2021 H3 Physics A Level Paper Suggested Solutions

1a) In vacuum, the object will fall at a constant acceleration of *g*.
Taking direction downward as positive, and applying the kinematics equation

$s = ut + \frac{1}{2}at^2$.

$100 = (0)t + \frac{1}{2}(9.81)t^2$

$t = 4.5152 = 4.5$ s (2 s.f.) [1]

1bi) time-of-flight $t' = 2 \times 4.5152 = 9.0304$ s

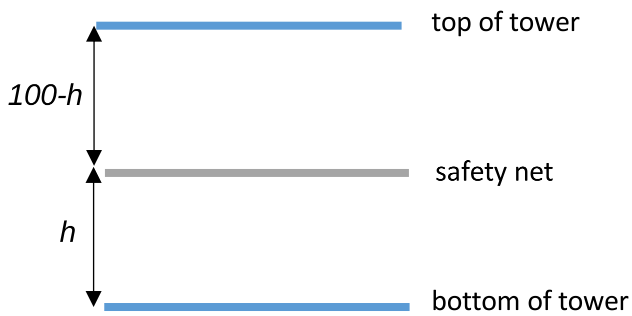
Taking direction downward as positive, and apply the kinematics equation

$s = ut + \frac{1}{2}at^2$.

$s = (0)(9.0304) + \frac{1}{2}(9.81)(9.0304)^2 = 399.96$ m = 400 m (2 s.f.) [1]

1bii) Instead of dropping the object from the top of the tower, the same double time-of-flight could be achieved by throwing the object up from the ground of the tower with the same speed as the speed of the object just before it reaches the ground from its fall from rest as indicated in (b)(i). [1]

1cii)



Let h be the initial height of the safety net and v_i be the final velocity of the 'weightless' (free-fall) phase and initial velocity at the safety net phase. Final speed at the safety net phase is zero.
Taking direction downward as positive, and apply the kinematics equation
$v^2 = u^2 + 2as$ [1]

[2]

For the 'weightless' phase, $v_i^2 = 0^2 + 2(g)(100 - h)$ …… (1)
For the safety net phase, $0 = v_i^2 + 2(-4g)(h) \Rightarrow v_i^2 = 8gh$……(2)

Subst (2) in eqn (1) $8gh = 2(g)(100 - h)$ [1]

$\therefore h = 20$ m (to 2 s.f.)

1ci) Taking direction downward as positive, and apply the kinematics equation $s = ut + \frac{1}{2}at^2$.

For the 'weightless' phase, $(100 - 20) = (0) + \frac{1}{2}(9.81)t^2$

$t = 4.0386$ s [1]

Hence the 'weightless' time of flight is reduced by 4.5152 - 4.0386 = 0.4766
= 0.5 s
(1 s.f.)

2a) By PCOLM,

$$4mv = 4mv_{4m} + mv_m$$
$$v_m = 4v - 4v_{4m}$$

By RSOA = RSOS,

$$v = v_m - v_{4m} \rightarrow v_m = v + v_{4m}$$
$$v + v_{4m} = 4v - 4v_{4m} \rightarrow 3v = 5v_{4m} \rightarrow v_{4m} = 3v/5$$ [1]
$$v_m = 8v/5$$

% of kinetic energy transferred to m,

$$\frac{\frac{1}{2}m\left(\frac{8v}{5}\right)^2}{\frac{1}{2}4mv^2} = \frac{64}{4(25)}$$ [1]

$$= 0.64 \rightarrow 64\%$$ [1]

Method II: Using zero-momentum frame (center-of-mass frame)

$$v_{cm} = \frac{4mv + m0}{5m} = \frac{4v}{5}$$

The velocities of particle in the zero-momentum frame before the collision,

$$v_{4m,cm} = v_{4m,Earth} + v_{Earth,cm} = v - \frac{4v}{5} = \frac{v}{5}$$

$$v_{m,cm} = v_{m,Earth} + v_{Earth,cm} = 0 - \frac{4v}{5} = -\frac{4v}{5}$$

After the collision, the sign of velocities will change, hence,

$v_{4m} = -\frac{v}{5}$ and $v_m = \frac{4v}{5}$

Rewriting the velocities after the collision in the Earth frame,

$$v_{4m,Earth} = v_{4m,cm} + v_{cm,Earth} = -\frac{v}{5} + \frac{4v}{5} = \frac{3v}{5}$$

