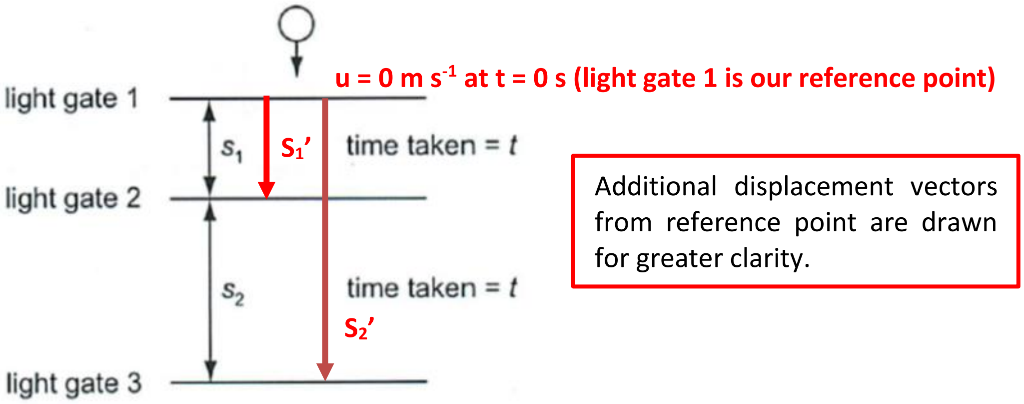


What is the acceleration in terms of s_1, s_2 and t?

A $\dfrac{(s_2 - s_1)}{t^2}$ **B** $\dfrac{(s_2 - s_1)}{2t^2}$ **C** $\dfrac{2(s_2 - s_1)}{3t^2}$ **D** $\dfrac{2(s_2 - s_1)}{t^2}$

Question analysis:

1. Identify the state of motion: **constant acceleration (stated in question)**
2. Draw a pictorial representation: already given in question, but details can be added on the diagram for clarity



3. Positive direction: **downwards** (this is chosen as the entire motion is downwards)
4. List down all known quantities: They have all been indicated in the diagram (***u*, *s* and *t)***. Notice that we may have to add some symbols for some unknown quantities
5. List the quantity that we need to find: **acceleration**
6. Choose the appropriate equation: $\boldsymbol{s = ut + \frac{1}{2} at^2}$ (this is chosen as most terms we need are given)

<u>Solution:</u>

Take direction downwards to be positive.

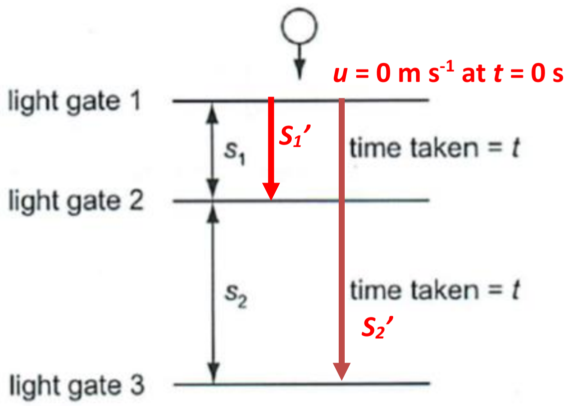
$S_1 = S_1' = ut + \frac{1}{2} at^2$ [*write the original equation*]

$= 0 + \frac{1}{2} a (t)^2$................ (1) [*show clearly the substitution with the context*]

$S_2' = ut + \frac{1}{2} at^2$

$= 0 + \frac{1}{2} a (t + t)^2$

$= \frac{1}{2} a (2t)^2$....................... (2)



Use (2) – (1),

$S_2' - S_1' = S_2 = \frac{1}{2} a (4t^2) - \frac{1}{2} a t^2$

From (1), $S_1 = \frac{1}{2} a t^2$

Thus, $S_2 - S_1 = \frac{1}{2} a (4t^2) - 2 (\frac{1}{2} a t^2)$

$a = 2(S_2 - S_1)/ (2t^2) = (S_2 - S_1)/ (t^2)$

Answer: A

Example 9

A car is speeding along a straight country road at a speed of 25 m s^{-1} when the driver sees a farm vehicle just starting to cross the road at a point 40 m ahead. The driver's reaction time (i.e. the time interval between seeing the obstacle and actually applying the brakes) is 0.80 s, and he decelerates with a constant maximum (brakes fully applied) acceleration of 5.5 m s^{-2}. If the car can be assumed to decelerate uniformly at this rate without swerving, calculate

(i) the distance travelled by the car during the driver's reaction time.

(ii) the car's velocity when it reaches the position of the farm vehicle.

(iii) the total time which has elapsed from first sighting until the car reaches the farm vehicle.

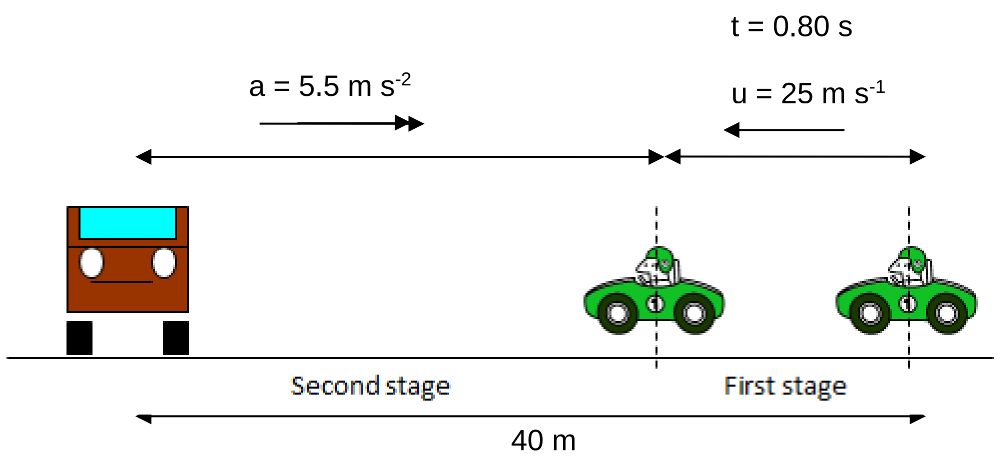
(iv) the minimum constant velocity of the farm vehicle so that the car does not collide with it, given the width of the road is 3.0 m and the length of the farm vehicle is 2.0 m.

Question analysis:

1. There are two stages of motion
   a) Constant speed during the driver's reaction time
   b) Constant deceleration after the driver applies the brake fully

   Therefore, the equation of motion have to be applied separately.

2. Clear label pictorial representation:



<u>Solution:</u>

(i) distance travelled by car during First stage = 25 x 0.80 = 20 m

(ii) Take direction to the left as positive.

In the Second stage of the car:
s = 40 – 20 = 20 m
u = 25 m s^{-1}
a = - 5.5 m s^{-2}

To find: v at s = 20 m

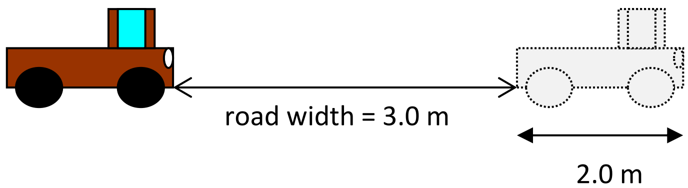
Use $v^2 = u^2 + 2as$
$v^2 = 25^2 + 2(-5.5)(20)$
v = 20.1 m s^{-1}

(iii) To find: t in Second stage

Use v = u + at
(20.1) = (25) + (-5.5)t
t = 0.891 s

Total time = 0.891 + 0.80 = 1.69 s

(iv) Maximum time allowed for farm vehicle to cross the road without being hit by car = 1.69 s

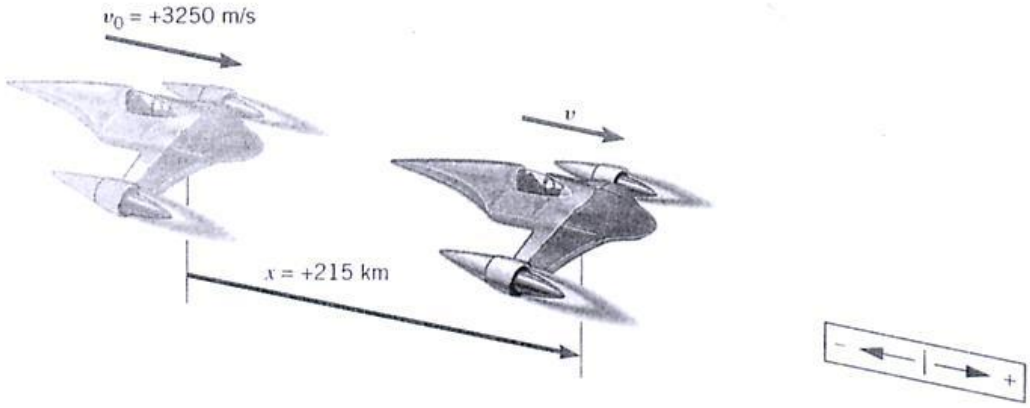


To completely clear-off the road, farm vehicle must travel a distance of (3.0 + 2.0) = 5.0 m

Minimum constant speed of farm vehicle required
= 5.0 / 1.69
= 2.96 m s^{-1} or 3.0 m s^{-1}

Example 10

The spacecraft shown is travelling forward with a velocity of 3250 m s^{-1}. Suddenly the retrorockets are fired in opposite direction to the velocity, and the spacecraft begins to slow down with a constant acceleration whose magnitude is 10 m s^{-2}. What is the velocity of the spacecraft when the displacement of the craft is 215 km in the direction of initial velocity, relative to the point where the retrorockets began firing?



Adapted from Pg 39 of Physics 7th edition, Cutnell & Johnson.

Solution:

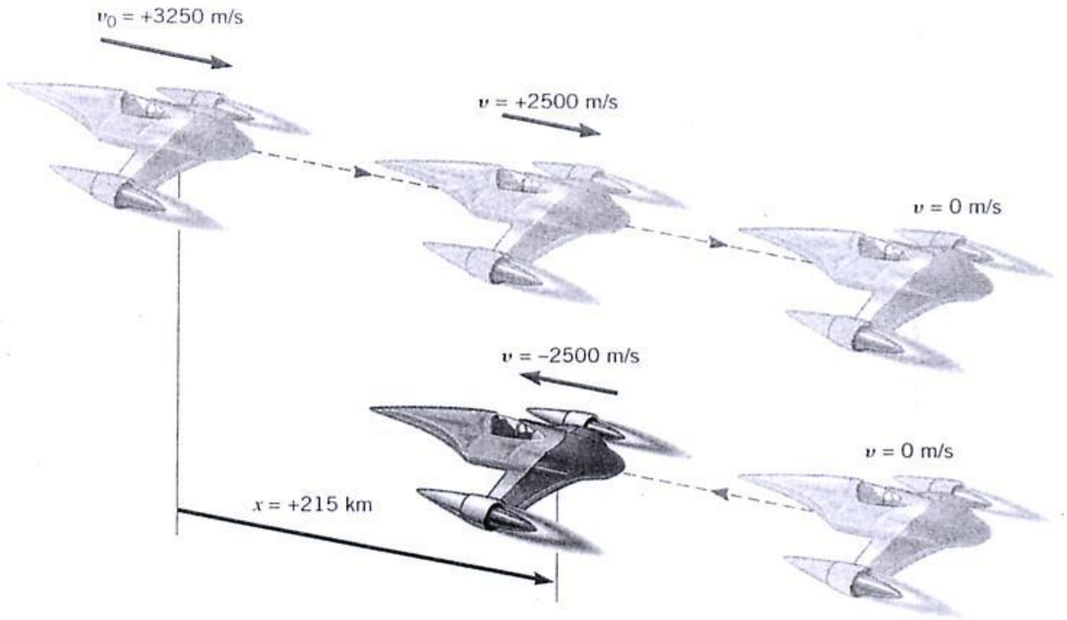
Since velocity is positive, and the object slows down, the acceleration is negative.

Use $v^2 = u^2 + 2as$

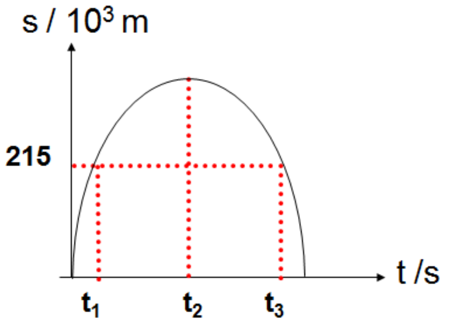
$v^2 = 3250^2 + 2(-10)(215\ 000)$

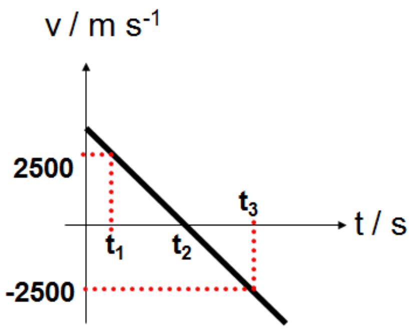
$v = \pm 2500$ m s^{-1}

Note: There are 2 possible answers to a variable. Visualize the different physical situations to which the answers correspond. -2500 m s^{-1} arises after the retro-rockets bring the craft to a momentary halt and the craft reverses in direction and moves back to the position where displacement is +215 km.



Graphical Representation of Motion:





2.1.6.3 Motion under gravity without air resistance

An object is said to be **free falling** if the ***only*** **force that acts on it is its own weight**. Air resistance is zero. The **acceleration of the free falling object is equal to *g* = 9.81 m s^{-2} downwards**. Since the **acceleration** of the object is **constant** in free fall, the equations of motion can be used.

"Free falling" does not necessarily mean an object is moving down. A free falling object is moving either upward or downward as long as it is solely under the influence of gravity alone. In other words, a free-falling object always experiences the same constant *downward acceleration* due to gravity.

Example 11
A lunar landing module is descending at a steady velocity of 10 m s^{-1} to the surface of the Moon. At an altitude of 120 m, a small object becomes detached from its landing gear. The gravitational acceleration of the Moon may be taken as 1.6 m s^{-2}.

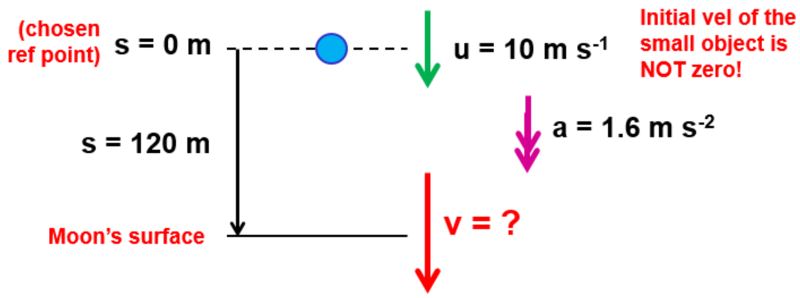
What is the speed of the object when it strikes the Moon?

Question analysis:

1. **Identify the state of motion:**
   Note that there are two objects in the question. As the question wanted us to find the speed of the object, we focus on the motion of the object.

   The motion of the object is initially together with the lunar landing which is at a constant velocity. After which, it falls at an acceleration of 1.6 m s^{-2}.

2. **Draw a pictorial representation with important details:**



3. Since the motion is entirely downwards, **we set downwards as positive.**
4. Quantity that we need to find is the speed of the object as it strikes the moon, that is **the final speed of the motion.** Therefore, we **use $v^2 = u^2 + 2as$**

<u>Solution:</u>

Let downwards be positive.

$v^2 = u^2 + 2as$ *[write down the equation to be used]*

$v^2 = (10)^2 + 2(1.6)(120)$ *[substitute in the quantities in the context of the question]*

$v = 22$ m s^{-1}

Example 12

A ball is thrown vertically upwards with an initial velocity of 30 m s^{-1}.

