Anderson Junior College
Preliminary Examination 2011
H2 Mathematics Paper 1 (Solutions)

1

$$\int x\cos^{-1}x^2\,\mathrm{d}x$$
$$=\frac{x^2}{2}\cos^{-1}x^2-\int\frac{x^2}{2}\left(\frac{-2x}{\sqrt{1-x^4}}\right)\mathrm{d}x$$
$$=\frac{x^2}{2}\cos^{-1}x^2-\frac{1}{4}\int\frac{-4x^3}{\sqrt{1-x^4}}\,\mathrm{d}x$$
$$=\frac{x^2}{2}\cos^{-1}x^2-\frac{1}{2}\sqrt{1-x^4}+C$$

2

Let the price of a X-box console be \$*x*, a Kinect sensor be \$*y* and a Game DVD be \$*z*.

$$x+y+z=499$$
$$0.9x+0.85y+0.9z=439.15$$
$$0.95x+0.75y+0.8z=426.30$$

$$\text{Aug matrix} = \begin{pmatrix} 1 & 1 & 1 & 499 \\ 0.9 & 0.85 & 0.9 & 439.15 \\ 0.95 & 0.75 & 0.8 & 426.30 \end{pmatrix} \Rightarrow \text{rref} = \begin{pmatrix} 1 & 0 & 0 & 247 \\ 0 & 1 & 0 & 199 \\ 0 & 0 & 1 & 53 \end{pmatrix}$$

$\underline{x = 247\ ,\ y = 199\ ,\ z = 53}$

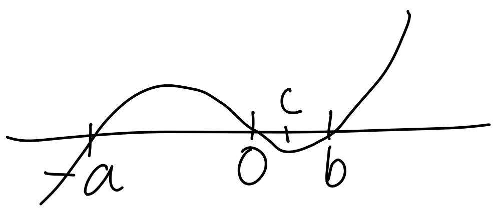
Employees of Company P will pay \$[0.9(247)+0.8(199)+0.85(53)] = \$426.55 > \$426.30.
No, it will not be more attractive for employees to purchase all the 3 items from own company.

3

$$\frac{(x+a)(x-b)}{(x)(x-c)^2}<0,\qquad x\neq c,\ x\neq 0$$

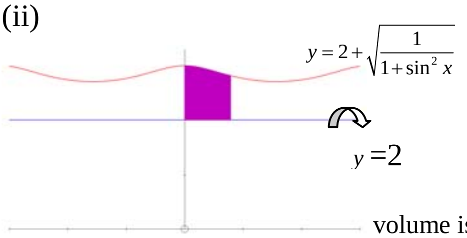
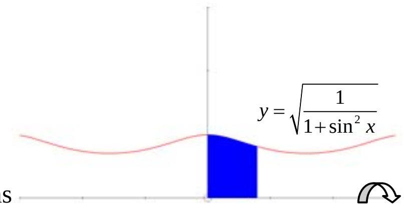
Since $(x-c)^2>0$ as $x\neq c$,

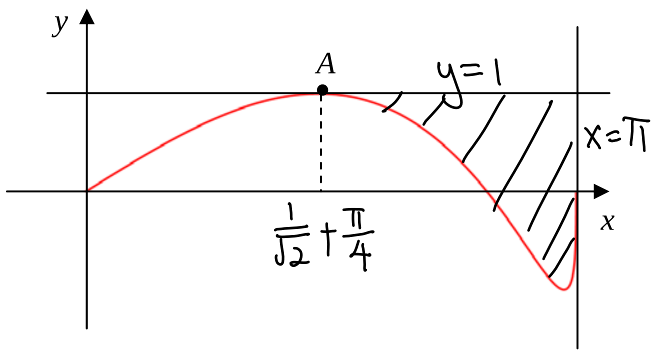
$$\frac{(x+a)(x-b)}{x}<0$$
$$x(x+a)(x-b)<0$$
$$x<-a\quad\text{or}\quad 0<x<b\quad\text{and}\quad x\neq c$$



Replace x by $\ln x$.

$$\ln x<-a\quad\text{or}\quad 0<\ln x<b\ \text{and}\ \ln x\neq c$$
$$0<x<e^{-a}\quad\text{or}\quad 1<x<e^{b}\quad\text{and}\quad x\neq e^{c}$$