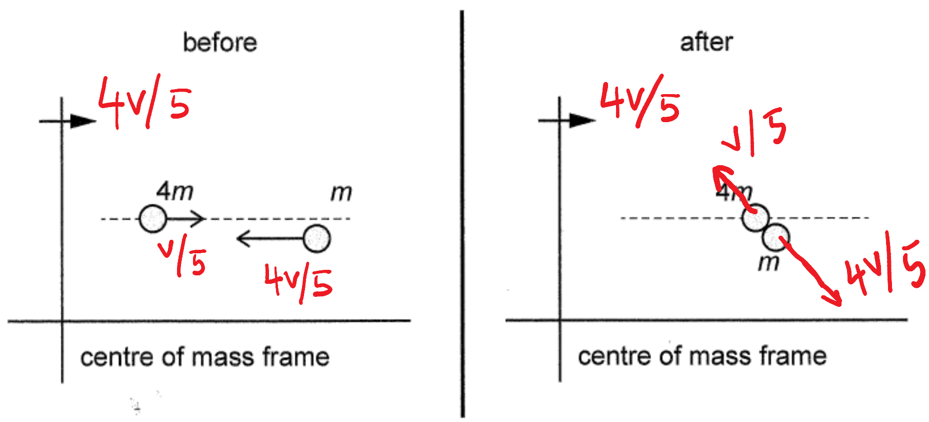
$$v_{m,Earth} = v_{m,cm} + v_{cm,Earth} = \frac{4v}{5} + \frac{4v}{5} = \frac{8v}{5}$$

% of kinetic energy transferred to m,

$$\frac{\frac{1}{2}m\left(\frac{8v}{5}\right)^2}{\frac{1}{2}4mv^2}=\frac{64}{4(25)}$$
$$=0.64 \rightarrow 64\%$$

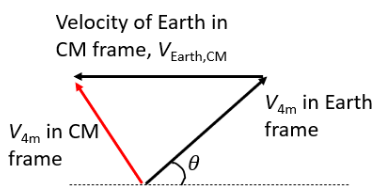
Note: There is a high chance that students might perform calculation error in solving simultaneous equations in the Earth frame (first method). The zero-momentum frame does not involve such equations.

2bi) [3]



$$v_{cm}=\frac{4mv}{5m}=\frac{4}{5}v$$

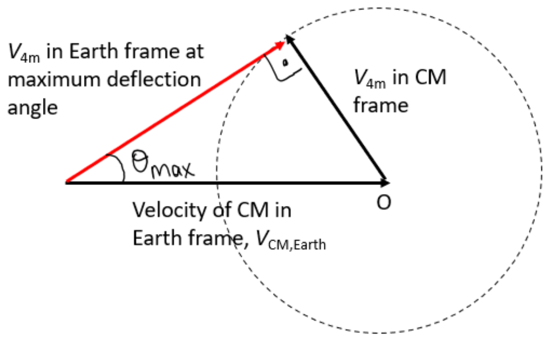
Using the vector diagram could help you find the direction of velocity of 4*m* in the center-of-mass frame readily.



$$V_{4m,CM}=V_{4m,Earth}+V_{Earth,CM}$$

Note: The diagram may help visualize the direction of velocities in earth and zero-momentum (CM) frames.

2bii)



[1]

$$\sin\theta_{\max} = \frac{V_{4m,CM}}{V_{CM,Earth}} = \frac{v/5}{4v/5} = \frac{1}{4}$$

$$\theta_{\max} = 14.5°$$

[1]

[1]

3a) Total energy of mass *m* in gravitational field by mass *M*

$$E_T = KE + PE$$
$$= \frac{1}{2}mv^2 + (-\frac{GMm}{r})$$
$$= \frac{1}{2}mv_t^2 - (\frac{GMm}{r})$$

[1]

Where v is the velocity of mass *m* and v_t is the tangential component of the velocity, noting that $v = v_t$ for a circular orbit.

Since gravitational force is a central force, $L = mrv_t$, where we can write [1]

$$\frac{1}{2}mv_t^2 = \frac{L^2}{2mr^2}$$

[1]

Thus,

$$E_T = -\frac{GMm}{r} + \frac{L^2}{2mr^2}$$

3bi) Consider the turning points such that $\frac{1}{2}mv_r^2 = 0$ [1]

$$E_T = \frac{L^2}{2mr^2} - \frac{GMm}{r}$$

$$E_T r^2 = \frac{L^2}{2m} - GMmr$$

$$r^2 + \frac{GMm}{E_T} r - \frac{L^2}{2mE_T} = 0$$ [1]

Since r_a and r_b are solutions, we can write

$$(r - r_a)(r - r_b) = 0$$

$$r^2 - (r_a + r_b) + r_a r_b = 0$$ [1]

Comparing coefficients,

$$-(r_a + r_b) = \frac{GMm}{E_T} = -2a$$ [1]

$$E_T = -\frac{GMm}{2a}$$

3bii) From part (b)(i),

$$E_T = -\frac{GMm}{2a}$$

Additionally, we know that

$$E_T = KE + PE$$

$$= \frac{1}{2}mv^2 + \left(-\frac{GMm}{r}\right)$$ [1]

Where v is the speed of the exoplanet.