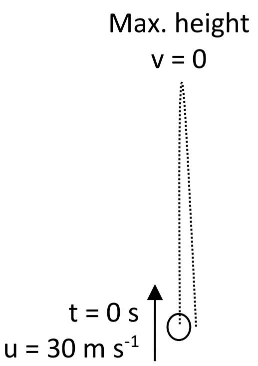
Find (i) the maximum height reached,

(ii) the time taken for the ball to return to the ground.

Question analysis:

Though this looks like a two-part motion, it is actually just ONE motion with constant acceleration, i.e. 9.81 m s^{-2} downwards.

<u>Solution:</u>



Take upwards as positive direction.

(i) Max. height = s when v reduces to zero

use $v^2 = u^2 + 2as$

$0 = 30^2 + 2(-9.81)(s)$

$s = 46$ m

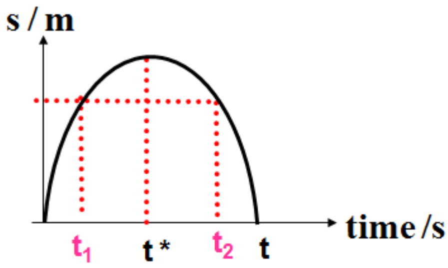
(ii) When ball returns to the ground, s = 0 again.

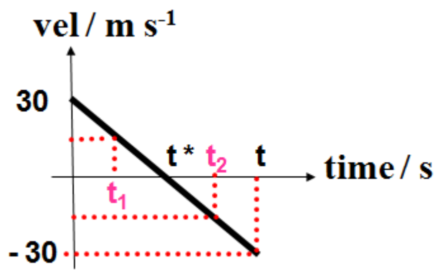
use $s = ut + \frac{1}{2}at^2$

$0 = (30)t + \frac{1}{2}(-9.81)t^2$

t = 0 s or 6.1 s

Hence time taken for ball to return to the ground is 6.1 s.





Note: For objects that are free falling, there is a type of symmetry for time and speed. The time it takes for the ball to reach its maximum height is equal to the time spent returning to its original position. The speed of the ball at any height above the ground on the upward trajectory is equal to the speed at the same height on the downward part. Although the two speeds are the same, the velocities are different because they point in different directions.

Quiz

1) A ball is thrown vertically upwards. While the ball is in free fall, its acceleration

A) increase,
B) decrease
C) remain constant

( )

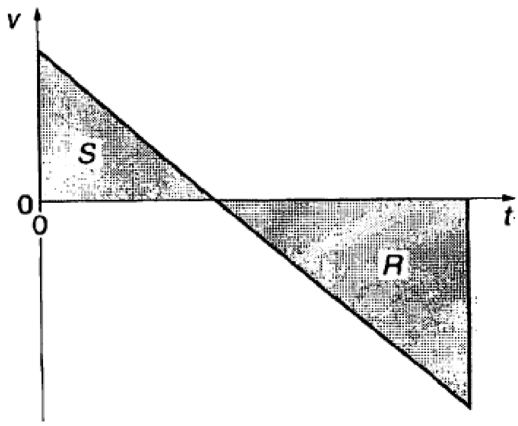
2) After a ball is thrown vertically upward and is in air, its speed

A) increases,
B) decreases,
C) decreases and then increases,
D) remains the same.

( )

Quiz (N98/I/3)

A stone is thrown upwards from the top of a cliff. After reaching its maximum height, it falls past the cliff-top and into the sea. The graph below shows how the vertical velocity v of the stone varies with time t after being thrown upwards. R and S are the magnitudes of the areas of the two triangles.



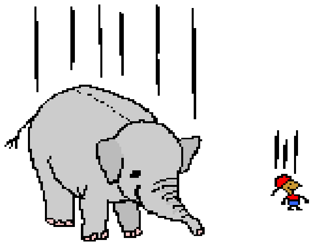
What is the height of the cliff-top above the sea?

A) R
B) S
C) R+ S
D) R - S

( )

Quiz

Would a more massive object (e.g. elephant) fall faster than a less massive object (e.g. mouse) in a vacuum?



MINI-TEST 2

A tennis ball is dropped from a height of 2.0 m above a hard level floor and it rebounds to a height of 1.5 m.

From the instant it is released to the point where it reaches maximum height after the first bounce,

(a) calculate the speed just after impact with the floor;
(b) calculate the total time taken and;
(c) sketch the velocity-time graph for this entire motion.

<u>My solution:</u>

2.1.6.4 Falling motion under gravity with AIR RESISTANCE

Consider a small spherical ball falling through a column of viscous oil in a long tube as shown in the schematic diagram of Fig. 2.1(a) of the set up as well as the motion diagram of the ball.

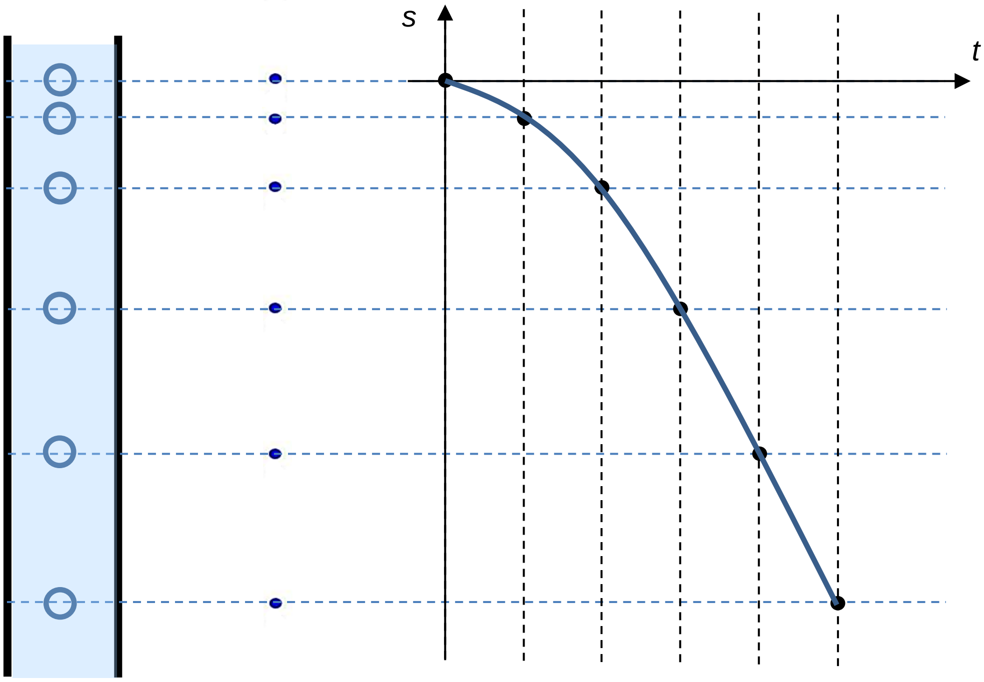


Fig. 2.1 (a)

Likewise, when an object falls under gravity, it moves through air in the atmosphere. Its motion diagram is similar to the diagram that represent the falling sphere in the column of oil in Fig. 2.1 (b). The motion diagram and the displacement-time (*s*-*t*) graph are translated to describe the velocity-time graph of the object,

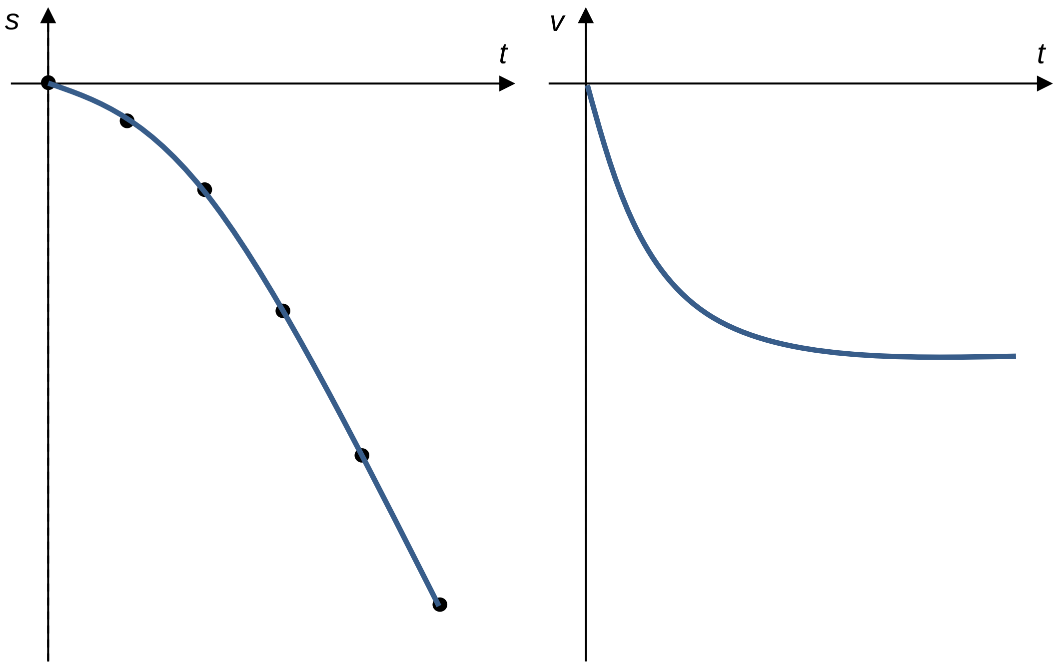


Fig. 2.1(b)

a) Gradient of the s-t graph is always negative and increasing. Therefore, the velocity of the object is getting larger in the negative direction to a maximum value.
b) Gradient of the v-t gradient at each point on the curve represents the object's acceleration, the velocity of the object increases downward with decreasing acceleration.
c) Since the gradient of the v-t graph is the steepest at t = 0, the acceleration of the object is the greatest when the object is just released.
d) When the time taken and distance fell for the object are long enough, the gradient of the v-t graph decreases to zero, indicating that the acceleration of the object reaches zero.
e) The object then moves with a constant maximum velocity – terminal velocity – for the rest of the falling motion.

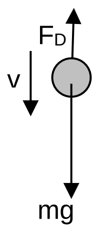
Explanation of falling motion under gravity with air resistance.
An explanation of a motion is only complete with the application of Newton's law of motion.

When an object moves through a fluid (e.g. gas, liquid, molten substances etc.), it experiences a resistive force in opposition to its motion. This resistive force, also known as the **drag force or viscous drag**.

Consider a body that is dropped from rest in air. Its initial velocity is zero, hence the drag force F_D is also zero initially. The body has an acceleration a = g initially.

The **direction of drag force is *always opposite* to the direction of velocity of the object and its magnitude increases with the speed of the object**.

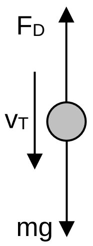
Free body diagram of body before reaching v_T



As the body moves downwards, the drag force acting upwards (opposite to velocity) on it increases with its speed. If mg is larger than F_D, then there is a resultant downward force and hence a downward acceleration. The resultant downward force gradually decreases as F_D increases. Thus the body speeds up at a decreasing rate.

$$mg - F_D = ma$$
$$a = (mg - F_D) / m$$

Free body diagram of body on reaching v_T



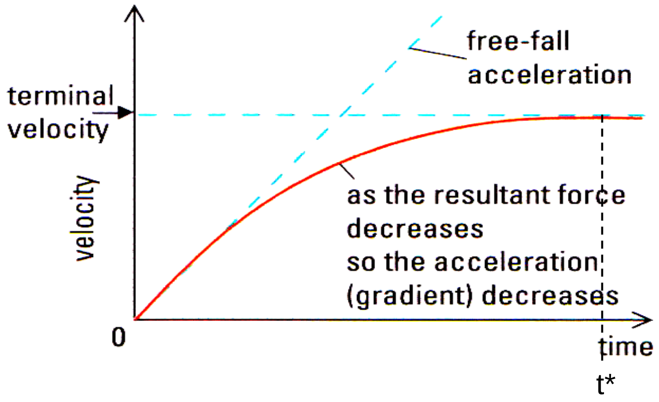
Eventually, the drag force reaches a magnitude equal to the weight of the body. Since drag force always acts opposite to the body's weight, the resultant force on the body becomes zero. Hence, acceleration of the body becomes zero. As a result, the velocity of the body reaches a maximum and constant magnitude, called the **terminal velocity** v_T.

$$mg - F_D = ma$$
$$mg - F_D = 0 \quad \text{(since } mg = F_D,\ a = 0\text{)}$$

This is a motion with **non-uniform acceleration**. The acceleration starts with value g, but acceleration decreases to zero at the time when terminal velocity is achieved.

The analysis of the same falling motion can be found by taking the opposite directions as positive. The following is a summary for the description of falling motion under gravity with air resistance when taking downwards as positive.

Velocity-time graph of the body

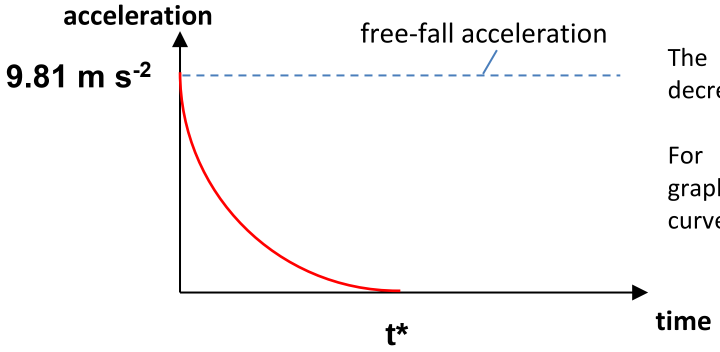


Taking downwards as positive

Velocity increases until it reaches a constant maximum magnitude. This is the terminal velocity of the body.

As seen from the gradient of the graph, acceleration decreases with time until it reaches 0. This occurs when terminal velocity is reached.

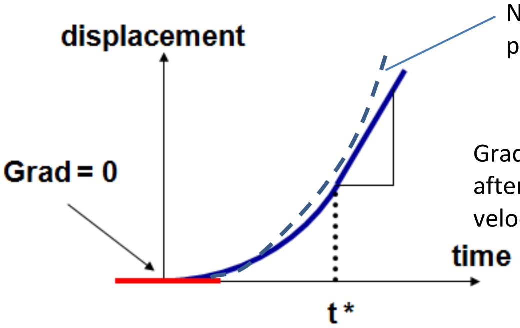
Acceleration-time graph of the body



The acceleration gradually decreases from g to 0.

For more info as to why a-t graph is not a straight line but a curve, refer to Appendix A.

Displacement-time graph of the body

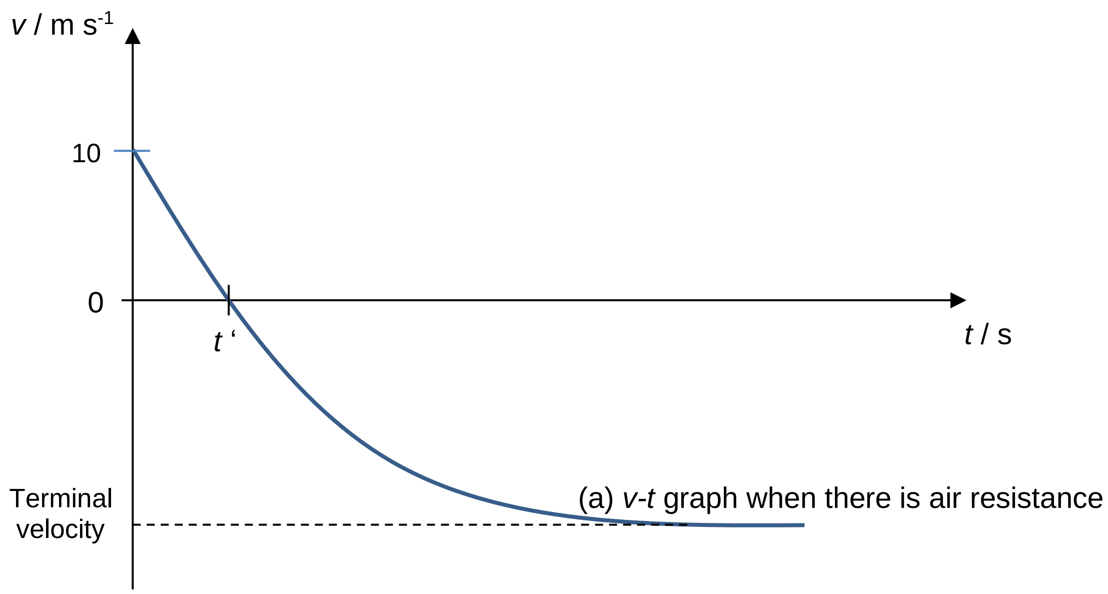


No air resistance. Curve is parabolic

Gradient becomes constant after t = t*, where terminal velocity is reached.