4	(i) $t=\tan x \Rightarrow \dfrac{dt}{dx}=\sec^2 x = 1+t^2$ $= \displaystyle\int \frac{1}{1+\frac{t^2}{1+t^2}}\left(\frac{1}{1+t^2}\right)dt$ $= \displaystyle\int \frac{1}{1+2t^2}dt$ $= \dfrac{1}{\sqrt{2}}\tan^{-1}\sqrt{2}t + c$ $= \dfrac{1}{\sqrt{2}}\tan^{-1}\sqrt{2}\tan x + c$
	(ii)  volume is same as  Exact volume = $\pi\displaystyle\int_0^{\frac{\pi}{4}}\left(\sqrt{\frac{1}{1+\sin^2 x}}\right)^2 \mathrm{d}x$ $=\pi\left[\dfrac{1}{\sqrt{2}}\tan^{-1}\sqrt{2}\tan x\right]_0^{\frac{\pi}{4}}$ $=\dfrac{\pi}{\sqrt{2}}\tan^{-1}\sqrt{2}$
5 (a)	Method 1: $\displaystyle\int \sin 2x \cos x \,\mathrm{d}x$ $=\dfrac{1}{2}\displaystyle\int \sin 3x + \sin x \,\mathrm{d}x$ $=\dfrac{1}{2}\left(-\dfrac{\cos 3x}{3}-\cos x\right)+C$ $=-\dfrac{1}{2}\left(\dfrac{\cos 3x}{3}+\cos x\right)+C$ Method 2: $\displaystyle\int \sin 2x \cos x \,\mathrm{d}x$

	$= \int 2\sin x \cos^2 x \,\mathrm{d}x \qquad [\text{use } \int \mathrm{f}(x)[\mathrm{f}(x)]^n \,\mathrm{d}x = \frac{[\mathrm{f}(x)]^{n+1}}{n+1} + c]$ $= -\frac{2}{3}\cos^3 x + C$
(b)	(i) $\frac{\mathrm{d}x}{\mathrm{d}t} = \cos t + 1, \quad \frac{\mathrm{d}y}{\mathrm{d}t} = 2\cos 2t$ $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}t}\frac{\mathrm{d}t}{\mathrm{d}x} = \frac{2\cos 2t}{\cos t + 1}$ When $\frac{\mathrm{d}y}{\mathrm{d}x} = 0, \ \cos 2t = 0 \ \Rightarrow \ 2t = \frac{\pi}{2}, \frac{3\pi}{2} \ \Rightarrow \ t = \frac{\pi}{4}, \frac{3\pi}{4} \ \Rightarrow \ x = \frac{1}{\sqrt{2}} + \frac{\pi}{4}, \ \frac{1}{\sqrt{2}} + \frac{3\pi}{4}$ At point A, $x = \frac{1}{\sqrt{2}} + \frac{\pi}{4}, \quad y = 1$ $\therefore y = 1$ is the equation of the tangent to the curve at point A. Or Since $0 \le t \le \pi$, the maximum and minimum values of y (i.e. $y = \sin 2t$) is 1 and -1. The y-coordinate of point A is 1 and since the tangent to this max pt is a horizontal line ($\frac{\mathrm{d}y}{\mathrm{d}x} = 0,$), therefore the equation of the tangent to the curve at point A is $y = 1$.
	(ii) 

$$\text{Area} = \int_{\frac{1}{\sqrt{2}}+\frac{\pi}{4}}^{\pi} 1-y\ \mathrm{d}x$$

$$= \frac{3\pi}{4}-\frac{1}{\sqrt{2}}-\int_{\frac{\pi}{4}}^{\pi}\sin 2t\left(\cos t+1\right)\mathrm{d}t$$

$$= \frac{3\pi}{4}-\frac{1}{\sqrt{2}}-\int_{\frac{\pi}{4}}^{\pi}\sin 2t\cos t+\sin 2t\ \mathrm{d}t$$

$$= \frac{3\pi}{4}-\frac{1}{\sqrt{2}}-\left[-\frac{1}{2}\left(\frac{\cos 3x}{3}+\cos x\right)\right]_{\frac{\pi}{4}}^{\pi}-\left[-\frac{\cos 2t}{2}\right]_{\frac{\pi}{4}}^{\pi}$$

$$= \frac{3\pi}{4}-\frac{1}{\sqrt{2}}-\frac{2}{3}-\frac{1}{3\sqrt{2}}+\frac{1}{2}$$

$$= \frac{3\pi}{4}-\frac{1}{6}-\frac{2\sqrt{2}}{3}$$

6

$$y=\frac{1}{\left(2+\sin 2x\right)} \Rightarrow \frac{dy}{dx}=\frac{-2\cos 2x}{\left(2+\sin 2x\right)^2}=-2y^2\cos 2x$$