Thus,

$$\frac{1}{2}mv^2 = \frac{GMm}{r} - \frac{GMm}{2a}$$

$$v^2 = \frac{2GM}{r} - \frac{GM}{a}$$

$$v = \sqrt{GM(\frac{2}{r} - \frac{1}{a})}$$ [1]

$$= \sqrt{6.67\times10^{-11}\times5.4\times10^{32}\times(\frac{2}{2.8\times10^{12}} - \frac{1}{3.8\times10^{12}})}$$

$$= 1.27\times10^{5} m\ s^{-1}$$ [1]

4a) Length of coil, $L = dN$ [1]

Resistance of coil, $R = \frac{\rho l}{\left(\frac{d}{2}\right)^2} = \frac{4\rho l}{d^2}$ [1]

$$k = \frac{B\rho l}{Vd} = \frac{\mu_0 NI}{L}\frac{\rho l}{Vd} = \frac{\mu_0 N}{L}\left(\frac{V}{\frac{4\rho l}{\pi d^2}}\right)\frac{\rho l}{Vd} = \mu_0\frac{1}{d}\frac{V\pi d^2}{4\rho l}\frac{\rho l}{Vd} = \frac{\mu_0\pi}{4}$$ [1]

$k = 10^{-7}$ [1]

Unit of k is $\frac{\text{(Tesla)}(\Omega\text{m})\text{m}}{(\text{Ampere})(\Omega)\text{m}} = \frac{\text{(Tesla)m}}{(\text{Ampere})} = \frac{\text{kgms}^{-2}\text{m}}{\text{A}^2\text{m}} = \text{kgms}^{-2}\text{A}^{-2}$ [1]

4bi)

$$B = \frac{\pi\,\mu_o}{4}\frac{V\,d}{\rho\,l} = \frac{\pi\left(4\pi\times10^{-7}\right)}{4}\frac{(3.0)\left(0.25\times10^{-3}\right)}{\left(1.7\times10^{-8}\right)(2.9)} = 0.015\,T$$ [2]

4bii) The thickness of the insulation is assumed to be negligible in the calculations. If the thickness of the insulation is considered, the diameter, *d*, of the wire itself will be less than 0.25 mm. Since the magnetic flux density is proportional to *d,* a small diameter will give a smaller magnetic flux density. [1]

5a) From first law of thermodynamics, $\Delta U = Q + W_{on}$

Since volume of gas is constant in the process of heat transfer, $W = 0$, therefore $\Delta U = Q$ [1]

Given that $C_V = \frac{Q}{\Delta T} \rightarrow Q = C_V(\Delta T) = \Delta U$

For a monatomic ideal gas, $\Delta U = \frac{3}{2}Nk\Delta T = C_V \Delta T \rightarrow C_V = \frac{3}{2}Nk$ [1]

5b) From first law of thermodynamics, $\Delta U = Q + W_{on}$

Given that $C_P = \frac{Q}{\Delta T} \rightarrow Q = C_P(\Delta T)$

For a monatomic ideal gas, $\Delta U = \frac{3}{2}Nk\Delta T$

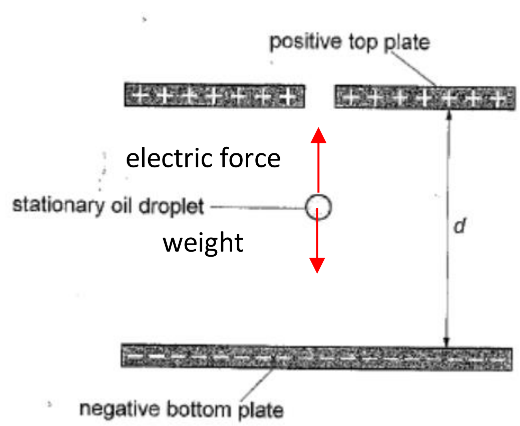
The temperature increases, so the work done on the gas is negative. Rewriting the first law of thermodynamics, [1]

$$\frac{3}{2}Nk\Delta T = C_P \Delta T - p\Delta V$$ [1]

$$C_P = \frac{\frac{3}{2}Nk\Delta T + p\Delta V}{\Delta T} = \frac{\frac{3}{2}Nk\Delta T + Nk\Delta T}{\Delta T} = \frac{5}{2}Nk$$ [1]

5c) For the same rise in temperature ΔT, under the constant pressure process, additional work must be done by the gas to the surroundings. Hence $C_P > C_V$. [1]

6ai)



[1]

Assume that the upthrust is negligible.