Example 13a
A stone is thrown *upwards* from a tall building with an initial velocity of 10 m s^{-1}. It then falls towards the ground. Sketch the v-t graph when air resistance is significant and taking upwards as positive.

When the stone moves upwards and then changes its direction downwards, there must be an instant where the stone's velocity is zero at the very top of its motion. As such, when the stone's velocity is zero, it is a context of a stone falling from rest and the graphs that represent such a motion have been discussed earlier.



1. As the stone moves upwards with initial speed of 10 m s^{-1}, it slows down with decreasing magnitude of acceleration until it comes to instantaneous rest (0 m s^{-1}) at some time $t = t'$.
2. After $t = t'$, the stone continues to fall, and its speed increases with decreasing magnitude of acceleration. The graph is similar to Fig. 2.1b.
3. Since the context of the question is a *tall* building, it suggests that the stone falls a distance long enough for it to reach terminal velocity. The terminal velocity is graphically represented as a horizontal straight line. Acceleration equals 0 m s^{-2} when terminal velocity is reached.
4. The stone's acceleration is the maximum in magnitude at the start, decreases to the acceleration of free fall at t = t' where v = 0 m s^{-1}, and continues to decrease to a = 0 m s^{-2} when the stone reaches terminal velocity.

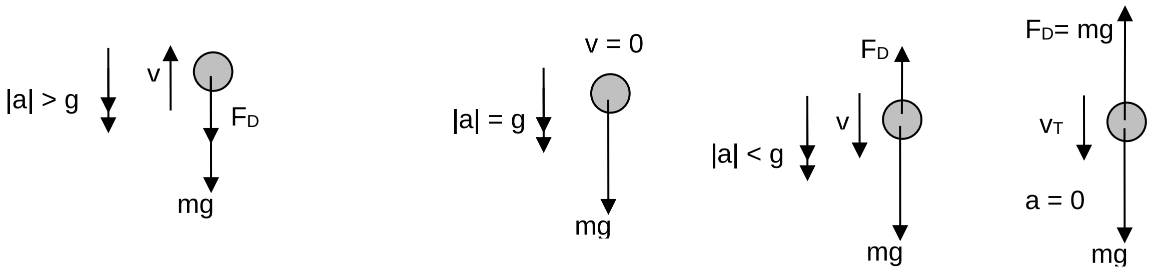
Note:
Why is the stone's speed decreasing with decreasing magnitude of acceleration from t = 0 to t = t' but increasing with decreasing acceleration from t = t' onwards?
→ Newton's laws provide an explanation of motion through the concept of net force.

While moving upwards from (t = 0 to t = t'):
While the stone is moving upwards, drag force acts downwards in the same direction as its weight, thus, the net force is larger than the weight. By Newton's 2nd law, the magnitude of acceleration is greater than g. Hence the gradient of the v-t graph is steeper than the value of g before v = 0. Since acceleration is opposite direction to the velocity, the stone slows down.

As the stone's speed decreases, the magnitude of

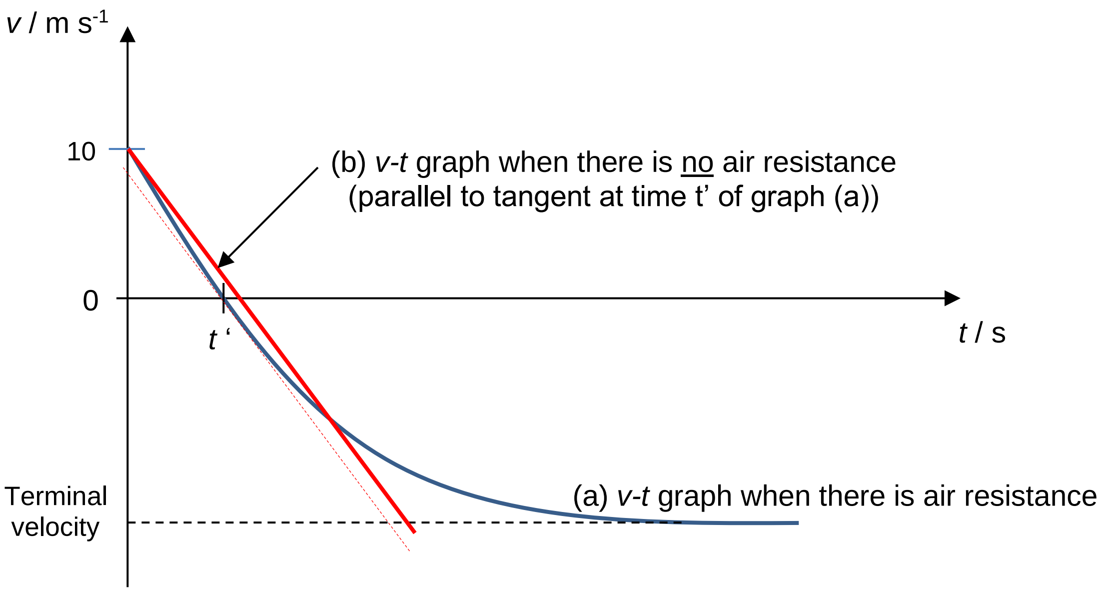
When stone is at the highest point at t = t':
Eventually, when v = 0, the stone is at its maximum height. Only force acting on stone is its weight. Drag force equals 0. Hence, acceleration (as seen from the gradient of the v-t graph) at that point equals to g.

After the maximum height beyond t = t':
Stone's weight increases the stone's speed from 0 m s^{-1}. Since the velocity is now downwards, the drag force acts upwards. Thus, drag force acts opposite in direction to its weight. By Newton's 2nd law, since the net force downwards is lesser than the stone's weight, the acceleration is lesser than g. As the stone's speed is increasing, drag force is increasing and the acceleration continues to decrease. Eventually, when drag force (which is acting opposite direction to the weight) equals to the magnitude of the weight, stone's acceleration ceases, and thus the stone continues moving downwards with constant maximum speed (i.e. terminal velocity reached).



Example 13b

If the stone in example 13a is falling through the same height in a vacuum with the same initial upward velocity, sketch the v-t graph when air resistance is negligible. Take upwards as positive.

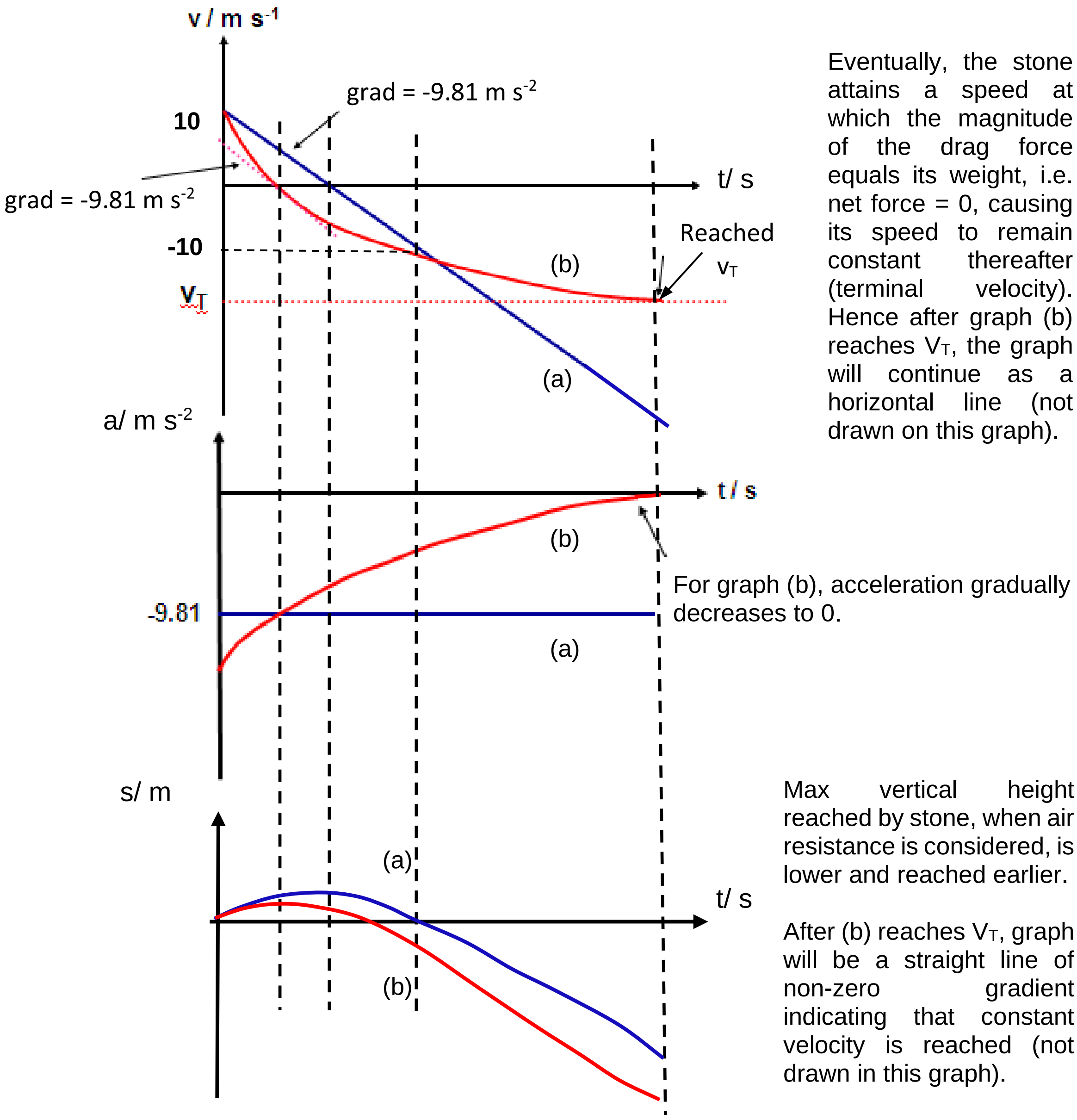


1. When there is no air resistance, the stone falls with a constant acceleration with a magnitude of g = 9.81 m s^{-2}. Hence the *v-t* graph (b) is a straight line with gradient equal to the gradient at *t* = *t'* of the graph (a) with the stone moving in air .
2. Before *v* = 0, area under the graph in (a) is smaller than area under the graph (b). This means that the maximum height reached with significant air resistance is smaller than the maximum height reached in vacuum.
3. Graph in (a) cuts the horizontal axis (where *v* = 0) earlier than graph in (b). This means that the stone reached its maximum height earlier when travelled with significant air resistance than the stone that travelled in vacuum.

Example 13c

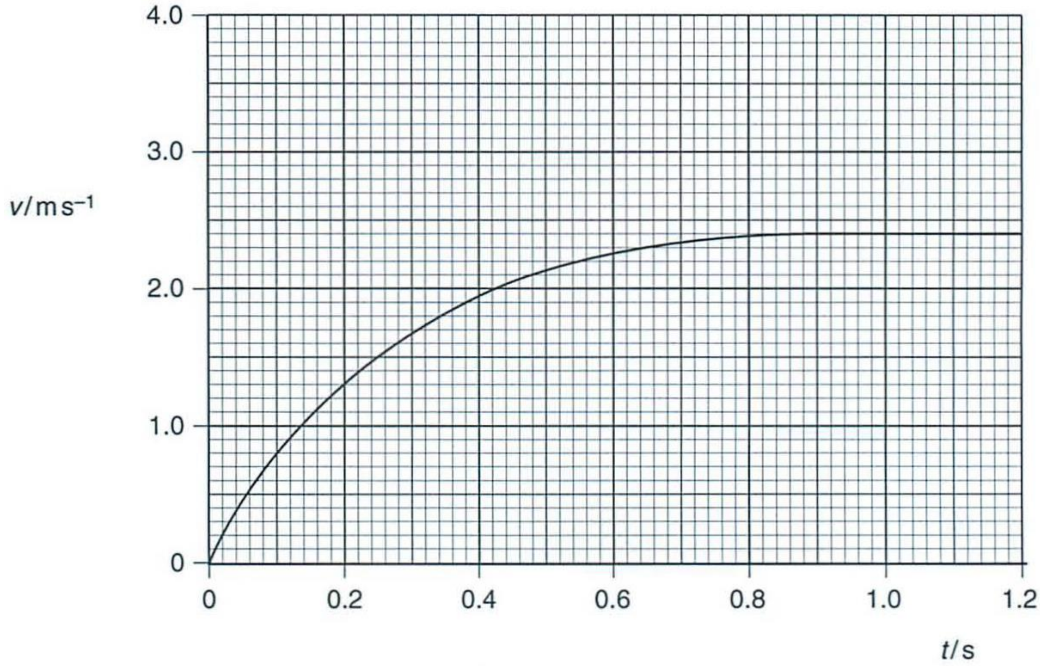
A stone is thrown upwards from a tall building with an initial velocity of 10 m s^{-1}. It then falls towards the ground. Sketch the v-t graphs, a-t graphs and s-t graphs when

(a) air resistance is ignored,
(b) air resistance is considered.



Example 14

The variation with time *t* of the vertical speed *v* of a light ball falling through air is shown.



The mass of the ball is 15 g.

(a) On the figure, draw a line to show the variation with time *t* of the vertical speed *v* of the ball falling from rest in a vacuum.

(b) Use the figure to determine the acceleration of the ball falling through air at time *t* = 0.20 s.

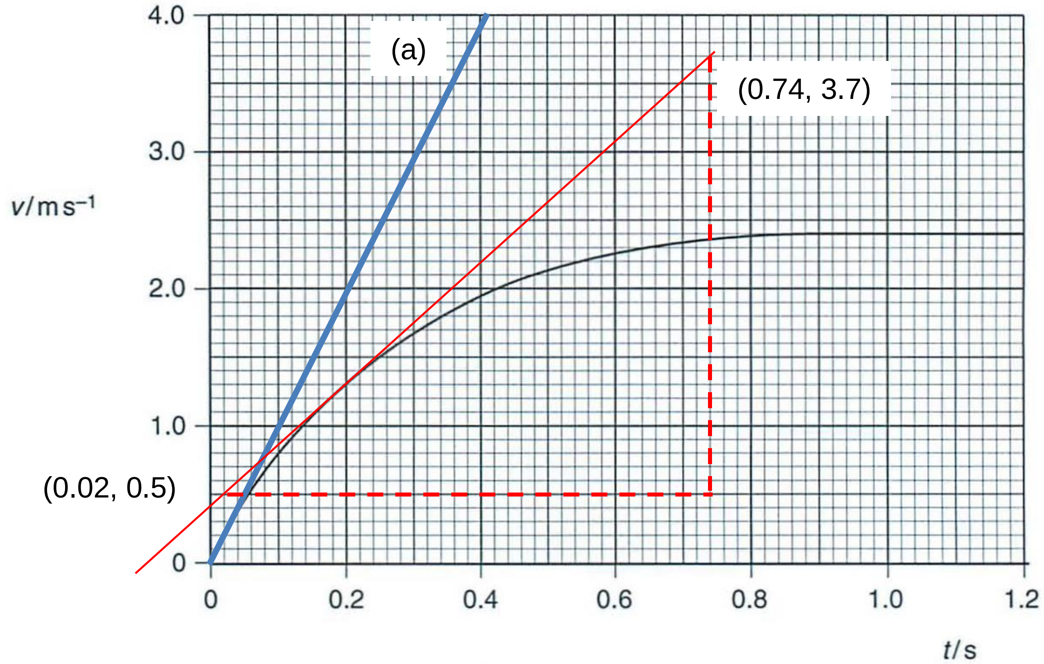
(c) For the air resistance acting on this ball,

(i) calculate the maximum resistive force,

(ii) show that the resistive force at *t* = 0.20 s is about 0.083 N.