Differentiating wrt x:

$$\frac{d^2y}{dx^2}=-4y\frac{dy}{dx}\cos 2x-2y^2\left(-2\sin 2x\right)$$

$$\frac{d^2y}{dx^2}=2(-2y\cos 2x)\frac{dy}{dx}+4y^2\sin 2x$$

$$\frac{d^2y}{dx^2}=2\frac{1}{y}\left(\frac{dy}{dx}\right)\left(\frac{dy}{dx}\right)+4y^2\sin 2x$$

$$\frac{d^2y}{dx^2}=\frac{2}{y}\left(\frac{dy}{dx}\right)^2+4y^2\sin 2x\ (\textit{shown})$$

(i) $$\frac{d^3y}{dx^3}=-\frac{2}{y^2}\left(\frac{dy}{dx}\right)^3+\frac{4}{y}\left(\frac{dy}{dx}\right)\left(\frac{d^2y}{dx^2}\right)+8y\left(\frac{dy}{dx}\right)\sin 2x+8y^2\cos 2x$$

At $x=0$, $y=\frac{1}{2}$, $\frac{dy}{dx}=-\frac{1}{2}$, $\frac{d^2y}{dx^2}=1$, $\frac{d^3y}{dx^3}=-1$

By Maclaurin's theorem, $y = \frac{1}{2}-\frac{1}{2}x+\frac{1}{2!}x^2+\frac{(-1)}{3!}x^3+\ldots$

$$= \frac{1}{2}-\frac{1}{2}x+\frac{1}{2}x^2-\frac{1}{6}x^3$$ (up to term in x^3)

(ii) $$\frac{1}{\left(2+\sin 2x\right)} = \left(2+\sin 2x\right)^{-1}=\frac{1}{2}\left(1+\frac{1}{2}\left(2x-\frac{8x^3}{6}+\ldots\right)\right)^{-1}$$

$$\approx \frac{1}{2}\left(1-\frac{1}{2}\left(2x-\frac{8x^3}{6}\right)+\left(\frac{1}{2}\left(2x+\ldots\right)\right)^2-\left(\frac{1}{2}\left(2x+\ldots\right)\right)^3+\ldots\right)$$

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$$= \frac{1}{2}\left(1-\frac{1}{2}\left(2x-\frac{8x^3}{6}\right)+\frac{1}{4}(2x+..)^2-\frac{1}{8}(2x+...)^3+...\right)$$

$$= \frac{1}{2}\left(1-x+\frac{2x^3}{3}+x^2-x^3+...\right)$$

$$= \frac{1}{2}-\frac{1}{2}x+\frac{1}{2}x^2-\frac{1}{6}x^3+..$$

which is the same as the series obtained by using Maclaurin’s theorem.

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(i) $2y+2z=48$, $x+2z=18$

Expressing z and x in terms of y,

$z=24-y$, $x=2y-30$
$V=xyz=(2y-30)y(24-y)$
$=-2y^3+78y^2-720y$

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(ii) $\frac{dV}{dy}=0$

$-6y^2+156y-720=0$
$y^2-26y+120=0$
Using G.C, $y=6$ or $y=20$
$y = 6$ is not a feasible solution as x will be negative.
$\frac{d^2V}{dy^2}=-12y+156$

When $y=20$, $\frac{d^2V}{dy^2}=-84<0$
Hence, when $y=20$,
Maximum volume = $20(2\times 20-30)(24-20)=800$

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(iii) Let t be the time in seconds when robot A starts to move.
$m = 2t$ and $n = t-1$
Distance between A and B = l,

$l^2=(21-3t)^2+10^2$
Differentiating wrt t,
$2l\frac{\mathrm{d}l}{\mathrm{d}t}=2(21-3t)(-3)$
At $n = 4$, $t = 5$

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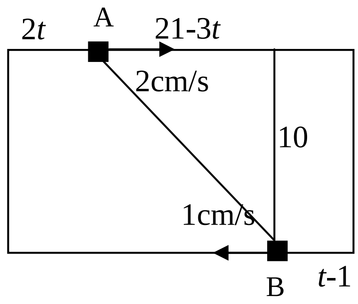


$$\frac{\mathrm{d}l}{\mathrm{d}t}=\frac{(6)(-3)}{\sqrt{6^2+10^2}}=-\frac{9}{\sqrt{34}}\text{ cm/s}$$

Method 2:

$$l^2=(20-m-n)^2+10^2$$

Since $m = 2n + 2$,

$$l^2=(18-3n)^2+10^2$$

Differentiating wrt n,

$$2l\frac{dl}{dn}=-6(18-3n)$$

At $n = 4$, $l^2 = 10^2 + 6^2$.

