Chapter 16 ELECTROMAGNETISM

Content

- Concept of a magnetic field
- Magnetic fields due to currents
- Force on a current-carrying conductor
- Force between current-carrying conductors
- Force on a moving charge [Not in H1 syllabus]

Learning Outcomes

[redacted]

Candidates should be able to:

(a) show an understanding that a magnetic field is an example of a field of force produced either by current-carrying conductors or by permanent magnets.

(b) sketch flux patterns due to currents in a long straight wire, a flat circular coil and a long solenoid.

(c) use $B = \dfrac{\mu_0 I}{2\pi d}$, $B = \dfrac{\mu_0 N I}{2r}$ and $B = \mu_0 n I$ for the flux densities of the fields due to currents in a long straight wire, a flat circular coil and a long solenoid respectively.

(d) show an understanding that the magnetic field due to a solenoid may be influenced by the presence of ferrous core.

(e) show an understanding that a current-carrying conductor placed in a magnetic field might experience a force.

(f) recall and solve problems using the equation $F = BIL \sin\theta$, with directions as interpreted by Fleming's left-hand rule.

[redacted] BIL sin θ
→ number of turns

(g) define magnetic flux density and the tesla.

(h) show an understanding of how the force on a current-carrying conductor can be used to measure the flux density of a magnetic field using a current balance.

(i) explain the forces between current-carrying conductors and predict the direction of the forces.

(j) predict the direction of the force on a charge moving in a magnetic field. [Not in H1 syllabus]

(k) recall and solve problems using $F = BQv \sin\theta$. [Not in H1 syllabus]

(l) describe and analyse deflections of beams of charged particles by uniform electric and uniform magnetic fields. [Not in H1 syllabus]

(m) explain how electric and magnetic fields can be used in velocity selection for charged particles. [Not in H1 syllabus]

16.1 Concept of a magnetic field

Properties of magnets

- Natural magnets have been discovered from ancient archaeological sites more than two thousand years old. The term 'magnet' comes from the name of one of the locations these stones were found.
- The magnetic properties of a magnet originate at certain regions in the magnet called **poles**.
- Experiments have shown that
  (i) Magnetic poles are of 2 kinds : North (**N**) or South (**S**)
  (ii) Like poles repel each other, unlike poles attract.
  (iii) Poles occur in equal and opposite pairs (dipoles).
  (iv) When no other magnet is near, a freely suspended magnet will align so that the line joining its poles is approximately parallel to the Earth's North–South axis.
  (v) The pole that points towards the north is called the north pole of the magnet and the other the south pole.

Concept of a magnetic field

- Forces between magnets can be explained using the concept of a magnetic field.
  - A magnet sets up a magnetic field in its vicinity.
  - The force exerted by one magnet on another magnet is due to the interaction between one magnet and the magnetic field of the other.

Definition

- A **magnetic field** is a region of space where a **magnetic pole** experiences a force.

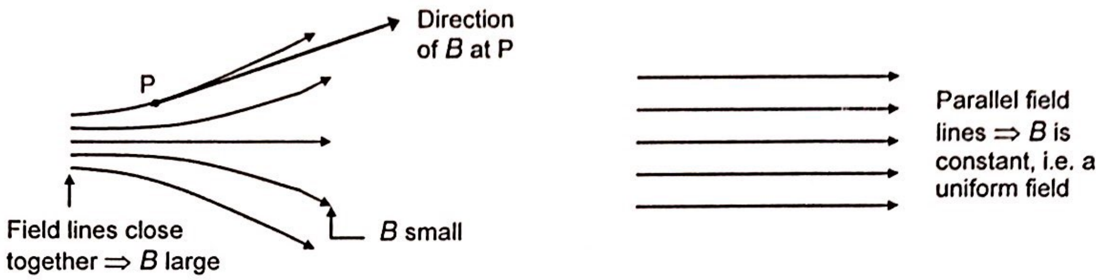
(not magnetic field strength)

- Quantitatively, the strength of a magnetic field is expressed by a quantity called the **magnetic flux density**. Its unit is the **tesla (T)**. (The magnetic flux density and its unit will be defined later.) Magnetic flux density is a vector.
  can be resolved
- It is important to note that **magnetic fields** are produced and experienced by **moving charges**, as opposed to <u>*electric fields*</u>, which are produced by <u>*static charges*</u>.

Representation of a magnetic field

A magnetic field can be represented by field lines drawn such that:

- the tangent to a field line at a point gives the direction of B at that point, and
- the number of lines per unit cross-sectional area is proportional to the magnitude of B. If the field is uniform, the field lines are evenly spaced.
- the arrows point away from the N-pole of a magnet toward the S-pole. This is because the direction of the field is given by the direction of the force that acts on the north magnetic pole of a magnet.