Solution:

(a)



When the ball falls <u>in vacuum</u>, its acceleration is constant and equals to the acceleration of free fall g. Therefore, the v-t graph for it is a straight line that is tangent to the given graph for motion in air at v = 0. This is because when in air and v = 0, air resistance equals zero. Hence the net force equals ball's weight and thus acceleration at that instant equals to g as well.

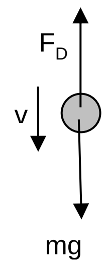
(b)
The gradient of the ball's velocity-time graph represents the ball's acceleration.
Drawing the tangent to the point at *t* = 0.20 s,

Gradient = $\dfrac{3.7-0.5}{0.74-0.02}=4.44$

Therefore, the acceleration is 4.44 m s^{-2}.

(c)(i)
The free-body diagram below shows the forces acting on the ball as it falls. The maximum resistive force occurs when the speed of the ball is maximum at the constant speed of 2.4 m s^{-1}. At this instant, the acceleration of the ball is 0 m s^{-2}, which means that the upward resistive force is numerically equal to the ball's weight. Therefore,

Maximum resistive force = $mg = 0.015 \times 9.81 = 0.15$ N



(c)(ii)
From (b), at t = 0.20 s, a = 4.44 m s^{-2}

Using F_{net} = ma

mg – F_D = ma

F_D = m (g – a)

= (15 x 10^{-3})(9.81 – 4.44)

= 0.081 N

which is approximately equal to 0.083 N.

Note: As graph reading and drawing of tangent line is subjective. As long as the application of physics principle is sound, full credit will be awarded.

2.2 NON-RECTILINEAR MOTION (2-D Motion)

2.2.1 Projectile Motion

When an object moves in both the x- and y-directions simultaneously under a constant acceleration, the object moves in a two-dimensional path that is parabolic in shape.

Consider a ball that is kicked at an angle above the ground. The motion picture of the ball is shown in Fig 2.2, where instantaneous pictures of the ball is taken at regular time intervals.

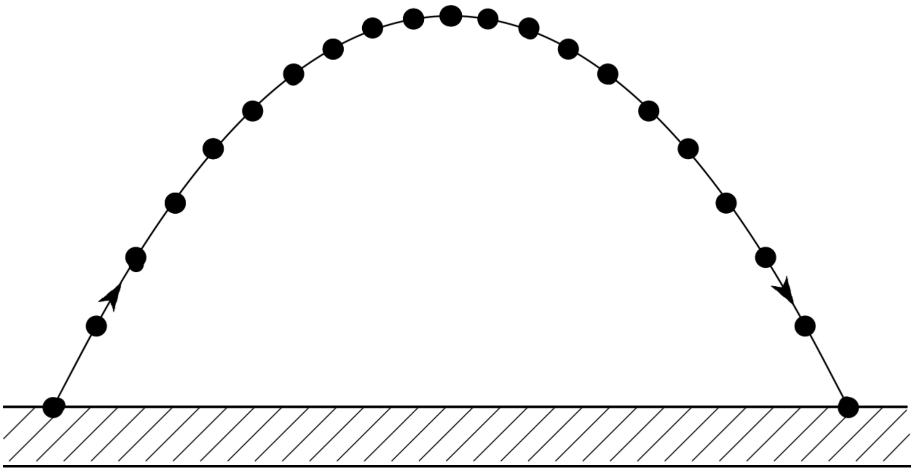


Fig. 2.2 Motion diagram showing a parabolic path moved by the ball

Consider the forces acting on the ball that was kicked. For the above motion, air resistance is negligible and so the only force acting on the ball during its flight is its own weight.

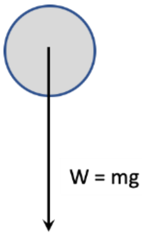


Fig. 2.3 Free Body Diagram of the ball

By Newton's Second Law, since the resultant force on the ball is its weight (mg) downwards, its acceleration is constant and equal to the acceleration due to gravity g directed downwards.

By definition, a *projectile* is any object upon which the only force is gravity, i.e. air resistance is considered negligible. An object dropped from rest, OR thrown vertically upward, OR thrown horizontally or upward/downward at an angle to the horizontal, are all called projectiles (provided air resistance is negligible). **The trajectory (path travelled) by a projectile is either linear or parabolic in shape.**

In general, for an object to move in a parabolic path, it fulfils the following two conditions:

(1) the acceleration remains constant (in both magnitude and direction) throughout the motion; and

(2) the INITIAL velocity is not parallel (nor anti-parallel) to the acceleration.

2.2.1.1 Displacement, Velocity and Acceleration vectors in 2-D projectile motion

The trajectory can be analysed at 5 separate points of the path: A, B, C, D and E, where A is the point *just after* the ball is kicked and point E is *just before* the ball touches the ground again as shown in Fig. 2.4.

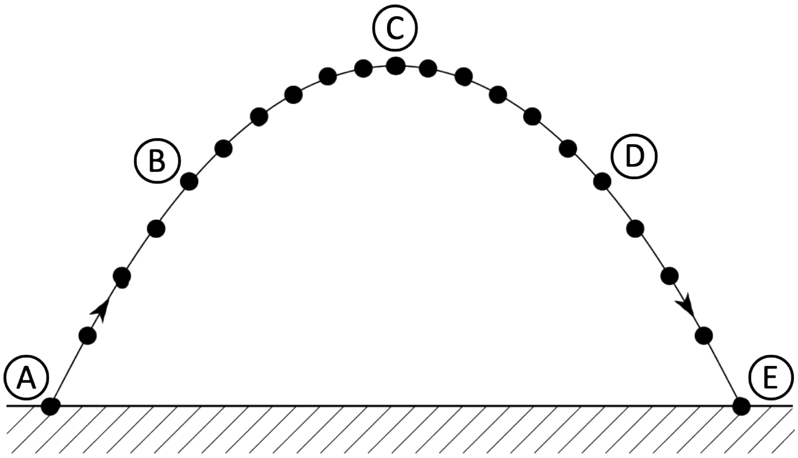


Fig. 2.4: The trajectory in 5 different points as in A, B, C, D and E

Setting up the stage

Set the positive x-direction to be horizontal and to the right, and the y-direction to be vertical and positive upward.

Resolve the motion into two perpendicular directions: *horizontal and vertical motions, which are independent of each other*. This means that motion in one direction has no effect on motion in the other direction.

Horizontal Axis

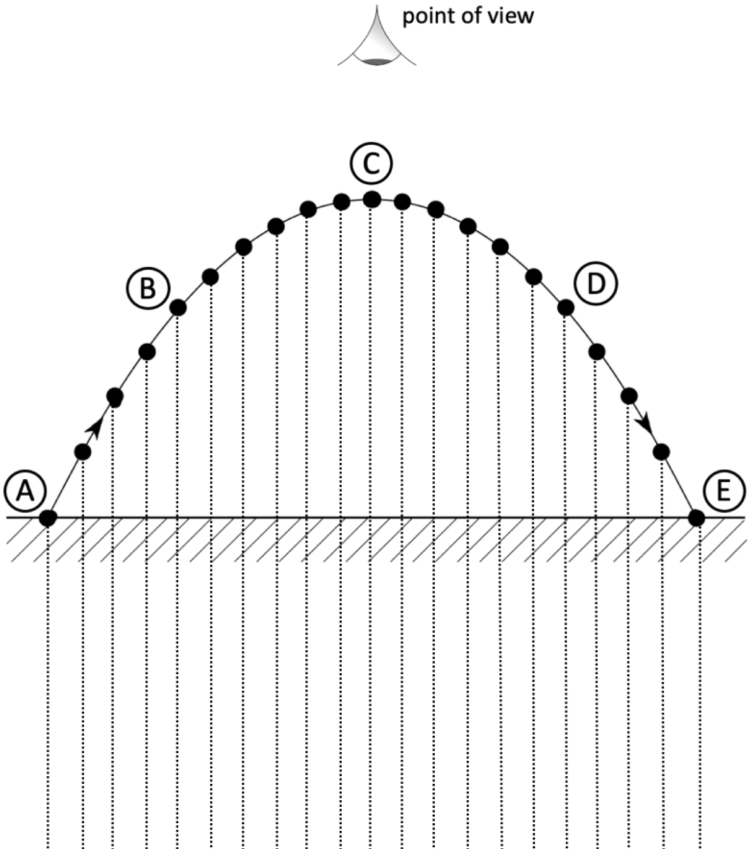


Fig. 2.5a: Point of view to analyse the horizontal motion of the ball's trajectory

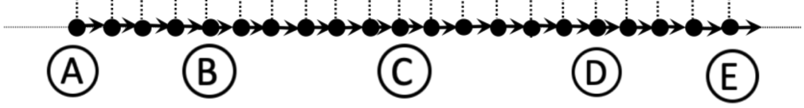


Fig. 2.5b: **Motion diagram for the horizontal axis**

Fig. 2.5a and 2.5b represents the horizontal movement of the ball. As shown in Fig. 2.3, the only acceleration on the ball is the acceleration due to gravity which acts in the vertical direction. Therefore, the horizontal velocity of the ball is unchanged (both in magnitude and in direction). As a result, from the point of view as in Fig. 2.5a, the ball seems to be moving in a horizontal straight line with a *constant velocity*. This will therefore give rise to the motion diagram as shown in Fig. 2.5b.

Vertical Axis

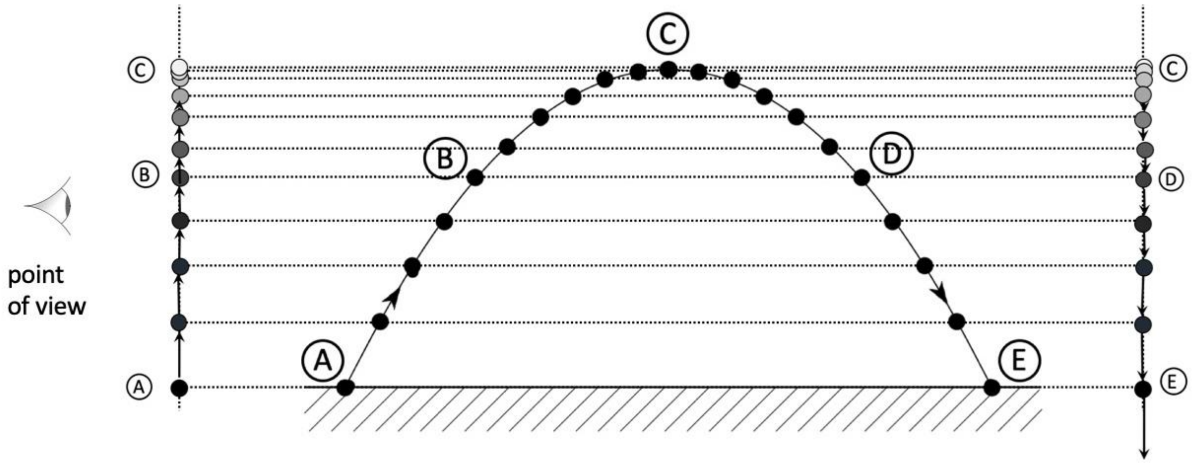


Fig. 2.6a: Motion diagram for the <u>vertical upward</u> path

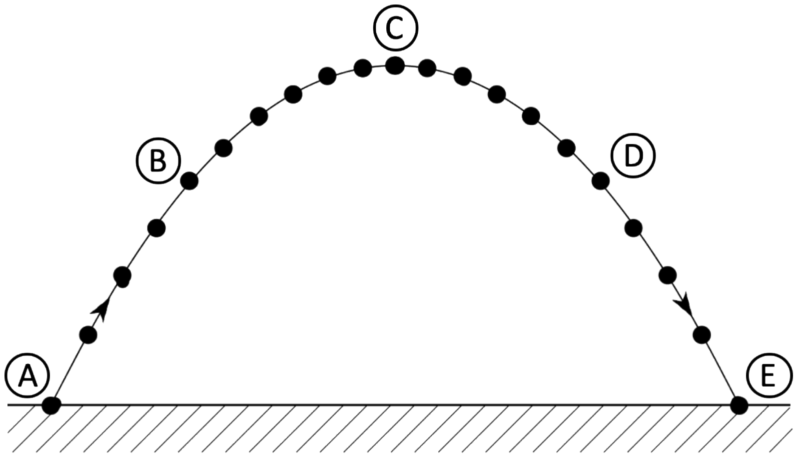
Fig. 2.6b: Motion diagram for the <u>vertical downward</u> path

If the ball is tossed in a parabolic path, as in Fig. 2.2, <u>from the point of view in Fig. 2.6, the motion in the *y*-direction will look like a ball tossed vertically upward under the influence of gravity (vertical motion with *constant acceleration*)</u>. This is depicted in Fig. 2.6a and Fig. 2.6b for the upward and downward flights.

In addition, in the absence of air resistance, the paths for the upward and downward flights are symmetrical. The vertical velocity vectors will decrease in length on the upward flight, whereas the velocity vectors increases in length on the downward flight. This is expected as the acceleration vector due to gravity is constant and directed downwards throughout the downward and upward flights.

Quiz

For the scenario of a ball being projected in a parabolic path from point A, draw vectors to represent the displacement, velocity and acceleration at positions A, B, C, D and E.



2.2.1.2 Mathematical Representation of the displacement, velocity and acceleration of a projectile

Following the pictorial representation in the above sections, we now delve deeper, analysing the projectile motion using the mathematical representation.

Fig 2.7 depicts the path of a projectile together with the velocity vectors along the trajectory.