$$\frac{\mathrm{d}l}{\mathrm{d}n}=\frac{-18}{\sqrt{10^2+6^2}}$$

$$\frac{\mathrm{d}l}{\mathrm{d}t}=\frac{\mathrm{d}l}{\mathrm{d}n}\frac{\mathrm{d}n}{\mathrm{d}t}=\frac{-18}{\sqrt{10^2+6^2}}(1)=\frac{-18}{\sqrt{10^2+6^2}}=-\frac{9}{\sqrt{34}}\text{ cm/s}$$

8

(i)

$$y=\frac{ax^2}{x+a}$$

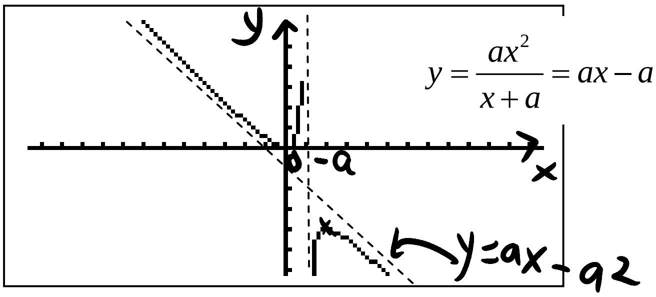
$$\frac{dy}{dx}=\frac{(x+a)2ax-ax^2}{(x+a)^2}=\frac{ax^2+2a^2x}{(x+a)^2}$$

Set $\frac{dy}{dx}=0 \Rightarrow ax^2+2a^2x=0 \Rightarrow ax(x+2a)=0 \Rightarrow x=0 \text{ or } x=-2a$

For all negative values of a, there will be two distinct values of x thus two stationary pts. (shown)

Or $B^2-4AC=4a^2>0$ for all negative values of a.

(ii)



$$y=\frac{ax^2}{x+a}=ax-a^2+\frac{a^3}{x+a}$$

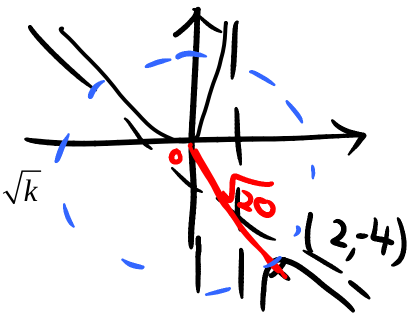
Max pt is $(-2a, -4a^2)$
Min pt is (0,0)

(iii) $x^4=\left(k-x^2\right)(x-1)^2 \Rightarrow a = -1$

$$\Rightarrow \left(\frac{-x^2}{x-1}\right)^2=k-x^2 \Rightarrow y^2=k-x^2$$

$\Rightarrow$ insert a circle, centre (0,0) , radius $\sqrt{k}$ to cut curve twice .

$\Rightarrow 0<k<20$



9

(i) $\begin{pmatrix}\mu\\2\mu\\2\mu\end{pmatrix}=\begin{pmatrix}-\lambda\\2\lambda\\-2+2\lambda\end{pmatrix}\Rightarrow \begin{matrix}\mu=-\lambda & \text{---(1)}\\ \mu=\lambda & \text{---(2)}\\ \mu=\lambda-1 & \text{---(3)}\end{matrix}$

The first and second equation has only 1 solution i.e. $\lambda=0$ and $\mu=0$ and it is obvious that equation (3) will be inconsistent for this solution; this implies that l_1 and l_2 are non-intersecting lines.

Since l_1 and l_2 are non-parallel lines as $\begin{pmatrix}1\\2\\2\end{pmatrix}\neq k\begin{pmatrix}-1\\2\\2\end{pmatrix}$ where k is a scalar

Since l_1 and l_2 are non-parallel and non-intersecting lines, l_1 and l_2 are skew lines.

(ii) Let $\overrightarrow{OX}=\begin{pmatrix}-\lambda\\2\lambda\\-2+2\lambda\end{pmatrix}$ and $\overrightarrow{OY}=\begin{pmatrix}\mu\\2\mu\\2\mu\end{pmatrix}$

$$\overrightarrow{OZ}=\frac{1}{2}\left(\overrightarrow{OX}+\overrightarrow{OY}\right)=\frac{1}{2}\left[\begin{pmatrix}-\lambda\\2\lambda\\-2+2\lambda\end{pmatrix}+\begin{pmatrix}\mu\\2\mu\\2\mu\end{pmatrix}\right]=\begin{pmatrix}0\\0\\-1\end{pmatrix}+\lambda\begin{pmatrix}-\frac{1}{2}\\1\\1\end{pmatrix}+\mu\begin{pmatrix}\frac{1}{2}\\1\\1\end{pmatrix}$$

Since λ and μ can be any real number, the locus of Z is a plane that passes through (0, 0, -