6aii) The oil droplet gained electrons when it went through the atomiser as it is attracted to the positively charged top plate and repelled by the negatively charged bottom plate. [1]

6aiii) Assume that the droplet has a spherical shape.

$$\rho = \frac{m}{V} \rightarrow m = \rho V = \rho \frac{4}{3}\pi \left(\frac{D}{2}\right)^3 = \frac{1}{6}\pi\rho D^3$$ [1]

6aiv)

$$m = \frac{1}{6}\pi\rho D^3 = \frac{1}{6}\pi(960)\left(0.5\times10^{-6}\right)^3 = 6.283\times10^{-17} \text{ kg}$$ [1]

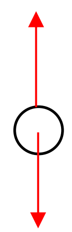
$$\frac{\Delta m}{m} = \frac{\Delta\rho}{\rho} + 3\frac{\Delta D}{D} = \frac{1}{100} + 3\frac{0.1}{0.5} = 0.61$$

$$\Delta m = 6.283\times10^{-17}(0.61) = 3.81\times10^{-17} \text{ kg}$$ [1]

$$\Delta m = 4\times10^{-17} \text{ kg}$$ [1]

$$m \mp \Delta m = \left(6\times10^{-17} \mp 4\times10^{-17}\right) \text{ kg}$$

6bi) [1]



Assume that the upthrust is negligible.

6bii) At terminal velocity, $F_D = W$

$$3\pi\eta D v_{\text{terminal}} = mg \rightarrow v_{\text{terminal}} = \frac{mg}{3\pi\eta D}$$

$$D^2 = B\frac{\eta v_{\text{terminal}}}{\rho g}$$

$$B = \frac{D^2\rho g}{\eta v_{\text{terminal}}} = \frac{D^2\rho g}{\eta\frac{mg}{3\pi\eta D}} = \frac{D^3 3\pi\rho}{m}$$

$$B = \frac{\left(0.5\times10^{-6}\right)^3 3\pi(960)}{6.238\times10^{-17}} = 18$$ [1]

6biii) When the electric field is switched on and the oil droplet is stationary,

Electric force = weight of oil droplet

[1]

$$q\frac{V}{d} = mg \rightarrow V = \frac{mgd}{q} = \frac{mgd}{ne} = \frac{1}{6}\frac{\pi\rho D^3 gd}{ne}$$

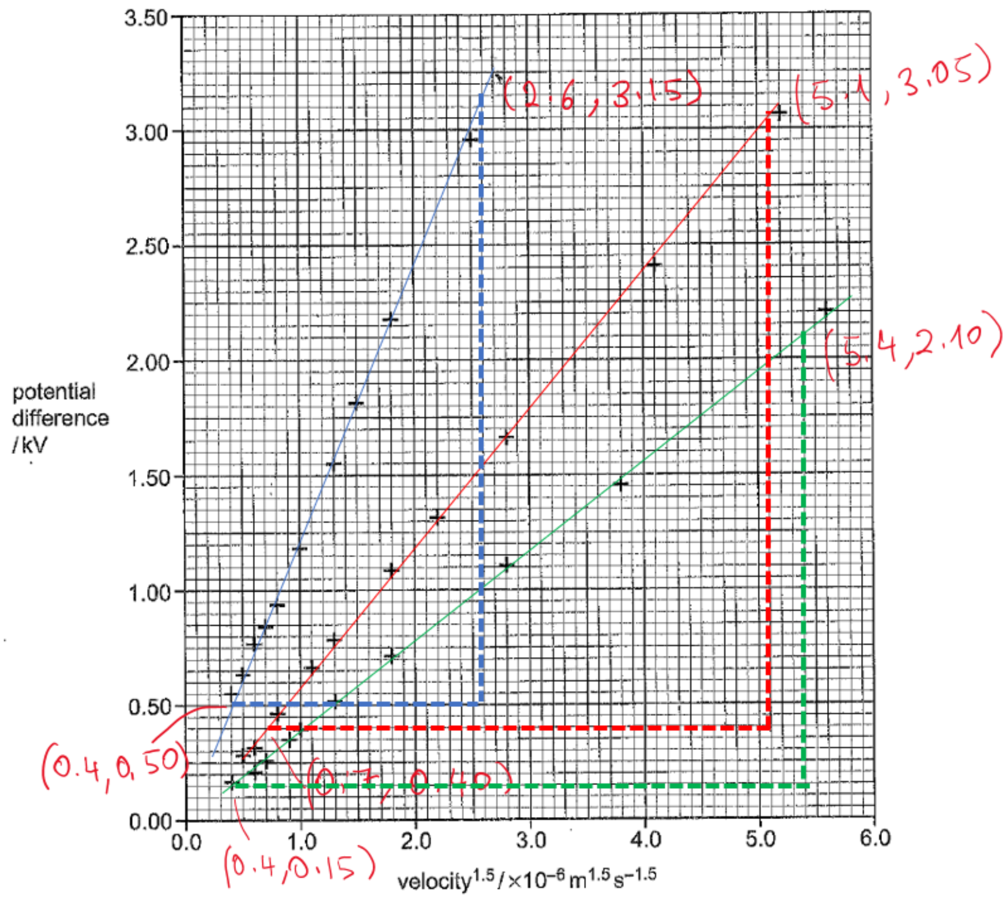
$$V = \frac{\pi\rho gd}{6ne}\left(\frac{B\eta}{\rho g}\right)^{3/2} v_{\text{terminal}}^{3/2}$$ [1]