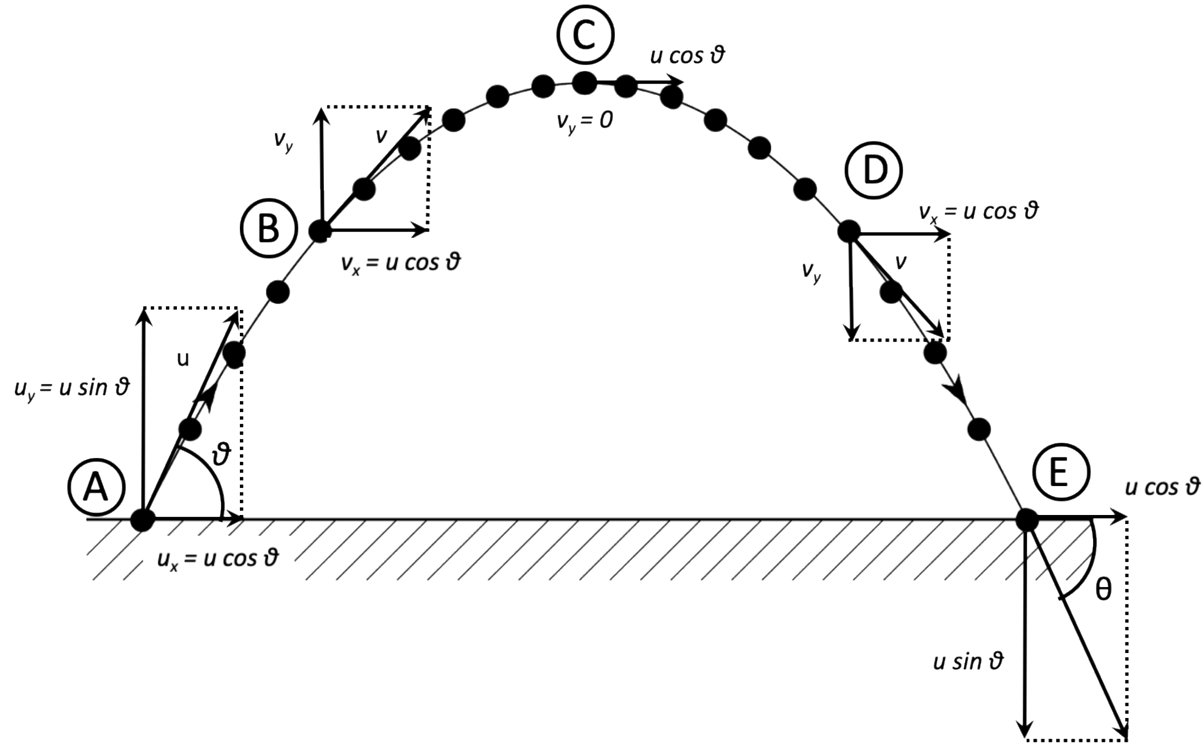


Fig. 2.7: Components of the velocity changing with the different positions of the trajectory

Consider a projectile, launched at point A at an angle θ to the horizontal with a velocity of *u*. Resolving velocity *u* into a horizontal x-component and vertical y-component gives:

$$u_x = u\cos\theta \qquad \text{and} \qquad u_y = u\sin\theta$$

When drawing the velocity vectors:

1. Subscript *"x"* denotes the horizontal component and subscript *"y"* denotes the vertical component.
2. The y-component of the velocity v_y changes in magnitude with time, caused by the acceleration due to gravity directed vertically downwards.
3. The *x*-component of the velocity v_x remains constant and equal to its initial velocity u_x.
4. v_y equals to zero at the peak of the trajectory.

Instantaneous velocity *v* is **tangential** to the parabolic path and it is the **vector sum** of its components. At any instant,

Magnitude of *v* is given by: $v = \sqrt{v_x^{\,2} + v_y^{\,2}}$

Direction of *v* can be given by: $\theta = tan^{-1}\left(\frac{v_y}{v_x}\right)$, measured from the horizontal vector.

Analysing the variation of the horizontal and vertical components of the displacement and velocity vectors

As the acceleration of the vertical motion and horizontal motion is constant in each case ($g = 9.81 \text{ m s}^{-2}$ for vertical motion and 0 m s^{-2} for horizontal motion), we are therefore able to apply the kinematics equations of motion learnt in the previous sections to determine the instantaneous velocity, acceleration and displacement of a projectile.

Kinematics Equation used	**Horizontal Motion:** Assume + → and reference point to be initial position of the object. $a_x = 0$, $u_x = u\cos\theta$	**Vertical Motion:** Assume + ↑ and reference point to be initial position of the object. $a_y = -g$, $u_y = u\sin\theta$
$v = u + at$	$v_x = u_x + a_x t$ $= u_x + (0)t$ $= u_x = u\cos\theta$ = constant	$v_y = u_y + a_y t$ $= u\sin\theta - gt$
$v^2 = u^2 + 2as$	$v_x^2 = u_x^2 + 2a_x s_x$ $= u_x^2 + 2(0)s_x = u_x^2$ $v_x = u_x = u\cos\theta$ = constant	$v_y^2 = u_y^2 + 2a_y s_y$ $= (u\sin\theta)^2 - 2gs_y$
$s = ut + \frac{1}{2}at^2$	$s_x = u_x t + \frac{1}{2}a_x t^2$ $= u_x t + \frac{1}{2}(0)t^2 = u_x t$ $= (u\cos\theta)t$	$s_y = u_y t + \frac{1}{2}a_y t^2$ $= (u\sin\theta)t - \frac{1}{2}gt^2$

In summary, defining the <u>reference point</u> to be <u>initial</u> position of the object,

<u>Horizontal motion: (Constant Velocity)</u>	**<u>Vertical motion: (Constant Acceleration)</u>**
Assume + → $\boldsymbol{a_x = 0}$ $\boldsymbol{u_x = u\cos\theta = \text{constant}}$ $\boldsymbol{s_x = u_x . t = u\cos\theta . t}$	**Assume + ↑** $\boldsymbol{v_y = u_y + a_y t}$ $\boldsymbol{v_y^2 = u_y^2 + 2a_y s_y}$ $\boldsymbol{s_y = u_y t + \frac{1}{2} a_y t^2}$ ***where:*** $\boldsymbol{a_y = -g}$ $\boldsymbol{u_y = u\sin\theta}$

Note:

1. The vertical and horizontal components are **independent** of each other. Thus, the equations of motion are applied to the vertical and horizontal motions separately.
2. At the top of the projectile motion, $v_y = 0$ and $v = v_x$, while acceleration is still g. (Acceleration remains constant and equal to g at every point in the projectile motion.)
3. a_y may not be equal to g in some cases (e.g. a charged particle moving non-parallel to a uniform electric field).

Problem-Solving Strategy for Projectile Motion

1. **Select** a coordinate system and **sketch** the path of the projectile, including initial and final positions, velocities, and accelerations.
2. **Resolve** the initial velocity vector into *x*- and *y*-components.
3. **Treat** the horizontal motion and the vertical motion independently.
4. **Follow** the techniques for solving problems with constant velocity to analyse the horizontal motion of the projectile.
5. **Follow** the techniques for solving problems with constant acceleration to analyse the vertical motion of the projectile.

Example 19

A long jumper leaves the ground at an angle of 20.0° to the horizontal and at a speed of 11.0 m s^{-1}.

(a) How long does it take for him to reach maximum height?
(b) What is the maximum height?
(c) How far does he jump? (Assume his motion is equivalent to that of a particle, disregarding the motion of his arms and legs.)

Solution:

[Thought processes]

We take the projectile equations, fill in the known quantities, and solve for the unknowns.

At the maximum height, the velocity in the y-direction is zero, so setting "v_y = = u sin θ – gt" equal to zero and solving gives the time it takes the jumper to reach his maximum height. By symmetry, his trajectory starts and ends at the same height, doubling this time gives the total time of the jump.

Part	Strategy	Solution
(a)	Find the time t_{max} taken to reach maximum height. Set v_y = 0 in "$v_y = u \sin\theta - gt$" and solve for t_{max}:	$v_y = u \sin\theta - gt = 0$ $t_{max} = \frac{u \sin\theta}{g}$ $= \frac{11 \sin 20.0}{9.81} = 0.384$ s
(b)	Substitute the time t_{max} into "$s_y = (u \sin\theta)t - \frac{1}{2}gt^2$":	$s_y = (u \sin\theta)t_{max} - \frac{1}{2}gt_{max}^2$ $= (11 \sin 20.0)(0.384) - \frac{1}{2}(9.81)(0.384)^2$ $= 0.722$ m
(c)	First find the time for the jump, which is twice t_{max}:	$t = 2t_{max}$
	Substitute this result into the equation for the *x*-displacement:	$s_x = (u \cos\theta)(2t_{max})$ $= (11 \cos 20.0)(2 \times 0.384)$ $= 7.94$ m

2.2.1.3 Other scenarios for Projectile Motion

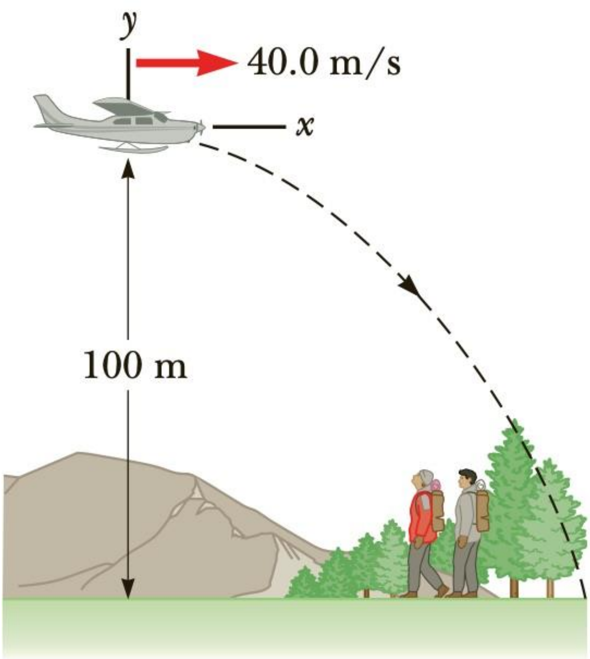
In the previous sections the trajectory of a ball kicked from a level ground is analysed and one of the many different scenarios that can use the kinematics equations is discussed. Further scenarios are discussed in this subsection.

Alternate Scenario 1: Object initially projected horizontally

A body is projected horizontally from an elevated platform.

Example 20

An Alaskan rescue plane drops a package of emergency rations to stranded hikers, as shown in the figure. The plane is travelling horizontally at 40.0 m s^{-1} at a height of 100 m above the ground.



Neglect air resistance.

(a) Where does the package strike the ground relative to the point at which it was released?

(b) What are the horizontal and vertical components of the velocity of the package just before it hits the ground?

(c) What is the angle of the impact?

Solution:

[Thought processes]

Here, we're just taking kinematics equations, filling in known quantities, and solving for the remaining unknown quantities.

In part (a), set the y-component of the displacement equations equal to -100 m — we take the s_y to be 0 at the point of release and the ground level where the package lands — and solve for the time it takes the package to reach the ground. Substitute this time into the displacement equation for the x-component to find the range. "Range" usually refers to the maximum horizontal displacement travelled by the object in projectile motion.

In part (b), substitute the time found in part (a) into the velocity components. Notice that the initial velocity has only an x-component, which simplifies the math. Solving part (c) requires the inverse tangent function.

Part	Strategy	Solution
(a)	Find the range of the package using the horizontal motion equation $s_x = u_x t$.	
	u_x is known: Since the package was 'dropped', its initial velocity is equal to the plane's velocity, i.e. u = 40.0 m s^{-1}. And since the plane's velocity was horizontal, it means that $u_y = 0$ and $u_x = u$ = 40.0 m s^{-1} .	$u_y = 0$ and $u_x = u$ = 40.0 m s^{-1}
	This leaves us to find t: Consider the vertical motion equations. Use "$s_y = (u \sin \vartheta)t - \frac{1}{2}gt^2$". Substitute s_y = -100 m and u_y = 0. Solve for t.	$s_y = u_y t - \frac{1}{2}gt^2$ $-100 = (0)t - \frac{1}{2}(9.81)(t)^2$ t = 4.52 s
	Substitute u_x and t into $s_x = u_x t$	$s_x = u_x t = (40.0)(4.52)$ = 181 m
(b)	Find the x-component of the velocity at the time of impact using the horizontal motion equation.	$v_x = u_x$ = 40.0 m s^{-1}
	Find the y-component of the velocity at the time of impact using a vertical motion equation.	$v_y = u_y - gt = 0 - (9.81)(4.52)$ = – 44.3 m s^{-1} *(The negative sign in the answer indicates that the vertical velocity is directed downwards, as expected.)*
(c)	Sketch the velocity vector at the instant just before it strikes the ground. Resolve this velocity vector into its x- and y- components. Using this vector diagram and the answers in (b), use a relevant trigo equation to find θ.	$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{44.3}{40.0}\right)$ = 48.0 ° Package strikes the ground at an angle of 48.0°, measured clockwise with respected to the positive x-direction. *(It is good practice to draw θ in the diagram to supplement the answer for greater clarity.)*

Alternate Scenario 2: Object initially projected at an angle on an elevated platform

When an object is projected at a angle on an elevated platform, there is a possibility to have the object finally land at a negative displacement, as shown in the following example.

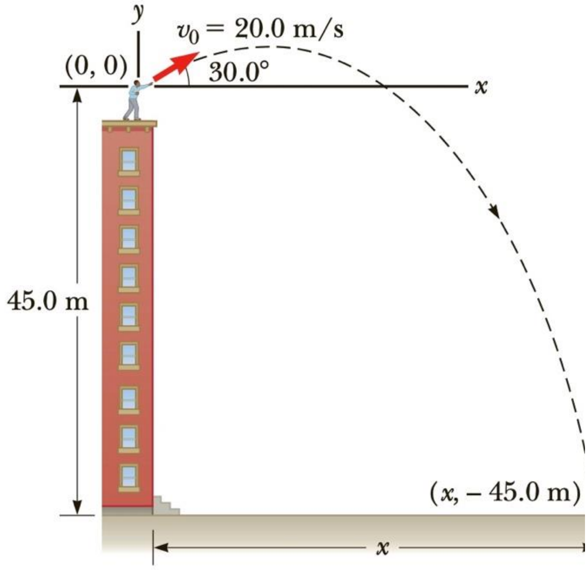
Example 21

A ball is thrown upward from the top of a building at an angle of 30.0° above the horizontal and with an initial speed of 20.0 m s^{-1}, as shown in the figure.
The point of release is 45.0 m above the ground.

(a) How long does it take for the ball to hit the ground?

(b) Find the ball's speed at impact.

(c) Find the horizontal range of the ball. Neglect air resistance.



Solution:

[Thought processes]

Choose the axes as in the figure, with the origin at the point of release.

*For **(a)**: fill in the constants of the kinematics equations for the y-displacement and set the displacement equal to -45.0 m, the y - displacement when the ball hits the ground.*

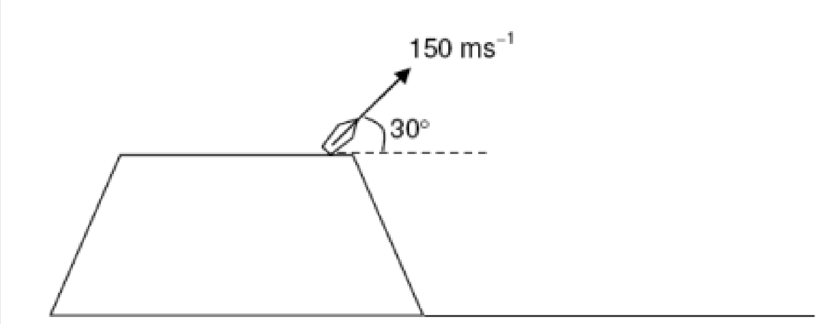
*Using the quadratic formula, solve for the time. To solve part **(b),** substitute the time from part **(a)** into the components of the velocity, and substitute the same time into the equation for the x-displacement to solve part **(c).***

Part	Strategy	Solution
(a)	Find the y-displacement, taking s_y = -45.0 m. Reorganize the equation into standard form to find the positive root of the time:	$s_y = u_y t + \frac{1}{2} a_y t^2$ $= (u \sin \theta)t - \frac{1}{2} g t^2$ $-45.0 = (20.0 \sin 30.0°)t - \frac{1}{2}(9.81)(t)^2$ $t = 4.22$ s
(b)	The speed is essentially the magnitude of the instantaneous velocity at that point.	
	Substitute the value of t found in part (a) into "$v_y = u_y + a_y t$" to find the y-component of the velocity at impact:	$v_y = u_y - gt$ $= u \sin \theta - gt$ $= 20.0 \sin 30.0 - (9.81)(4.22)$ $= -31.4$ m s^{-1}
	Use this value of v_y, and the Pythagorean theorem "$v = \sqrt{v_x^2 + v_y^2}$", and the fact that $v_x = u_x$ to find the speed of the ball at impact:	$v = \sqrt{u_x^2 + v_y^2}$ $= \sqrt{(20.0 \cos 30.0)^2 + (-31.4)^2}$ $= 35.9$ m s^{-1}
(c)	Substitute the time of flight into the range equation:	$s_x = u_x t$ $= (20.0 \cos 30.0)(4.22)$ $= 73.1$ m (Note to calculate range, the time used should be the total flight time of the projectile.)