1) and parallel to both –½ ***i*** + **j** + ***k*** and ½ i + j + k ,

Therefore $\begin{pmatrix} -\frac{1}{2} \\ 1 \\ 1 \end{pmatrix} \times \begin{pmatrix} \frac{1}{2} \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$ is a normal to the plane *p*. The equation in scalar product form is

$$p: \ \mathbf{r} \cdot \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} = 1$$

(iii) Let $\overrightarrow{OS} = \begin{pmatrix} -\lambda \\ 2\lambda \\ -2+2\lambda \end{pmatrix}$ and $\overrightarrow{OS'} = \begin{pmatrix} \mu \\ 2\mu \\ 2\mu \end{pmatrix}$

Method 1:

$$\overrightarrow{S'S} = \begin{pmatrix} -\lambda \\ 2\lambda \\ -2+2\lambda \end{pmatrix} - \begin{pmatrix} \mu \\ 2\mu \\ 2\mu \end{pmatrix} = \begin{pmatrix} -\lambda-\mu \\ 2\lambda-2\mu \\ -2+2\lambda-2\mu \end{pmatrix}$$

This vector will be parallel to the normal of *p*.

$$\overrightarrow{S'S} = \begin{pmatrix} -\lambda-\mu \\ 2\lambda-2\mu \\ -2+2\lambda-2\mu \end{pmatrix} = k\left[\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}\right] = k\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} \Rightarrow \begin{array}{c} \lambda+\mu=0 \\ 2\lambda-2\mu=k \\ -2+2\lambda-2\mu=-k \end{array}$$

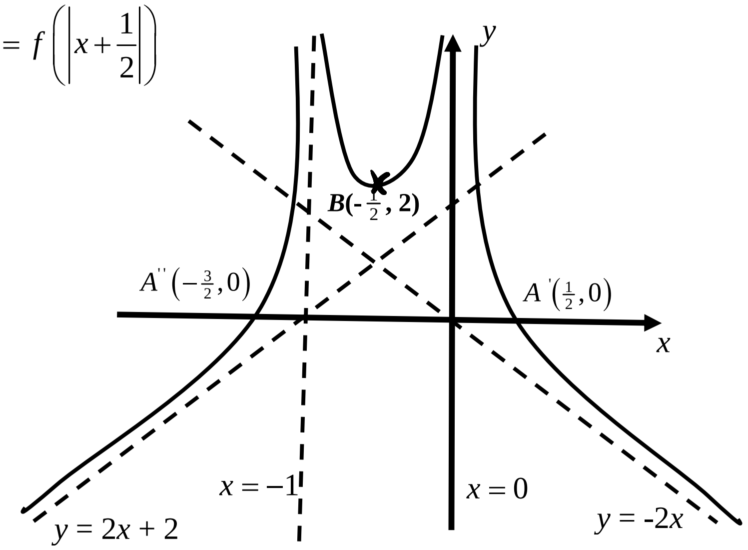
Solving, $\lambda = \frac{1}{4}$ => $\overrightarrow{OS} = \begin{pmatrix} -\frac{1}{4} \\ \frac{1}{2} \\ -\frac{3}{2} \end{pmatrix}$

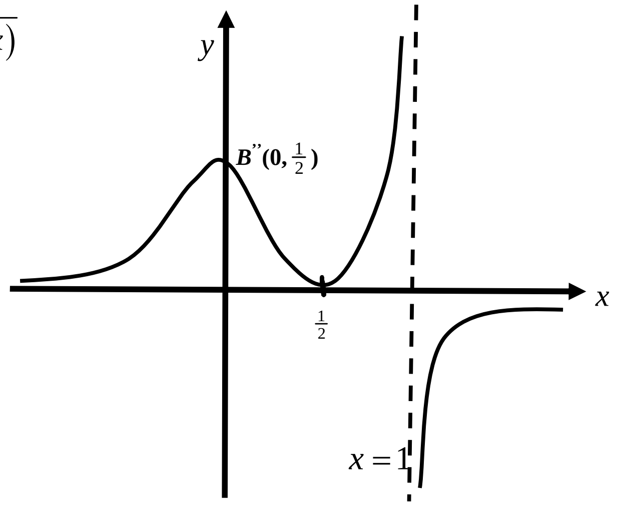
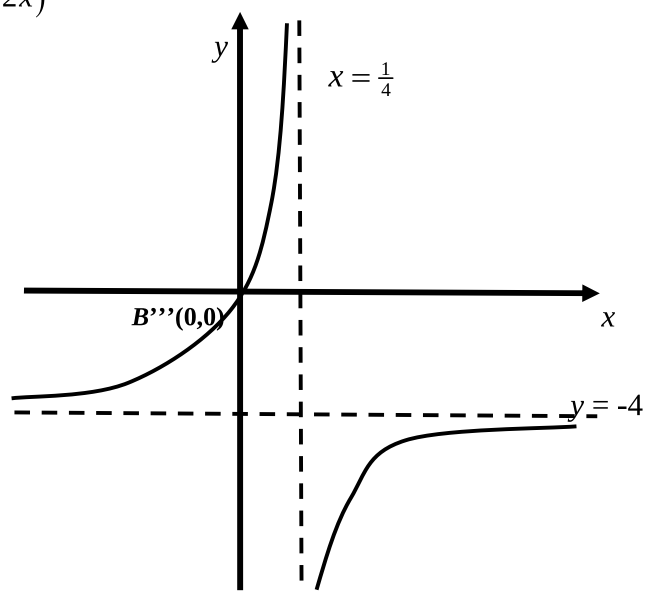
Coordinates of S is $\left(-\frac{1}{4}, \frac{1}{2}, -\frac{3}{2}\right)$