$$k = \frac{\pi d}{6}\frac{(B\eta)^{3/2}}{(\rho g)^{1/2}} = \frac{\pi 6.0\times10^{-3}}{6}\frac{\left(18\times1.8\times10^{-5}\right)^{3/2}}{(960\times9.81)^{1/2}}$$ [1]

$$k = 1.89\times10^{-10}\ \text{kg m}^{1/2}\ \text{s}^{-1/2}$$

6ci) The Fig. 6.2 shows that there are three clusters of data points with different gradients (k/ne). Hence, three straight lines should be drawn. [1]
Since k and e are constant, oil droplets must carry different quantum charges, e, $2e$, $3e$ etc. [1] [1]

6cii)



$V = \frac{k}{ne}v_{\text{terminal}}^{3/2}$ and the gradient is $\frac{k}{ne}$

For blue linear fit: $\text{gradient} = \frac{(3.15-0.50)10^6}{(2.6-0.4)10^3} = 1.20\times10^9$

$$n_{blue}e = \frac{k}{(\text{gradient})} = \frac{1.89 \times 10^{-10}}{(1.2045 \times 10^{9})} = 1.57 \times 10^{-19}\ \text{C}$$

For red linear fit: $\text{gradient} = \dfrac{(3.05 - 0.40)10^{6}}{(5.1 - 0.7)10^{3}} = 0.602 \times 10^{9}$

$$n_{red}e = \frac{k}{(\text{gradient})} = \frac{1.89 \times 10^{-10}}{(0.602 \times 10^{9})} = 3.14 \times 10^{-19}\ \text{C}$$ [1]

For green linear fit: $\text{gradient} = \dfrac{(2.10 - 0.15)10^{6}}{(5.4 - 0.4)10^{3}} = 0.39 \times 10^{9}$

$$n_{green}e = \frac{k}{(\text{gradient})} = \frac{1.89 \times 10^{-10}}{(0.39 \times 10^{9})} = 4.85 \times 10^{-19}\ \text{C}$$

$$\frac{n_{red}}{n_{blue}} = 2,\ \frac{n_{green}}{n_{blue}} = 3.09,\ \frac{n_{green}}{n_{red}} = 1.5$$ [1]

From blue linear fit, $e = 1.57 \times 10^{-19}$ C
From red linear fit, $e = 1.57 \times 10^{-19}$ C
From green linear fit, $e = 1.62 \times 10^{-19}$ C

$e_{avg} = 1.59 \times 10^{-19}$ C [1]

Maximum uncertainty = $\frac{1}{2}\left(1.62 \times 10^{-19}\ \text{C} - 1.57 \times 10^{-19}\ \text{C}\right) = 0.03 \times 10^{-19}\ \text{C}$ [1]

Charge = (1.59 ± 0.03) x 10^{-19} C [1]

7ai)

$$g_{moon} = \frac{GM_{moon}}{r_{moon}^2}$$ [1]

$$1.62 = \frac{6.67 \times 10^{-11} M_{moon}}{(1.74 \times 10^6)^2}$$ [1]

$$M_{moon} = 7.35 \times 10^{22} \text{ kg}$$ [1]