MINI-TEST 3

1. A grasshopper jumps at an angle of 30° to the horizontal with a take-off speed of 3.0 m s^{-1}.
   (a) What is the height of its jump?
   (b) How long is it above the ground?
   (c) What is the range of its jump?

2. A projectile is fired from the top of a cliff with a velocity of 150 m s^{-1} at 30° to the horizontal as shown in the diagram. If the time of flight is 22.5 s, what is the height of the cliff?



3. A tennis ball is thrown horizontally at 10 m s^{-1} from a height of 2.0 m above flat horizontal ground. How far away from the thrower does it first hit the ground?

My solutions:

Quiz

The diagram shows the parabolic path of a ball thrown in air. On the same diagram, draw the path of the ball if air resistance is **not** negligible.

Solution:

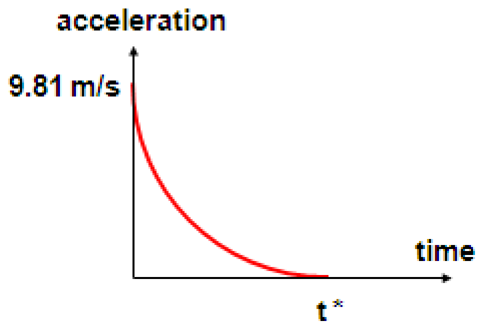
- ➔ *Knowledge that both horizontal & vertical velocities are affected. Horizontal velocity decreases to ZERO (if flight time is long enough).*
- ➔ *Asymmetric (horizontal distance travelled per unit time DECREASES due to horizontal deceleration), reaches maximum height earlier, lower maximum height, shorter range*

CHAPTER SUMMARY / SELF-CHECK

1. What is the difference between scalar quantities and vector quantities?
2. What is the difference between displacement and distance?
3. What is the difference between speed and velocity?
4. What is the difference between 'average' and 'instantaneous' values?
5. What graphical approach could be used to determine 'average' and 'instantaneous' values?
6. What does gradient represent on a
    a. Displacement-time graph?
    b. Velocity-time graph?
7. What does the area under the graph represent on a
    a. Velocity-time graph?
    b. Acceleration-time graph?
8. Given any one of the graphs (displacement-time, velocity-time and acceleration-time), how do I go about sketching the other two graphs?
9. What are the cause-and-effect relationship between force, acceleration, velocity and displacement?
10. For an object that is speeding up, what is the relationship between the acceleration and velocity vectors? What about for an object that is slowing down?
11. State the three equations of motion.
12. How are the three equations of motion derived?
13. Under what conditions are the equations of motion applicable?
14. In the equations of motion, from which point in a body's motion is time t measured from?
15. What is a general problem solving strategy for kinematics problems?
16. Consider the example where an object is projected vertically upwards. With the aid of free-body diagrams, describe and explain how the displacement, velocity and acceleration of a free-falling body in the presence of air resistance varies with time. Sketch the graphs when air resistance is considered, and, when air resistance is ignored.
17. What is 'terminal velocity'? How does it arise?
18. What is a 'projectile'?
19. What is the acceleration of a 'projectile'? Does its acceleration vary with time?
20. How can a 2-D projectile motion be analysed?
21. Write down the equations that can be applied to solve 2-D projectile motion problems.
22. When an object is projected horizontally or obliquely in a gravitational field (ignore air resistance), what is the shape of its trajectory?
23. Sketch vectors to illustrate the displacement, velocity and acceleration vectors at different positions of a 2-D projectile motion.

APPENDIX A

Acceleration-time graph for an object moving under gravity WITH air resistance



Notice that the a-t graph is a curve not a straight line, why is this so?

- Taking downwards as positive:
  From Newton's 2nd law:
  mg – kv = ma ------ (1)

- Differentiate equation (1) with respect to time t
  -k(dv/dt) = m(da/dt) [dv/dt = a]
  -ka = m(da/dt)
  da/dt = -ka/m

- Note that da/dt represents the gradient of the a-t graph. The negative sign indicates that the graph is downward sloping.

- As t increases with time, a decreases from g to 0.

- Hence the value of gradient da/dt decreases with time, and ultimately gradient = 0 when terminal velocity is reached.

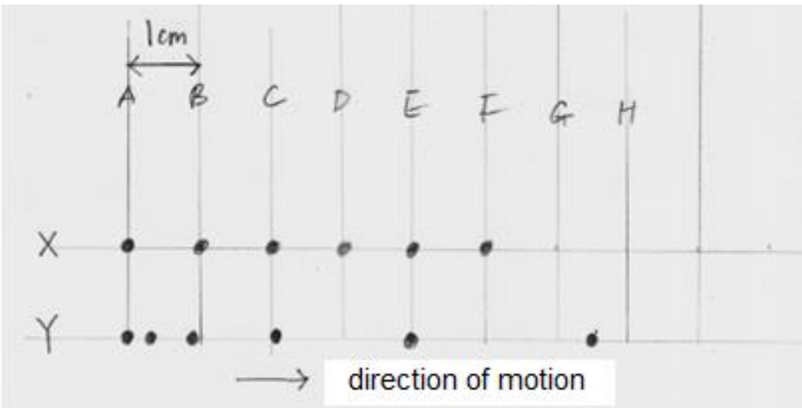
Hence a-t graph is a curve whose gradient is negative and decreases with time.

Kinematics Tutorial

Rectilinear Motions

Interpreting Motion Diagram

Q1. Two spheres X and Y are rolling along a plane in the direction shown in the diagram below. The line markers from A to H are equally spaced at 1 cm apart. A series of stroboscopic images of the two spheres are recorded on a single photograph. The stroboscope captures an image every 2s. Sphere Y starts from rest at the line marker A.



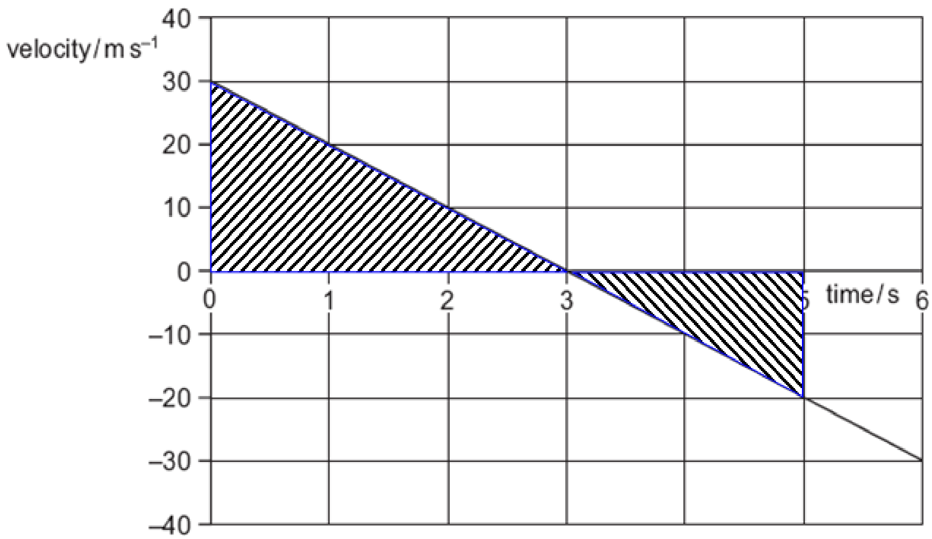
(i) Plot a displacement versus time graph for the spheres X and Y. [3]

(ii) At what time are the two spheres travelling at the same speed? [1]

(iii) Where is each sphere when the two spheres have the same speed? [1]

Graphical Representations of Motion

Q2. A stone is thrown vertically upwards. A student plots the variation with time of its velocity.

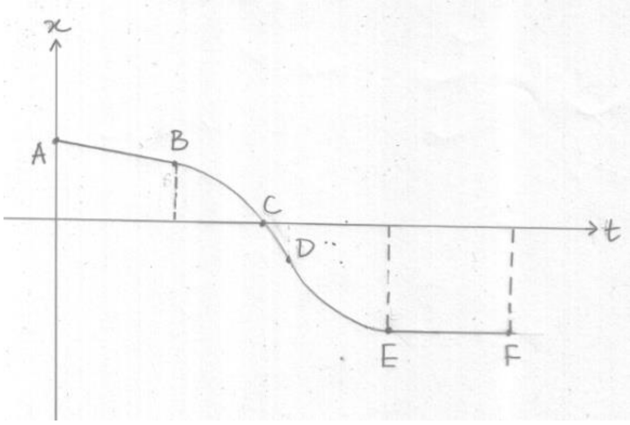


What is the vertical displacement of the stone from its starting point after 5 seconds?

A 20 m **B** 25 m **C** 45 m **D** 65 m

Q3. The displacement-time graph of a moving object is shown below. Sketch the velocity-time graph and acceleration-time graph for this motion. Label times A-F on your time axis.

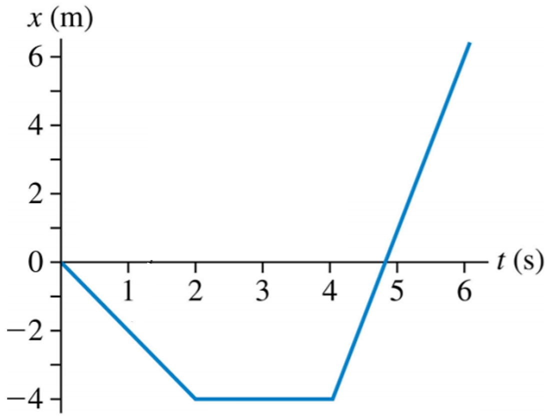
- Note that: the line from A to B is straight.
- Assume that the velocity from B to D is changing at a constant rate.
- Assume that the velocity from D to E is changing at a constant rate. [4]



Displacement-time graph

Describe Motion

Q4. The variation with time *t* of a car's displacement *x* is shown on the graph below.



(a) Draw the car's velocity v_x against time *t* graph. [2]

(b) Describe the car's motion, assuming the car moves along a straight line. (Take the forward direction as positive.) [4]

Equations of Motion (for constant accelerated motion)

Q5. N08/I/4

A metal ball is dropped from rest over a bed of sand. It hits the sand bed one second later and makes an impression of maximum depth 8.0 mm in the sand.

Air resistance is negligible.

On hitting the sand, what is the average deceleration of the ball?

A 6.0×10^2 m s^{-2}
B 1.2×10^3 m s^{-2}
C 6.0×10^3 m s^{-2}
D 1.2×10^4 m s^{-2}

Rectilinear Motion under Gravity (without air resistance)

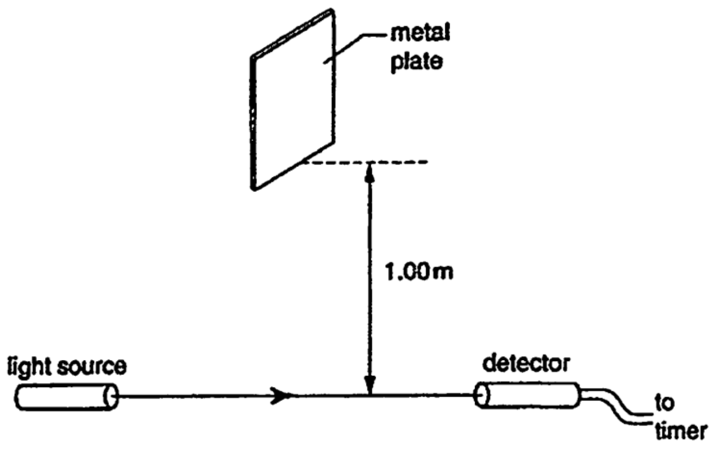
Q6. A skydiver jumps out of a helicopter. A few seconds later, another skydiver jumps out, so that they both fall along the same vertical line. Ignore air resistance, so that both skydivers fall with the same acceleration.

(i) Does the vertical distance between them
A) increase,
B) decrease,
C) stay the same? [1]

(ii) Does the difference in their velocities
A) increase,
B) decrease,
C) stay the same? [1]

Q7. A balloon is 30.0 m above the ground and is rising vertically with a uniform speed when a coin is dropped from it. If the coin reaches the ground in 4.00 s, what is the speed of the balloon? [12.1 m s^{-1}] [2]

Q8. N05/II/1
A student wishes to measure the length of a metal plate. The only equipment available is an electronic timer controlled by a light beam and a rod of 1.00 m long. Using the rod, the student positions the plate so that its lower edge is 1.00 m above the light beam, as shown below.



The metal plate is release and the timer starts to record as the light beam is cut. The total time for the plate to pass through the beam is 0.052s.
The student is told that the local value for acceleration of free fall is 9.79 m s^{-2}.

(a) (i) Show that the time for the bottom edge of the plate to reach the light beam is 0.452 s. [1]

(ii) Calculate the length of the metal plate, giving your answer to an appropriate number of significant figures. [0.24 m] [4]

(b) Suggest two reasons why the time for the bottom edge of the plate to reach the light beam may differ from that quoted in (a)(i). [2]

Rectilinear Motion under Gravity (<u>with</u> air resistance)

Q9. A ball is thrown vertically upwards from ground level. Air resistance is **not** negligible. The variation with time t of the vertical velocity v of the ball is shown in Fig. 6.1.

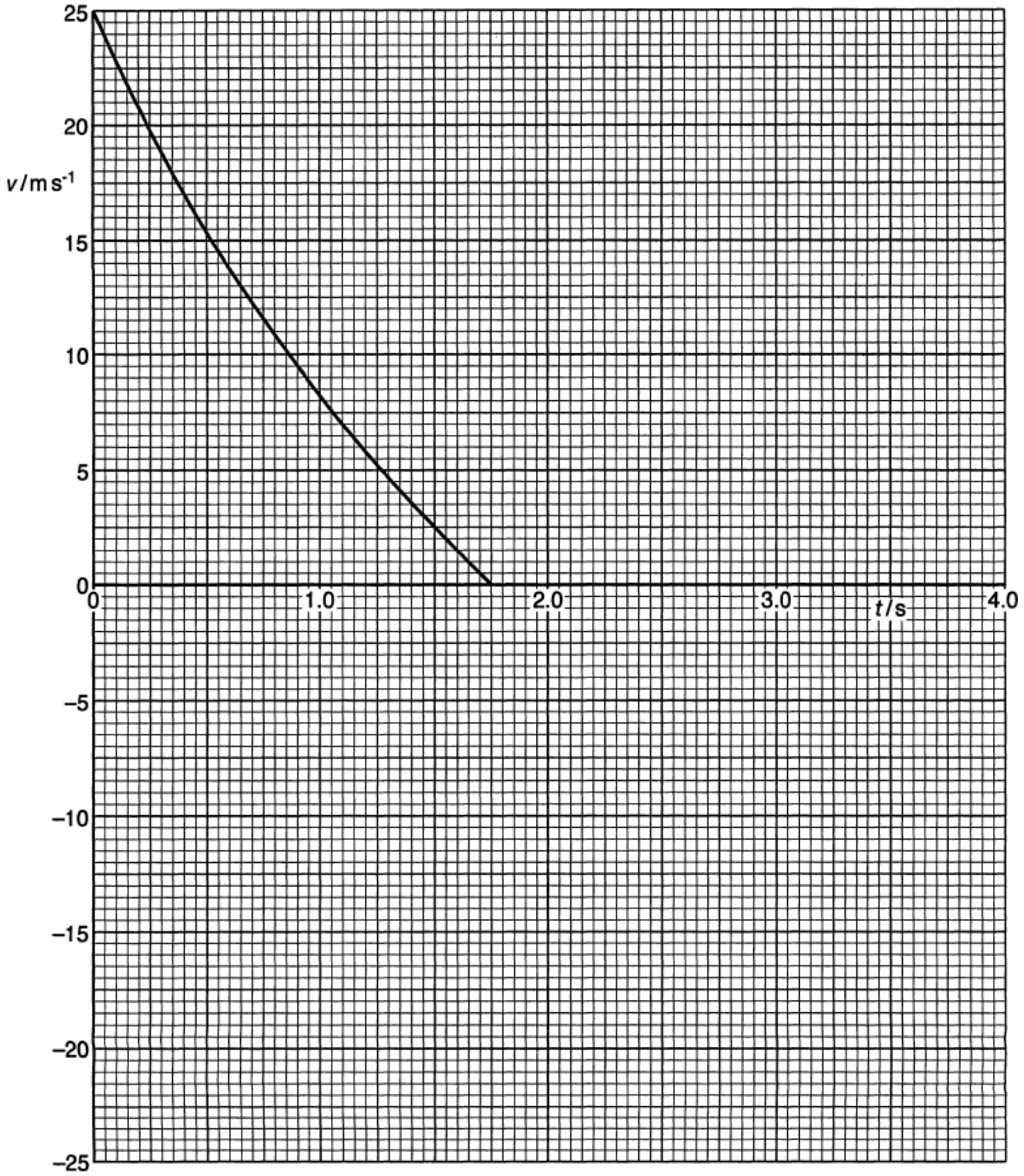


Fig. 6.1

(a) Use Fig. 6.1 to explain how it may be deduced that air resistance varies with speed. [2]