Method 2:

Let F be the midpoint between S and S',

$$\overrightarrow{OF} = \frac{1}{2}\left(\overrightarrow{OS} + \overrightarrow{OS'}\right) = \frac{1}{2}\begin{pmatrix} -\lambda-\mu \\ 2\lambda-2\mu \\ -2+2\lambda-2\mu \end{pmatrix}$$

	and $\overrightarrow{OF} = \overrightarrow{OS} + k\,\underset{\sim}{n} = \begin{pmatrix} -\lambda \\ 2\lambda \\ -2+2\lambda \end{pmatrix} + k\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} -\lambda \\ 2\lambda + k \\ -2+2\lambda - k \end{pmatrix}$ Equating the position vector of point F, $\frac{1}{2}\begin{pmatrix} -\lambda-\mu \\ 2\lambda-2\mu \\ -2+2\lambda-2\mu \end{pmatrix} = \begin{pmatrix} -\lambda \\ 2\lambda+k \\ -2+2\lambda-k \end{pmatrix} \Rightarrow \begin{matrix} \lambda+\mu=0 \\ 2\lambda-2\mu=k \\ -2+2\lambda-2\mu=-k \end{matrix}$ Solving, $\lambda = \frac{1}{4}$ => $\overrightarrow{OS} = \begin{pmatrix} -\frac{1}{4} \\ \frac{1}{2} \\ -\frac{3}{2} \end{pmatrix}$ Coordinates of S is $\left(-\frac{1}{4}, \frac{1}{2}, -\frac{3}{2}\right)$
10	(i) $y = f\left(\left\|x+\frac{1}{2}\right\|\right)$ 

	(ii) $y=\dfrac{1}{f(x)}$ 
	(iii) $y=f'(2x)$ 
11	(i) Let P_n be the proposition: $u_n = na+(n-1)$ for $n\in\mathbf{Z}^+$. When n = 1, LHS = u_1 = a RHS = $(1)a+0=a$ = LHS Since LHS = RHS $\therefore$ P_1 is true Assume that P_k is true for some $k\in\mathbf{Z}^+$. i.e. $u_k = ka+(k-1)$ To prove that P_k is true $\Rightarrow P_{k+1}$ is true . i.e. to prove that $u_{k+1}=(k+1)a+(k+1)-1$ RHS $=(k+1)a+(k+1)-1=(k+1)a+k$ LHS = $u_{k+1}=\dfrac{k+1}{k}u_k+\dfrac{1}{k}$

$$= \frac{k+1}{k}(ka + (k-1)) + \frac{1}{k}$$
$$= (k+1)a + \frac{(k+1)(k-1)}{k} + \frac{1}{k}$$
$$= (k+1)a + \frac{k^2-1}{k} + \frac{1}{k}$$
$$= (k+1)a + k - \frac{1}{k} + \frac{1}{k}$$
$$= (k+1)a + k \quad = \text{RHS}$$

$\therefore$ P_k is true $\Rightarrow P_{k+1}$ is true

As P_1 is true and P_k is true $\Rightarrow P_{k+1}$ is true , by the principle of mathematical induction P_n is true for all $n \in \mathbf{Z}^+$.