7ii) Angular momentum of the moon: $p_{moon} = Mvr_{moon} = M\omega r_{moon}^2$ [1]

$$= (7.35 \times 10^{22}) \left(\frac{2\pi}{28 \times 24 \times 60 \times 60}\right) (3.84 \times 10^8)^2$$

$$= 2.82 \times 10^{34} \quad \text{kg m}^2\text{s}^{-1}$$ [1]

7b) The gravitational force by the Earth provides the centripetal force for the Moon.

$$\frac{GM_E M_{moon}}{r^2} = M_{moon} v_{moon}^2 / r$$ [1]

$$\frac{GM_E}{R^2} = g$$

$$\frac{R^2 g}{r^2} = \frac{v_{moon}^2}{r}$$ [1]

$$v_{moon} = \sqrt{\frac{gR^2}{r}}$$

7c)

$$L = I_{solid}\omega$$

$$L = \frac{2}{5} MR^2 \omega = \frac{4\pi MR^2}{5T}$$ [1]

$$L = \frac{4\pi \left(\frac{gR^2}{G}\right) R^2}{5T} = \frac{4\pi g R^4}{5GT} = k\frac{gR^\beta}{GT}$$ [1]

$$k = \frac{4\pi}{5} \quad \text{and} \quad \beta = 4$$ [1]

7di) The moon's gravitational pull (away from centre of Earth) on the water on the near side of Earth to the Moon, is stronger than the average moon's pull on water on Earth causing the water to bulge creating a high tide. [1]

The moon's gravitational pull (towards centre of Earth) on the water on the far side of Earth to the Moon, is weaker than the average moon's pull on water on Earth, causing the water to bulge creating a high tide. [1]

Note: You may learn more about high tides here

7dii)

$$\omega_f = \frac{2\pi}{T_f} \quad and \quad \omega_i = \frac{2\pi}{T_i}$$

$$(\omega_f - \omega_i) = \frac{2\pi}{T_f} - \frac{2\pi}{T_i}$$ [1]

$$\Delta\omega = \frac{2\pi(T_i - T_f)}{T_i T_f}$$

$$\Delta\omega = \frac{2\pi(\Delta T)}{T_i T_f}$$

Annual **decrease** in angular velocity

$$\Delta\omega \approx \frac{2\pi(\Delta T)}{T_{ave}^2}$$ [1]

7diii) In 4 million years, the period decreased by 4 hours. So, $\Delta T = \frac{4\text{ h}}{4\text{ million years}}$ [1]

Annual decrease in angular velocity

$$\Delta\omega = \frac{2\pi(24-20)(60)^2/(4\times10^6)}{(22\times60\times60)^2}$$

$$\Delta\omega = 3.6\times10^{-12} \text{ rad s}^{-1}$$ [1]

7div)

$$\Delta L = I_{solid}\,\Delta\omega = \frac{2}{5}MR^2\Delta\omega$$

$$= \frac{2\left(\frac{gR^4}{G}\right)}{5}\Delta\omega$$ [1]

$$= \frac{2}{5}\left(\frac{9.81\times(637800)^4}{6.67\times10^{-11}}\right)3.6\times10^{-12}$$

$$= 3.51\times10^{26}\text{ kg m}^2\text{ s}^{-1}$$ [1]

7dv) Time for Earth to stop = $7.1\ \times 10^{33} / 3.51\times10^{26}$

$$= 2.02\text{ years}$$ [1]

7dvi) Friction from the tidal movement is not constant over time. Since the angular rotation of Earth reduces, the frictional forces decrease too. Therefore, the rotational period of the Earth does not increase at a constant rate. [1]

8ai) A capacitance of 320 μF would mean that 320 μC of charge per Volt can be stored on the plates of the capacitor. [2]

8aii) As the capacitor is initially uncharged, when the switch is on,

$$V_b = ir + q/C$$ [1]

Differentiate with respect to time,

$$\frac{di}{dt} = -\frac{1}{rC}\frac{dq}{dt} = -\frac{i}{rC}$$ [1]

8aiii) At $t = 0$, the initial current $I_0 = \frac{V_b}{r} = \frac{1.5}{0.15} = 10A$ [1]

