1 Without the use of the graphic calculator, solve the inequality

$$\frac{4x^2+4x+1}{x^2+x+1}>0.$$

Hence solve the inequality $\dfrac{x^2+4x+4}{x^2+x+1}>0$. [5]

2 Given that points A, B, and C have position vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ respectively, relative to the origin O, and the point D is such that $ABCD$ is a parallelogram.

(i) Write down an expression for the position vector of D, in terms of $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$. [1]

(ii) Prove that the area of the parallelogram $ABCD$ is given by

$$\left|\mathbf{a}\times\mathbf{b}+\mathbf{b}\times\mathbf{c}+\mathbf{c}\times\mathbf{a}\right|.$$ [3]

(iii) Show that shortest distance of D to AC is given by

$$\frac{\left|\mathbf{a}\times\mathbf{b}+\mathbf{b}\times\mathbf{c}+\mathbf{c}\times\mathbf{a}\right|}{\left|\mathbf{c}-\mathbf{a}\right|}.$$ [1]

3 By using the substitution $t=\mathrm{e}^{1-x}$, show that

$$\int_0^1 \mathrm{e}^{1-x}\tan^{-1}\left(\mathrm{e}^{1-x}\right)\mathrm{d}x=\int_1^{\mathrm{e}}\tan^{-1}t\,\mathrm{d}t.$$

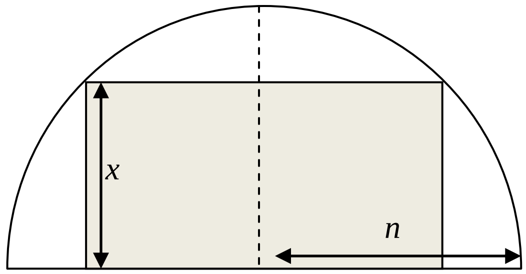
Hence find the exact value of $\int_0^1 \mathrm{e}^{1-x}\tan^{-1}\left(\mathrm{e}^{1-x}\right)\mathrm{d}x$. [6]

4 Prove by the method of mathematical induction that

$$\cos\theta+\cos2\theta+...+\cos n\theta=\frac{\sin\left(n+\frac{1}{2}\right)\theta-\sin\left(\frac{1}{2}\theta\right)}{2\sin\left(\frac{1}{2}\theta\right)}$$

for all positive integers n. [6]

5



The diagram shows the cross sectional view of a cylinder open on both ends inscribed in a hemisphere with fixed radius n cm. If the diameter of the cylinder is x cm, show that the surface area A of the cylinder is

$$2\pi x\sqrt{n^2 - x^2}\ \text{cm}^2. \qquad [2]$$

Given that as x varies, the maximum value of A occurs when the ratio of the diameter of the cylinder to the height of the cylinder is $\frac{1}{k}$. Find the value of k. [5]

6 A curve C is defined by the parametric equations

$$x = t - \cos t, \quad y = 3 + \sin 2t, \text{ for } -\pi \le t \le 0.$$

Find

(i) the exact values of the turning point(s) to the curve, [3]

(ii) the equation of the tangent where $t = -\frac{\pi}{2}$, and [2]

(iii) the area of the region bounded by C, the tangent in **(ii)**, and the line $x = -\frac{\pi}{4} - \frac{\sqrt{2}}{2}$. [2]

7 Use the method of difference to find $\sum_{r=1}^{N} \frac{2}{r(r+2)}$ in terms of N. [4]

Hence express

$$\sum_{r=6}^{N+3} \frac{2}{(r-4)(r-2)}$$

in the form of $a + \mathrm{f}(N)$, where a is a constant and $\mathrm{f}(N)$ is an expression in terms of N, are to be determined, and deduce the value of

$$\sum_{r=6}^{\infty} \frac{2}{(r-4)(r-2)}. \qquad [4]$$

8 A piece of meat is taken from an oven when it reaches a temperature of $20^\circ C$ and put in a refrigerator where the temperature is set to a constant value of $-20^\circ C$. The rate of cooling in t minutes is proportional to the difference between the body temperature $F^\circ C$ and its immediate surrounding temperature. It was observed that the meat cools to $-10^\circ C$ in 10 minutes.

(i) By forming and solving a differential equation, show that

$$F = 40\left(\frac{1}{4}\right)^{\frac{t}{10}} - 20.$$ [6]

(ii) How many more minutes are required for the meat to cool further to $-15^\circ C$? [2]

9 A hamster farm has 500 hamsters to sell. The farmer sells k hamsters at the end of every week, where k is a constant factor of 500 (e.g. 5, 100, etc.). The selling price of a hamster is $10 in the first week and it drops by 5% in each subsequent week. It is assumed that there is neither birth nor death of hamsters in the farm.

(i) State the total number of weeks for the farmer to sell all his hamsters in terms of k. [1]

(ii) Show that the total proceeds from selling all the hamsters is

$$200k\left(1-0.95^{\frac{500}{k}}\right).$$ [3]

(iii) Given that the cost of rearing a hamster is $0.50 a week, find the total cost incurred in rearing the hamsters, when the farmer has sold all his hamsters. [3]

(iv) If all other costs are negligible, find the least value of k for the farmer to make a profit. [2]

10 **(a)** Solve the simultaneous equations

$$2p - q\mathrm{i} = 2\,,$$
$$p^2 - q + 8 + 2\mathrm{i} = 0\,,$$

for the complex numbers p and q in Cartesian form. [3]

(b) Express $\dfrac{-1+\mathrm{i}}{\sqrt{3}-\mathrm{i}}$ in both Cartesian and trigonometric forms.

Hence show that

$$\cos\left(\frac{11\pi}{12}\right) = -\frac{\left(\sqrt{6}+\sqrt{2}\right)}{4}\quad.$$ [5]

Deduce the exact value of $\tan\left(\dfrac{11\pi}{12}\right)$. [1]

11 The curve C has equation

$$c^2x^2 - b^2y^2 - a^2 = 0\,,$$

where a, b and c are constants such that $a > b > c > 0$.

(i) Show that C has no turning points. [3]

(ii) Obtain the equations of the asymptotes of C. [2]

(iii) State the restriction of x and the axes of symmetry of C. [2]

(iv) Sketch C, stating the equations of the asymptotes and the coordinates of the points of intersection with the x-axis. [3]

(v) It is given that the equation

$$(c - bk)(c + bk)x^2 = a^2$$

has 2 real roots. By using your sketch in **(iv)**, show that $-1 < k < 1$. [3]

12 The Cartesian equation of the plane p_1 is given by $x - y = 5$. The position vectors of the points A and B are given by $\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$ and $4\mathbf{i} - \mathbf{j} + 2\mathbf{k}$ respectively.

(i) Find the acute angle between the position vector of A and the plane p_1. [2]

(ii) Find the position vector of the foot of the perpendicular from A to plane p_1. [3]

(iii) Show that B lies in p_1 and hence find the vector equation of the reflection of line AB in the plane p_1. [4]

(iv) Find an equation for the plane p_2, in the form $\mathbf{r} \bullet \mathbf{n} = p$, which contains the points A and B and is perpendicular to p_1. Hence, or otherwise, find the vector equation of l, the line of intersection of p_1 and p_2. [5]

(v) The plane p_3 has a Cartesian equation $2x + 3y + az = d$, where a and d are constants. It is given that p_1, p_2 and p_3 have no point in common and the distance of B from p_3 is $\sqrt{13}$. Find the value of a, and determine the value(s) of d. [3]

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