(ii) $a = 1$, $u_n = n + (n-1) = 2n-1$, $u_{n-1} = 2(n-1) - 1 = 2n-3$

$$\sum_{n=2}^{N} \frac{1}{u_n u_{n-1}}$$
$$= \sum_{n=2}^{N} \frac{1}{(2n-1)(2n-3)}$$
$$= \sum_{n=2}^{N} \left(\frac{1}{2(2n-3)} - \frac{1}{2(2n-1)} \right)$$
$$= \frac{1}{2} \sum_{n=2}^{N} \left(\frac{1}{(2n-3)} - \frac{1}{(2n-1)} \right)$$
$$= \frac{1}{2} \Bigg[\frac{1}{1} - \frac{1}{3}$$
$$+ \frac{1}{3} - \frac{1}{5}$$
$$\vdots$$
$$+ \frac{1}{2N-5} - \frac{1}{2N-3}$$
$$+ \frac{1}{2N-3} - \frac{1}{2N-1}$$
$$= \frac{1}{2}\left(1 - \frac{1}{2N-1}\right) \qquad \text{(Shown)}$$

Since $\sum_{n=2}^{N} \frac{1}{(2n-1)(2n-3)} = \frac{1}{2}\left(1 - \frac{1}{2N-1}\right)$,

$$\sum_{n=2}^{N} \frac{1}{(2n+9)(2n+7)}$$

	$= \sum_{k=7}^{N+5} \frac{1}{(2k-1)(2k-3)}$ $= \sum_{k=2}^{N+5} \frac{1}{(2k-1)(2k-3)} - \sum_{k=2}^{6} \frac{1}{(2k-1)(2k-3)}$ $= \frac{1}{2}\left(1 - \frac{1}{2(N+5)-1}\right) - \frac{1}{2}\left[1 - \frac{1}{2(6)-1}\right]$ $= \frac{1}{2}\left(\frac{1}{11} - \frac{1}{2N+9}\right)$
12	Given $S_n = \frac{1}{a}\left[1-(a-1)^n\right]$, $T_n = S_n - S_{n-1}$, $n \geq 2$ $= \frac{1}{a}\left[1-(a-1)^n\right] - \frac{1}{a}\left[1-(a-1)^{n-1}\right]$ $= -\frac{1}{a}(a-1)^n + \frac{1}{a}(a-1)^{n-1}$ $= \frac{1}{a}(a-1)^{n-1}(-a+1+1)$ $= \frac{1}{a}(a-1)^{n-1}(2-a)$ $T_n = \frac{1}{a}(a-1)^{n-1}(2-a), \quad n \in \mathbb{Z}^+$ $\frac{T_n}{T_{n-1}} = = \frac{\frac{1}{a}(a-1)^{n-1}(2-a)}{\frac{1}{a}(a-1)^{n-2}(2-a)} = (a-1) =$ constant (Shown)
	(i) the total number of terms in the first n brackets $= 1+3+5+\cdots(1+(n-1)2)$ $= 1+3+5+\cdots(2n-1)$ $= \frac{n}{2}(1+(2n-1)) = n^2$
	(ii) number of terms in the 11^{th} bracket = 1 + (11-1)2 = 21 terms (middle term will be the 11^{th} term) Number of term from 1^{st} bracket to 10^{th} bracket = 10^2 = 100 $T_{111} = \frac{1}{a}(a-1)^{110}(2-a)$
	(iii) For the sum to infinity of the series to exist, $\|a-1\| < 1$

$0 < a < 2$

Method 1:

when $a = \frac{39}{20}$ (i.e. $|a-1| < 1$),

sum to infinity of the series $= \frac{1}{a}\left[1-0\right] = \frac{1}{a}$

For the sum of all the terms in the first *n* brackets to be within 0.1% of the sum to infinity of the series,

$\left|S_{n^2} - S_\infty\right| < 0.1\% S_\infty$

$\left|\frac{1}{a}\left[1-(a-1)^{n^2}\right] - \frac{1}{a}\right| < 0.1\% \frac{1}{a}$

$\frac{1}{a}\left|-(a-1)^{n^2}\right| < 0.001(\frac{1}{a})$

$(a-1)^{n^2} < 0.001 \quad \Rightarrow (\frac{19}{20})^{n^2} < 0.001$

Using GC, $n > 11.605$

At least 12 brackets.

Method 2:

(iii) when $a = \frac{39}{20}$, first term $= \frac{2-a}{a} = \frac{1}{39}$, $r = (a-1) = \frac{19}{20}$

sum to infinity of the series $= \frac{\frac{1}{39}}{1-\frac{19}{20}} = \frac{20}{39}$

the sum of all the terms in the first n brackets

= the sum of n^2 terms in the GP

$= \frac{1}{a}\left[1-(a-1)^{n^2}\right] = \frac{20}{39} - \frac{20}{39}(\frac{19}{20})^{n^2}$

For the sum of all the terms in the first n brackets to be within 0.1% of the sum to infinity of the series,

$\left|\frac{20}{39} - \frac{20}{39}(\frac{19}{20})^{n^2} - \frac{20}{39}\right| < 0.1\% \frac{20}{39}$

$\left|-\frac{20}{39}(\frac{19}{20})^{n^2}\right| < 0.001 \times \frac{20}{39}$

$(\frac{19}{20})^{n^2} < 0.001$

Using GC , $n > 11.605$

At least 12 brackets.