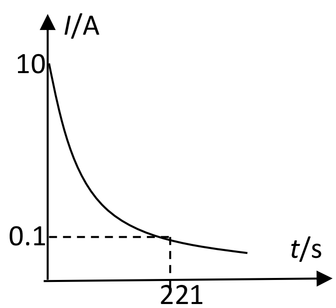
From the given expression,

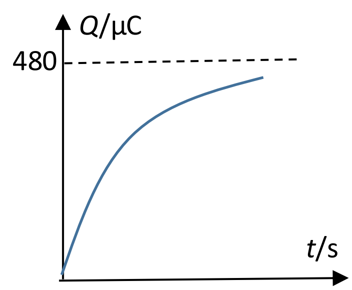
$$0.10 = 10e^{-t/(0.15)(320\times10^{-6})}$$

$$t = -(0.15)(0.00032)ln\left(\frac{0.1}{10}\right) = 2.21 \times 10^{-4}s$$ [1]

8aiv)

$$Q = CV = 320 \times 10^{-6}(1.5) = 480 \times 10^{-6}$$ [1]





Horizontal asymptote for *Q* – [1]
Correct shape for both graphs – [2]

8av) Energy dissipated in the battery,

$$U_{battery} = \int_0^{\infty} i^2 r dt = \int_0^{\infty} r I_0^2 e^{-2t/rC} dt = r I_0^2\left(\frac{-rC}{2}\right)(0-1) = \frac{1}{2}r^2 I_0^2 C$$ [2]

$$U_{battery} = \frac{1}{2}(0.15)^2(10)^2(320 \times 10^{-6}) = 0.00036J$$ [1]

8bi) As the capacitor is being discharged,

$$\frac{Q}{C} + iR + L\frac{di}{dt} = 0$$ [1]

Substitute $i = \frac{dQ}{dt}, \frac{di}{dt} = \frac{d^2Q}{dt^2}$ [1]

$$\frac{d^2Q}{dt^2} + \frac{R}{L}\frac{dQ}{dt} + \frac{Q}{LC} = 0$$

8bii) Differentiate the given expression with respect to time, we obtain

$$\frac{dQ}{dt} = (A - A\alpha t)e^{-\alpha t}$$ [1]

$$\frac{d^2Q}{dt^2} = (-2A\alpha + A\alpha^2 t)e^{-\alpha t}$$ [1]

Substituting these two expressions into the second order differential equation,

$$(-2A\alpha + A\alpha^2 t) + \frac{R}{L}(A - A\alpha t) + \frac{At}{LC} = 0$$

$$\left(\frac{A}{LC} - \frac{RA\alpha}{L} + A\alpha^2\right)t + \left(\frac{RA}{L} - 2A\alpha\right) = 0$$ [1]

Comparing coefficients of t^0, we obtain $\alpha = \frac{R}{2L}$ [1]

Comparing coefficients of t, we obtain

$$\left(\frac{A}{LC} - \frac{RA}{L}\frac{R}{2L} + A\left(\frac{R}{2L}\right)^2\right) = 0$$

Divide all the terms by A/L:

$$\frac{1}{C} - \frac{R^2}{2L} + \frac{R^2}{4L} = 0$$

$$\frac{1}{C} = \frac{R^2}{4L}, \qquad C = \frac{4L}{R^2}$$ [1]

9ai) The total momentum in a center of mass frame is zero.

$$0 = (m + dm)dv + (-dm)(-u) \rightarrow dv = -\frac{udm}{m + dm}$$ [1]

$(dm)(dv)$ is very small [1]

$$dv \approx -\frac{u}{m}dm$$

9aii)

$$\int_{v_0}^{v_f} dv = -\frac{u}{m}\int_{M_0}^{M_f} dm$$ [1]

$$v_f - v_0 = -\frac{u}{m}\ln\left(\frac{M_f}{M_0}\right) \rightarrow \Delta v = u\ln\left(\frac{M_0}{M_f}\right)$$ [2]

9bi)

$$\Delta v = u\ln\left(\frac{M_0}{M_f}\right) \rightarrow 9700 = 4500\ln\left(\frac{M_0}{M_f}\right) \rightarrow M_0 = 8.63M_f$$ [1]

M_0: initial total mass with fuel = mass of fuel + mass of (fuel tank + engines)
M_f : final total mass without fuel: mass of (fuel tank + engines)

mass of fuel: $M_0 - M_f$, which is $8.63M_f - M_f = 7.63M_f$

$$\frac{7.63}{8.63} \rightarrow 88.4\%$$ [1]

9bii) M_0: the initial total mass of the rocket = m_{fuel_1} + M_1
M_1: M_{stage2} + 0.08 M_0
0.08 M_0 is the mass of (fuel tank + engines) of the 1st stage rocket

$$5000 = 4500\ln\left(\frac{M_0}{M_1}\right)$$

$$M_0 = 3.038M_1 = 3.038\left(M_{stage2} + 0.08M_0\right)$$

$$M_{stage2} = 0.249M_0$$

$$m_{fuel\_1} = M_0 - 0.08M_0 - 0.249M_0$$

$$m_{fuel\_1} = 0.671M_0$$ [1]