(b) State and explain, without any calculation, the feature of Fig. 6.1 that enables the magnitude of the acceleration of free fall to be determined. [2]

(c) (i) Use Fig. 6.1 to determine the maximum height reached by the ball. [4]

Use your answer in (i) to calculate the ratio

(ii) $\dfrac{\text{energy lost from the ball due to air resistance during the ball's upward motion}}{\text{initial kinetic energy of the ball}}$ [4]

(d) The ball has mass 350 g.
For the instant when this ball is travelling at 10 m s^{-1},
(i) Use Fig. 6.1 to show that the acceleration of the ball is approximately -13 m s^{-2}, [2]
(ii) calculate the magnitude of the force due to air resistance on the ball. [3]

(e) The ball returns to the ground at t = 4.0 s. On Fig. 6.1., sketch the graph to show the variation with time *t* of the velocity *v* of the ball as it falls back to ground level. [3]

<u>Non-Rectilinear Motions</u>

Projectile Motion in 2-D

Q10. N04/II/1a
A student throws a ball from point S to a friend at point F. The path of the ball is shown in the figure.

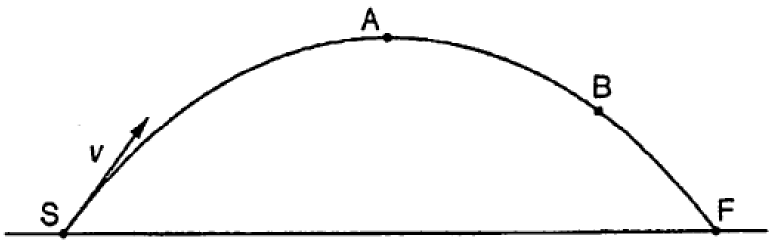
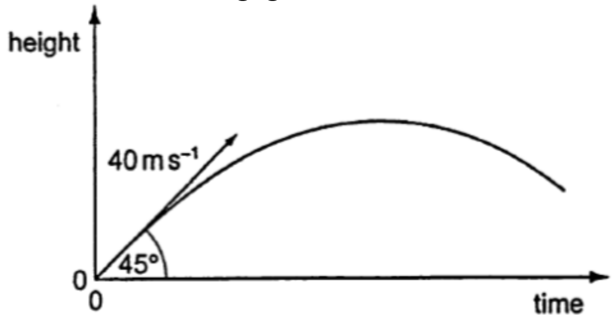


Fig. 11

The points S and F are on the same horizontal level. Air resistance is negligible. The ball is thrown from point S with velocity *v*, represented by the vector arrow shown on Fig. 11.
On Fig. 11,
(i) draw arrows from point S to represent the initial horizontal and vertical components of the velocity *v* (label these components v_H and v_V respectively), [1]
(ii) draw arrows at A and at B to represent the horizontal and vertical components of the velocity of the ball at these two points. [3]

Q11. 2009/I/5

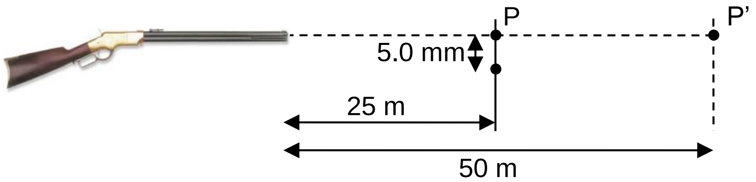
An object is projected with velocity 40 m s^{-1} at an angle of 45° to the horizontal. Air resistance is negligible.



What is the speed of the object after 5.0 s?

A 21 m s^{-1} **B** 28 m s^{-1} **C** 35 m s^{-1} **D** 49 m s^{-1}

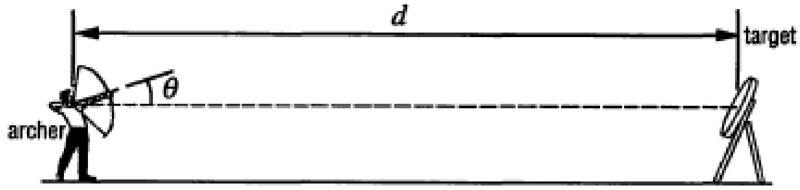
Q12. When a rifle is fired horizontally at a target P on a screen at a range of 25 m, the bullet strikes the screen at a point 5.0 mm below P. The screen is now moved to a distance of 50 m and the rifle again fired horizontally at P in its new position. Assuming that air resistance may be neglected, what is the new distance below P at which the screen would now be struck?



[3]

Q13. 2000/II/2c

The archer fires the arrow with an initial speed *v* and hits a target which is a distance *d* away and on the same horizontal level as the bow, as illustrated in the figure below.



The arrow is aimed so that, on release, it makes an angle ϑ with the horizontal.

(i) Assuming air resistance to be negligible, write down an expression for

1. *d* in terms of *v*, θ and the time of flight *t*, [1]
2. the time of flight *t* in terms of *v*, θ and the acceleration of free fall *g*. [2]

(ii) The distance *d* is given by the expression

$$d = \frac{v^2 \sin 2\theta}{g}$$

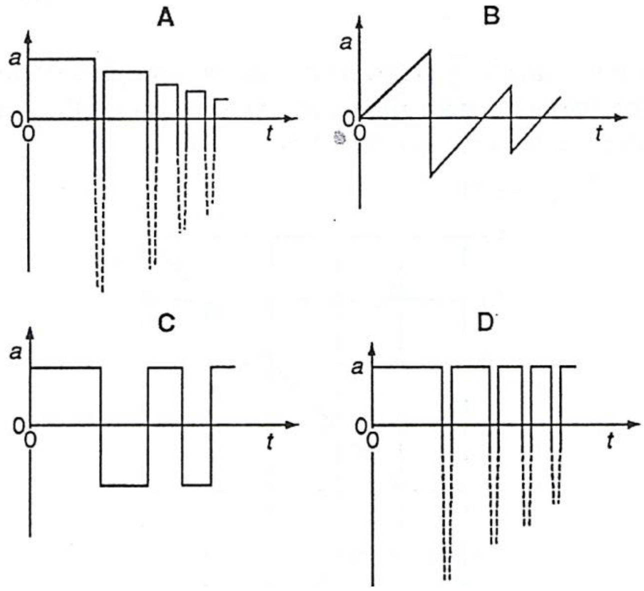
Calculate the angle θ for an arrow with initial speed v = 32 m s^{-1} and a target at a distance *d* of 94 m from the bow. [32.1° or 57.9°] [2]

(iii) Suggest with a reason, whether the angle θ would, in practice, be larger or smaller than that calculated in (ii) for the arrow to hit the target. [3]

<u>Assignment Questions</u>

A1 (a) N2000/I/4; N05/I/3

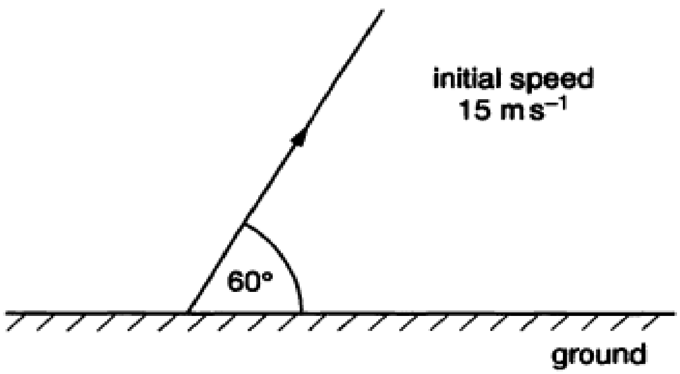
A steel ball is released from rest a distance above a rigid horizontal surface and is allowed to bounce. Which graph best represents the variation with time t of acceleration a?



(b) **Extension:** Sketch, describe & explain how the displacement-time and velocity-time graph look like for the same motion.

A2. N97/III/1b

A ball is thrown from horizontal ground with an initial velocity of 15 m s^{-1} at an angle of 60° to the horizontal, as shown below.



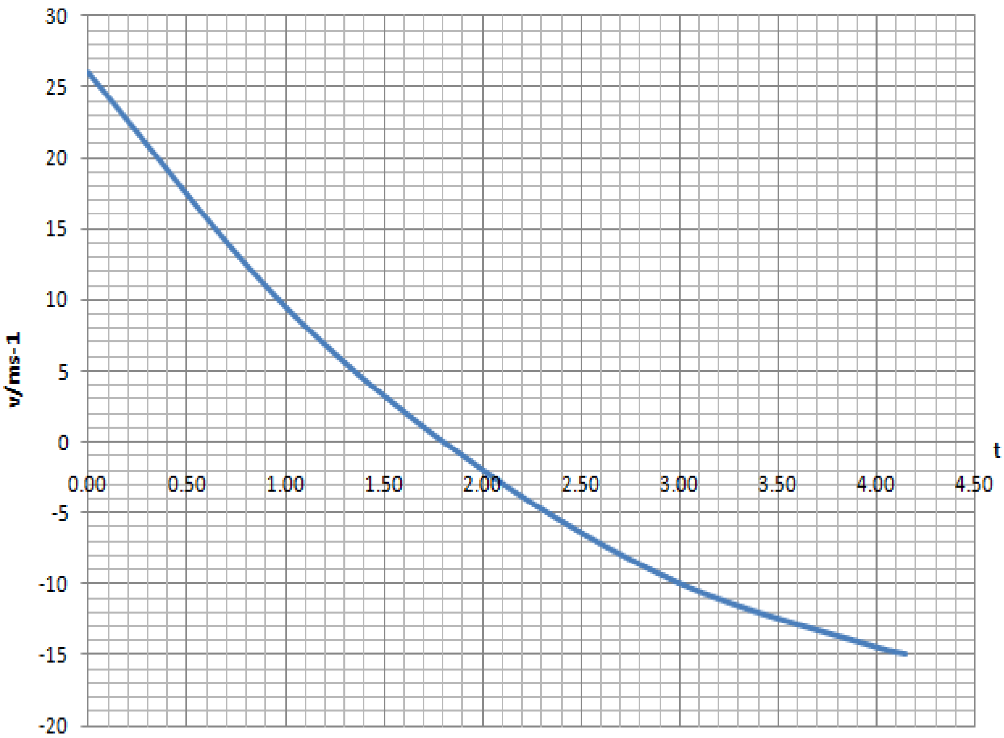
(i) Calculate, for this ball, the initial values of

1. the vertical component of the velocity, [13.0 m s^{-1}] [1]
2. the horizontal component of the velocity. [7.5 m s^{-1}] [2]

(ii) Assuming that air resistance can be neglected, use your answers in (i) to determine

1. the maximum height to which the ball rises, [8.6 m] [2]
2. the time of flight, i.e. the time interval between the ball being thrown and returning to ground level, [2.65 s] [2]
3. the horizontal distance between the point from which the ball was thrown and the point where it strikes the ground. [19.9 m] [2]

A3. The graph below shows the variation with time t of the velocity of a ball from the moment it is thrown with a velocity of 26 m s^{-1} vertically upwards.



(i) State the time at which the ball reaches its maximum height. [1]

(ii) State the feature of a velocity time graph that enables the acceleration to be determined. [1]

(iii) Just after the ball leaves the thrower's hand, it has a downward acceleration of approximately 20 m s^{-2}. Explain how this is possible.

(iv) State the time at which the acceleration is g. Explain why the acceleration has this value only at this particular time. [2]

(v) Sketch the acceleration time graph for the motion. Show the value of g on the acceleration axis. [2]

(vi) Explain why, for all vertical throws, the time taken to reach maximum height must be shorter than the time taken to return to starting point. [2]

[4]

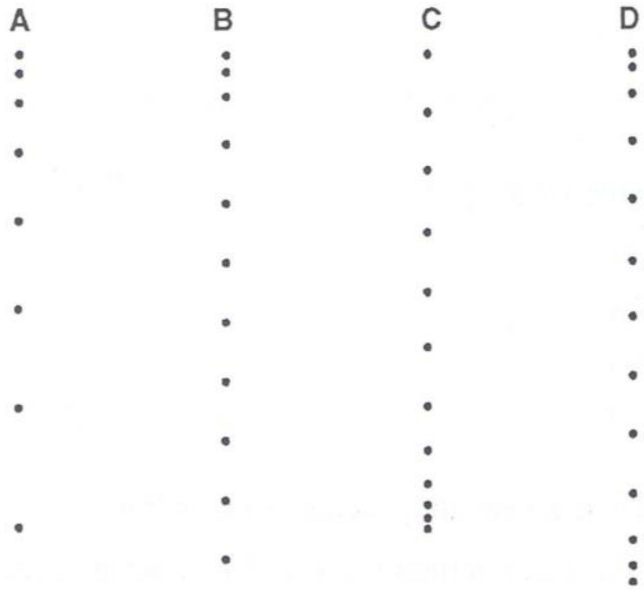
Supplementary Questions

Rectilinear Motions

Interpreting Motion Diagram

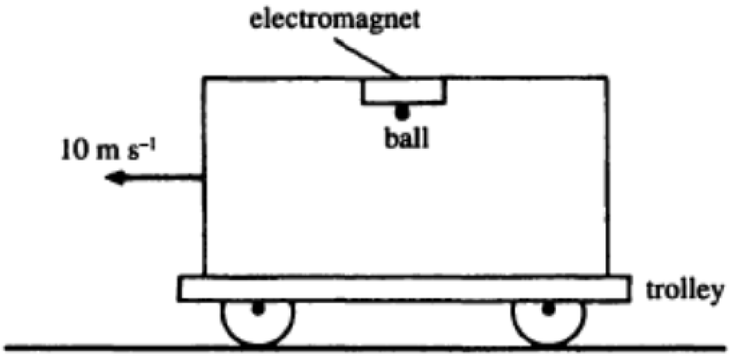
S1. 2004/I/4

A steel sphere is released from rest at the surface of a deep tank of viscous oil. A multiple exposure photograph is taken of the sphere as it falls.
The time interval between exposures is always the same.
Which of the following could represent this photograph?

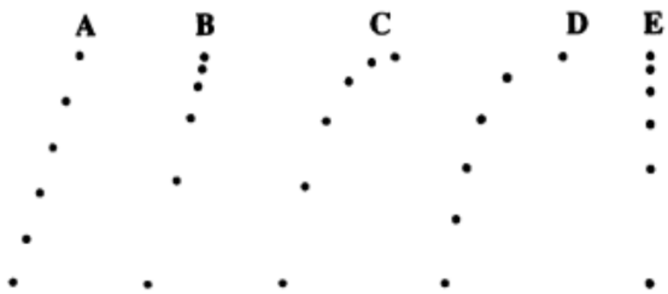


S2. 1991/I/4

A ball is suspended from an electromagnet attached to a trolley which is travelling at a constant speed of 10 m s^{-1} to the left. The trolley is illuminated by a stroboscope flashing at a constant rate. The diagram represents the viewpoint of a stationary camera.