13 (a)

(i) $p^* + 10i = qi + 5$ --------- (1)

$|p|^2 - q - 5 + 2i = 0 \Rightarrow q = |p|^2 - 5 + 2i$

Substitute into (1): $p^* + 10i = \left(|p|^2 - 5 + 2i\right)i + 5$

Let $p = x + yi$

$x - yi + 10i = \left(x^2 + y^2 - 5 + 2i\right)i + 5$

Equating real parts: $x = -2 + 5 = 3$

Equating imaginary parts: $-y + 10 = x^2 + y^2 - 5$

$\Rightarrow -y + 10 = 9 + y^2 - 5$

$\Rightarrow y^2 + y - 6 = 0$

$\Rightarrow y = -3$ or 2 (rejected as Im(p) < 0)

Therefore $p = 3 - 3i$.

$|p| = \sqrt{3^2 + 3^2} = \sqrt{18}$ and $\arg(p) = -\frac{\pi}{4}$

$$p^{2n} = \left(\sqrt{18}e^{-i\frac{\pi}{4}}\right)^{2n}$$

$$= \left(\sqrt{18}\right)^{2n}\left(\cos\frac{2n\pi}{4} - i\sin\frac{2n\pi}{4}\right)$$

p^{2n} is purely imaginary $\Rightarrow \cos\frac{n\pi}{2} = 0$

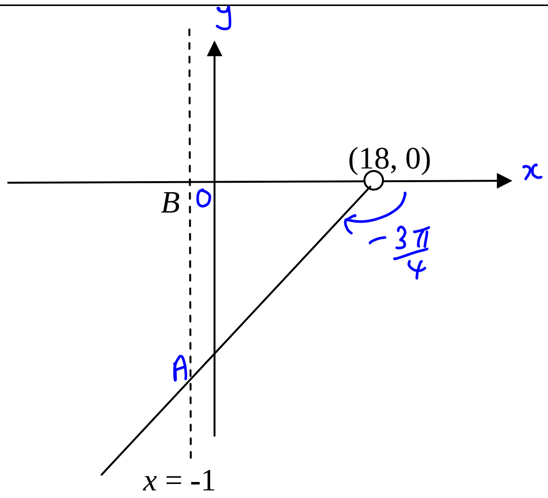
$\Rightarrow n = 2k + 1$, where $k \in \mathbb{Z}$

(ii) $\arg\left(\frac{w}{p} - p^*\right) = -\frac{\pi}{2} \Rightarrow \arg\left(\frac{w - pp^*}{p}\right) = -\frac{\pi}{2}$

$\Rightarrow \arg\left(w - pp^*\right) - \arg(p) = -\frac{\pi}{2}$

$\Rightarrow \arg(w - 18) - \left(-\frac{\pi}{4}\right) = -\frac{\pi}{2}$

$\Rightarrow \arg(w - 18) = -\frac{3\pi}{4}$

	$w+w^*=-2$ $\Rightarrow x=-1$ $\Rightarrow w$ is represented by point A $\tan\dfrac{\pi}{4}=\dfrac{BA}{19} \Rightarrow BA=19$ $\Rightarrow w=-1-19i$ 
13 (b)	$\left(z-\sqrt{2}\right)^6=8$ $\Rightarrow \left(z-\sqrt{2}\right)^6=8e^{i2k\pi}$ $\Rightarrow z-\sqrt{2}=\sqrt{2}e^{\frac{2k\pi}{6}i}$ $\Rightarrow z=\sqrt{2}\left(1+e^{\frac{2k\pi}{6}i}\right)$ $\Rightarrow z=\sqrt{2}e^{\frac{k\pi}{6}i}\left(e^{-\frac{k\pi}{6}i}+e^{\frac{k\pi}{6}i}\right)$ $\Rightarrow z=\sqrt{2}e^{\frac{k\pi}{6}i}\left(2\cos\dfrac{k\pi}{6}\right)=2\sqrt{2}\left(\cos\dfrac{k\pi}{6}\right)e^{\frac{k\pi}{6}i}$ where $k=0,\pm1,\pm2\,,\,3$ $\left(z-\sqrt{2}\right)^6=8 \;\Rightarrow \left\|z-\sqrt{2}\right\|=\sqrt{2}$ $\therefore$ the roots lie on a circle centre at $\left(\sqrt{2},0\right)$ and radius = $\sqrt{2}$