M_{02}: the initial total mass of the rocket in stage 2 = M_{stage2}
M_{f2}: M_{stage2} - m_{fuel_2}

$$4700 = 4500\ln\left(\frac{M_{02}}{M_{12}}\right) \rightarrow e^{4700/4500} = \frac{0.249M_0}{0.249M_0 - m_{fuel\_2}}$$ [1]

$$m_{fuel\_2} = 0.161M_0$$

$$\frac{m_{fuel\_1} + m_{fuel\_2}}{M_0}100\% = 83.2\%$$ [1]

9biii) To increase the orbital range of the rockets. They could reach much further distances when multistage rockets are used. [1]

9ci) Average thrust = $F_{ave} = \Delta m \dfrac{u}{\Delta t} \rightarrow u = \dfrac{(35.1)10^6(152)}{2.16\times10^6} = 2470$ m s^{-1} [2]

9cii) $\Delta v = u\ln\left(\dfrac{M_0}{M_f}\right) = 2470\ln\left(\dfrac{2.97\times10^6}{(2.97-2.16)\times10^6}\right) = 3.2$ km s^{-1} [1]

9ciii) The shape of rockets minimizes the surface area that are in contact with air particles. [1]

The fuel, which is about 80% of the mass of rocket, is burnt in a short period of time. The much lighter rocket reaches a very high speed at that time. The effects of air resistance are greatly reduced during that time. [1]

9civ) $\Delta v = u\ln\left(\dfrac{M_0}{M_f}\right) - g't_{burn}$ [1]

$2300 = 3200 - g'(152) \rightarrow g' = 5.92$ m s^{-2} [1]

9di) For the rocket to be launched, the thrust must be greater than *mg*. The value in **(c)(iv)** is over-estimation. [1]

Note: In usual rocket launches, the upward acceleration is about 0.20*g*.

9dii) To get a boost from the rotational speed of the Earth so that less fuel would be used to reach the same speed for the rocket. [1]

You may read the worked example below, which is similar to Q9, from the book Kleppner and Kolenkow (page 142).

Example 4.18 Saturn V

The Saturn V ("Five") three-stage rocket, one of the most powerful expendable launch vehicles ever constructed, fulfilled its purpose by sending Apollo astronauts to land on the Moon on six different missions. The first stage was powered by five enormous F-1 rocket engines (each nearly 6 m tall and 4 m diameter at the outlet). The F-1 engines burned a hydrocarbon similar to kerosene, with liquid oxygen as the oxidizer. All of these materials had to be carried by the rocket; a fully fueled Saturn V had a total mass of 3.0×10^6 kg, of which 2.1×10^6 kg was the fuel for the first stage. All the first-stage fuel was expended in 168 seconds.

The rocket equation with constant gravity is

$$M\frac{d\mathbf{v}}{dt} = \mathbf{u}\frac{dM}{dt} + M\mathbf{g}. \tag{1}$$

The first term on the right-hand side of Eq. (1) is called the "thrust." The second term is the weight of the rocket, directed vertically downward. At launch, the weight was 2.9×10^7 N, but this decreased rapidly as the first stage burned its fuel. The five F-1 engines in the first stage produced a total thrust of 3.4×10^7 N, somewhat greater than the initial weight. The initial upward acceleration, as you can easily verify, was only about 0.17 g.

Where does the thrust come from? Because $\mathbf{u}\Delta M$ is the momentum carried off by the expelled gases in time Δt, the thrust is the rate at which momentum is carried off by the burning fuel. Because both $\mathbf{u}$ and dM/dt are negative, the thrust is positive, opposite to $\mathbf{g}$.

Fuel is a precious commodity on rockets. To minimize the fuel mass required for a given thrust, the exhaust velocity must be as large as possible. The exhaust velocity for the first stage F-1 engines was 2600 m/s, but the second and third stages used liquid hydrogen and liquid oxygen, giving an exhaust velocity of 4100 m/s.

Evaluating the right-hand side of Eq. (1) for the first stage gives $(2600\,\mathrm{m/s})(2.1 \times 10^6\,\mathrm{kg})/168\,\mathrm{s} = 3.4 \times 10^7$ N, in good agreement with the thrust.

Rocket data tables often do not list the exhaust velocity but instead a quantity called the "specific impulse," which is the exhaust velocity divided by g. Specific impulse has units of seconds, and is therefore independent of whether we use SI, CGS, or English units.