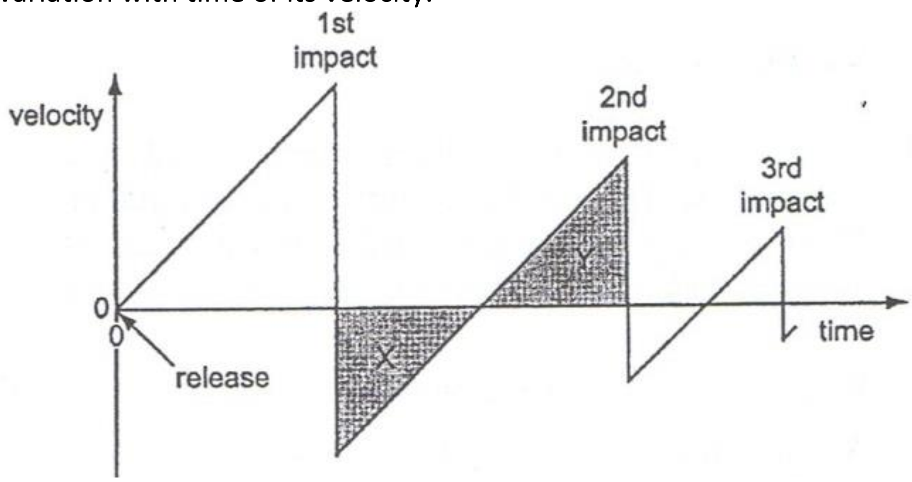
The ball is released and a series of stroboscopic images of the ball are recorded on a single photographic plate.
Which diagram best represents what is seen on the photographic plate?



Graphical Representations of Motion

S3. 2006/I4

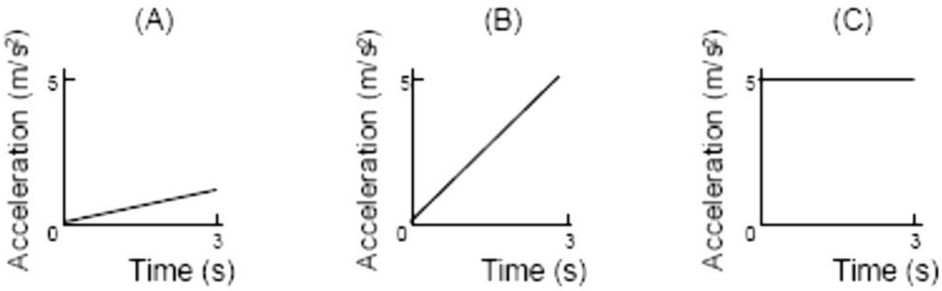
A ball is released from rest above a horizontal surface. The graph shows the variation with time of its velocity.

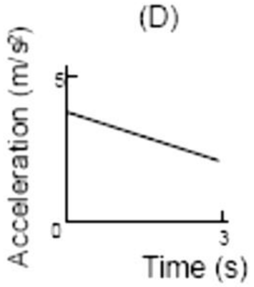


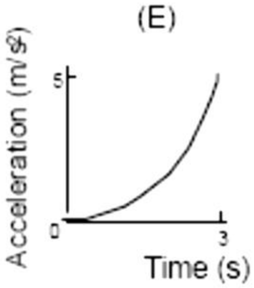
Why are areas X and Y equal?

A. For one impact, the speed at which the ball hits the surface equals the speed at which it leaves the surface.
B. The ball rises and falls through the same distance between impacts.
C. The ball's acceleration is the same during its upward and downward motion.
D. The speed at which the ball leaves the surface after an impact is equal to the speed at which it returns to the surface for the next impact.

S4. Five objects move according to the following acceleration versus time graphs. Which has the smallest change in velocity during the three second interval?

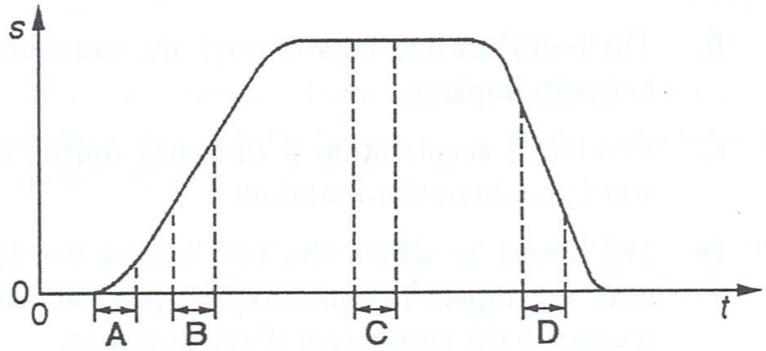






S5. 2002/I/3

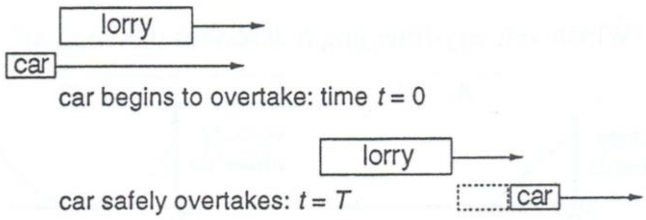
The graph shows the variation with time t of the displacement s of a vehicle moving along a straight line.



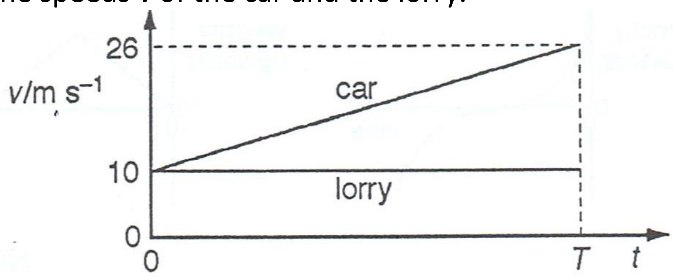
During which time interval does the acceleration of the vehicle have its greatest numerical value?

S6. 2002/1/4

The minimum time T required for a car safely to overtake a lorry on the motorway is measured from the time the front of the car is level with the rear of the lorry, until the rear of the passing car is a full car-length ahead of the lorry.



The car is 3.5 m long and the lorry is 17.0 m long. The graph shows the variation with time t of the speeds v of the car and the lorry.



What is the value of T?

A 0.86 s

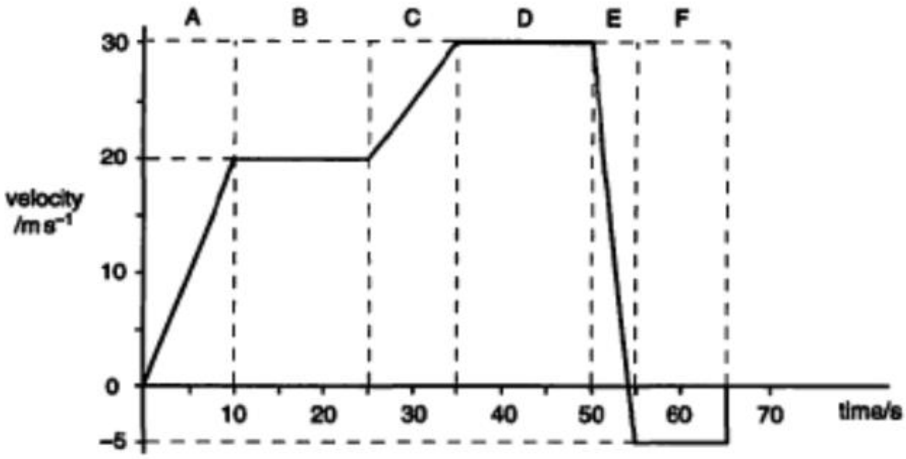
B 1.2 s

C 2.6 s

D 3.0 s

S7. 1997/II/1 (modified)

The figure shows a velocity-time graph for a journey lasting 65 s. It has been divided up into six sections for ease of reference.



(a) Using information from the graph obtain

(i) the acceleration in section A, [1]

(ii) the acceleration in section E, [1]

(iii) the total distance travelled, [2]

(iv) the displacement at time = 65 s. [2]

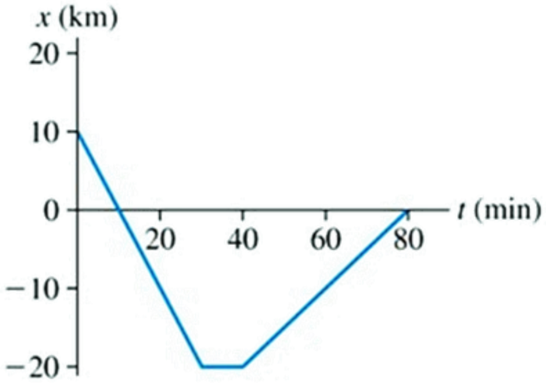
(b) Describe qualitatively in words what happens in sections E and F of the journey. [4]

(c) Sketch the shape of the corresponding displacement-time graph. Label the vertical axes with the values at time = 10s, 25s, 35s, 50s, 55s, 65s. [6]

(d) Sketch the shape of the corresponding acceleration-time graph. Label the vertical axes with the values at time = 10s, 25s, 35s, 50s, 55s, 65s. [6]

Describe Motion

S8. The graph below represents the variation with time *(t)* of a car's displacement *(x)* along a straight road. Direction to the right is the positive direction.



q

Describe the motion of the car. Use a motion diagram to visualize the car's position at various times.

S9. 2004/III/1b
Explain why it is technically incorrect to define speed as distance travelled per second. Include in your answer the correct statement defining speed. [2]

Equations of Motion (for constant accelerated motion)

S10. What are the necessary assumptions for usage of the equations of motion?

I. The particle must move in a straight line
II. The particle must move with constant velocity
III. The particle must move with increasing velocity
IV. The particle must move with constant acceleration

A III **B** I and IV **C** I, II and IV **D** I, III and IV

S11. 2013/1/3
A car accelerates uniformly from rest along a level road. The effects of air resistance on the car are negligible. The car travels 12 m in the time between 1 s and 2 s after starting. How far does it travel in the time between 3 s and 4 s after starting?

A 28 m **B** 35 m **C** 48 m **D** 64 m

S12. 2012/1/3
Two cars, initially next to each other and at rest, accelerate in the same straight line at different uniform rates. After 3 s. they are 12m apart. If they continue to accelerate at the same rate, how far apart will they be 6 s after they started?

A 18 m **B** 24 m **C** 36 m **D** 48 m

Rectilinear Motion under Gravity (without air resistance)

S13. An elevator ascends with an upward acceleration of 1 m s^{-2}. At the instant its upward speed is 2 m s^{-1}, a loose bolt drops from the ceiling of the elevator 3 m from the floor.

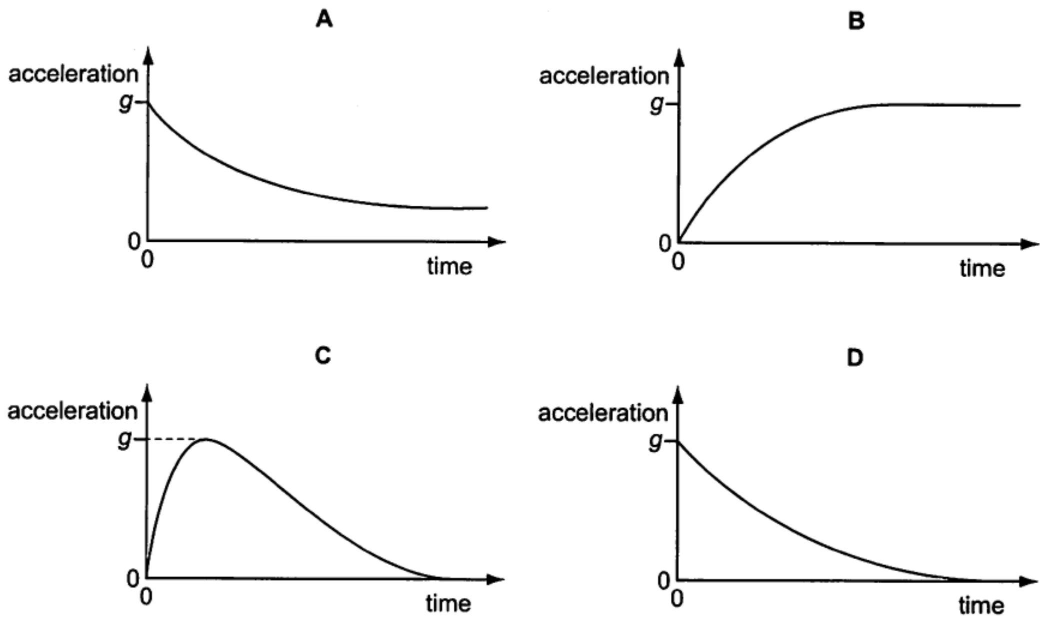
(a) Calculate the time of flight of the bolt from the ceiling to the floor. [0.74 s] [5]

(b) Calculate the distance it has fallen relative to the elevator shaft. [1.26 m] [2]

Rectilinear Motion under Gravity (<u>with</u> air resistance)

S14. 2010/1/4

An object is dropped from a great height so that air resistance becomes significant. Which graph shows how its acceleration varies with time?

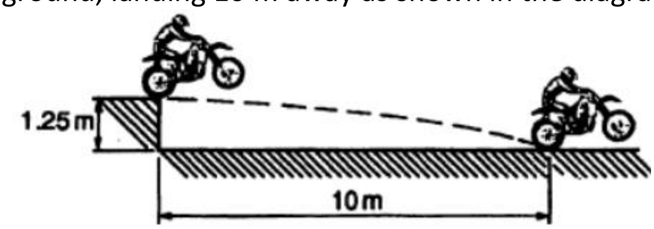


<u>Non-Rectilinear Motions</u>

Projectile Motion in 2-D

S15. 1993/1/3

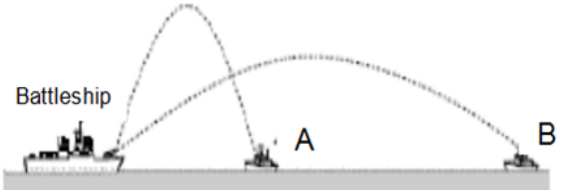
A motorcycle stunt-rider moving horizontally takes off from a point 1.25 m above the ground, landing 10 m away as shown in the diagram.



What was the speed at take-off?

A 5 ms^{-1} **D** 20 ms^{-1}
B 10 ms^{-1} **E** 25 ms^{-1}
C 15 ms^{-1}

S16. A battleship simultaneously fires two shells at enemy ships. If the shells follow the parabolic trajectories shown, which ship gets hit first?



A. A
B. Both at the same time
C. B
D. Need more information

S17. A tennis ball is served 2.5 m above the ground at an angle 5° above the horizontal direction with an initial speed of 30 m s^{-1}.
(a) When will it hit the ground? [1.0 s] [2]
(b) How far will it travel horizontally? [31 m] [2]
(c) At what velocity will the ball strike the ground? [3]
[31 m s^{-1}, 14.1° below the horizontal]

S18. 2014/3/1
An object is thrown horizontally from the top of a tall building.
(a) Assuming that air resistance is negligible, describe qualitatively the variation, if any, of the component of the velocity in
(i) the horizontal direction, [1]
(ii) the vertical direction. [2]
(b) In practice, air resistance is not negligible. Describe qualitatively the effect of air resistance on the variation, if any, of the component of the velocity in
(i) the horizontal direction, [1]
(ii) the vertical direction. [2]
(c) The path of the object, assuming no air resistance, is shown in the figure below. On the same figure, sketch the path of the object as it falls to the ground, allowing for air resistance. [2]