Class	Index Number	Name

ANG MO KIO SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2019
SECONDARY FOUR EXPRESS / FIVE NORMAL ACADEMIC

ADDITIONAL MATHEMATICS 4047/01
Paper 1

Tuesday **17 September 2019** **2 hours**

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is **80**.

For Examiner's Use
80

This document consists of **18** printed pages.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

1 The curve $y = \mathrm{f}(x)$ is such that $\mathrm{f}'(x) = px^2 - 8x + 1$. Given that $\mathrm{f}''(2) = 4$ and the curve passes through the point (3, 0), find

(i) the value of p, [2]

(ii) an expression for $\mathrm{f}(x)$. [3]

2 When the graph of y^2 against $\sqrt{x}$ is drawn, a straight line is obtained which passes through the points (10, 16) and (16, 25). Find the value of x when $y = 2$. [4]

3 The equation $2x^2 - 3x + p = 0$ has roots α and β. Given that the equation $8x^2 + qx + 64 = 0$ has roots α^3 and β^3, find the value of p and of q. [7]

4 **(i)** Sketch the graph $y = 2x^{\frac{2}{3}}$ for $x > 0$. [1]

(ii) **(a)** In order to solve the equation $2\sqrt[3]{x} = \sqrt{x}$, a graph of a suitable straight line is drawn on the same set of axes as the graph of $y = 2x^{\frac{2}{3}}$. Find the equation of this straight line. [2]

(b) On the same axes as **(i)**, sketch the graph of the straight line for $x > 0$, indicating the coordinates of the points of intersection clearly. [3]

5 **Without using a calculator,** solve the equation $\sqrt{18}x = 2x + 6$, giving your answer in the form $\dfrac{a\sqrt{2}+b}{7}$, where a and b are integers. [5]

6 It is given that $f(x) = 2x^3 + 11x^2 + 12x - 9$.

(i) Show that $(2x - 1)$ is a factor of $f(x)$. [1]

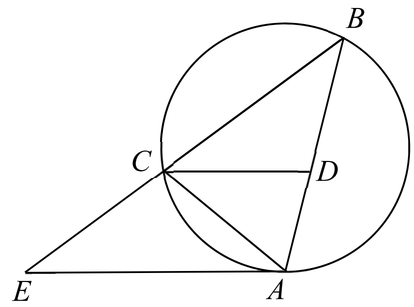
(ii) Hence, or otherwise, factorise $f(x)$ completely. [3]

(iii) Hence express $\dfrac{15x+17}{2x^3+11x^2+12x-9}$ as a sum of 3 partial fractions. [4]

7 **(i)** Prove that $\left(\operatorname{cosec}\theta - \cot\theta\right)^2 = \dfrac{1-\cos\theta}{1+\cos\theta}$. [3]

(ii) Hence find, for $0 \le \theta \le 2\pi$, the values of θ in radians for which $\left(\operatorname{cosec}\theta - \cot\theta\right)^2 = \dfrac{1}{4}$. [3]

8 The diagram shows a triangle ABC whose vertices lie on the circumference of a circle. The tangent to the circle at A meets BC produced at E such that $AB = AE$. D is on a point on AB such that CD is parallel to EA.



Prove that

(i) triangles ABC and ACD are similar, [3]

(ii) CD bisects $\angle ACB$. [3]

9 An open cylinder has radius r cm and external surface area 75π cm^2.

(i) Show that the volume of the cylinder, V cm^3, is given by $V = \frac{\pi}{2}\left(75r - r^3\right)$. [3]

(ii) Given that r can vary, find the stationary value of V, giving your answers in terms of π. [3]

10 A particle P moves along a straight line so that its displacement, s m, from a fixed point O, t seconds after motion has begun, is given by $s = t^3 + 4t^2 - 3t - 14$.

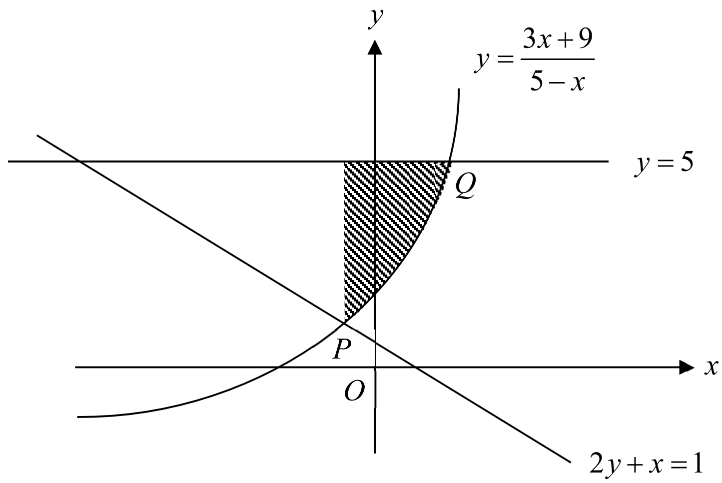
(i) Find an expression for the velocity of particle P, in terms of t. [1]

Another particle Q moves along the same straight line as P and passes O at the same instant that P begins to move. The initial velocity of Q is −2 m/s and its acceleration, a m/s^2, t seconds after passing O, is given by $a = 6t + 2$.

(ii) Find the value of t when P and Q collide. [6]

(iii) Determine if P and Q are travelling in the same direction at the point of collision. [2]

11 The diagram below shows part of the curve $y = \dfrac{3x+9}{5-x}$. The curve meets the line $2y + x = 1$ at point P and the line $y = 5$ at point Q.



(i) Show that the coordinates of point P are $(-1, 1)$. [3]

(ii) By showing that $\frac{3x+9}{5-x}=\frac{24}{5-x}-3$, find the area of the shaded region bounded by $y=\frac{3x+9}{5-x}$, the line $y=5$ and the line from *P* perpendicular to the *x*-axis . [7]

12 The line $3y - 4x = -48$ is a tangent to a circle at point $A(12, 0)$.

(i) Find the equation of the normal to the circle at A. [3]

(ii) Given that the centre of the circle is (a, b) where $a > 0$, $b > 0$ and radius of the circle is 5 units, find the value of a and of b. [5]

END OF PAPER