(a) non-uniform field (b) uniform field

Fig. 1

For a **two-dimensional view**, magnetic fields can be represented by dots and crosses, depending on whether it is perpendicularly pointing out or into the plane of the paper respectively.

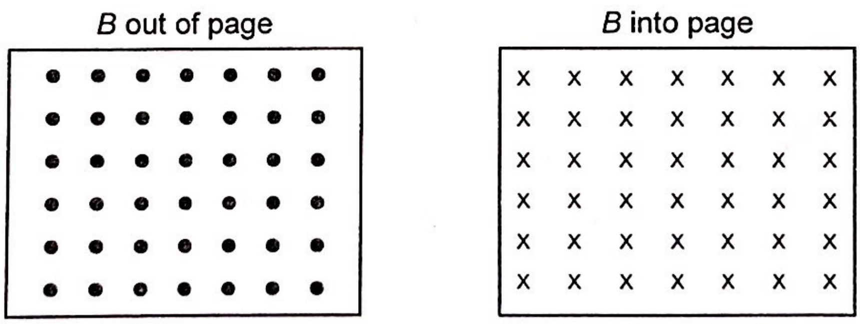
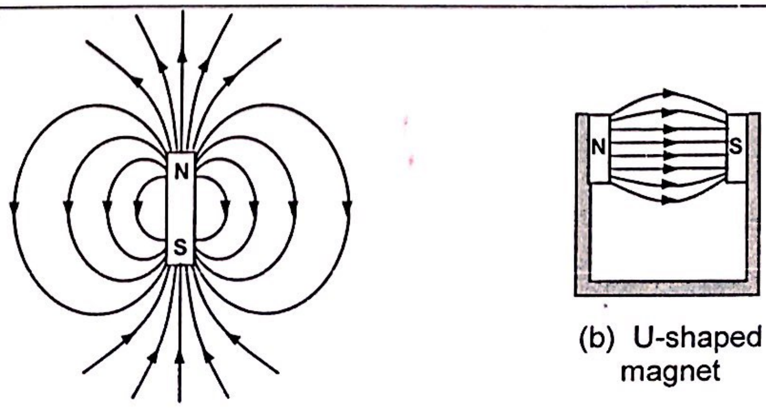


Fig. 2

Examples of field patterns

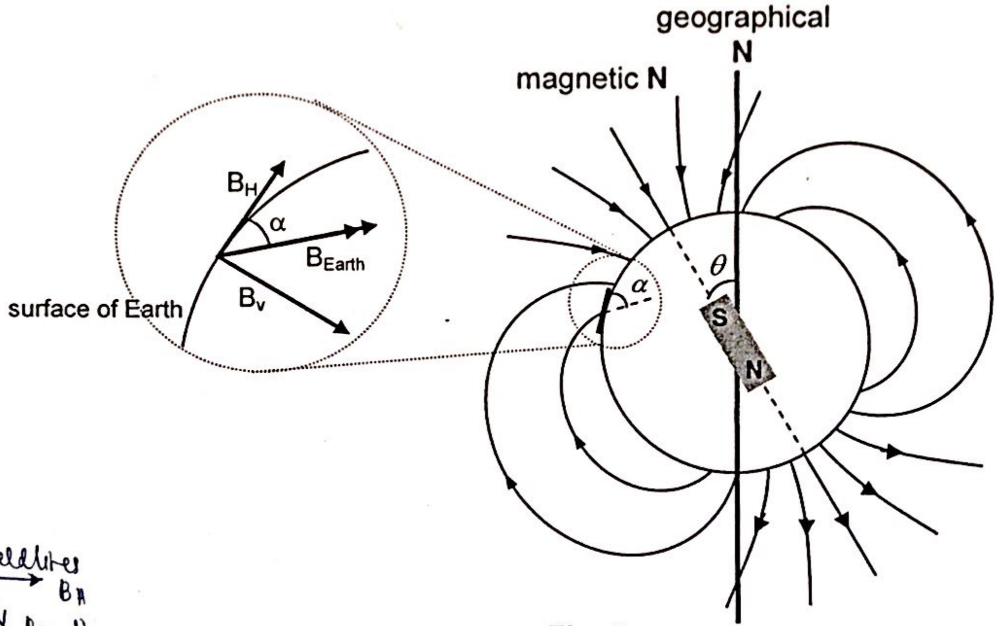


(a) Bar magnet (b) U-shaped magnet

Fig. 3

Earth's magnetic field

- The Earth's magnetic field is a weak magnetic field believed to be caused by electric currents circulating within the core of the Earth.
- The magnitude and direction of this field varies with position over the Earth's surface and changes gradually with time.
- It is known that the axis of the Earth's magnetic field is approximately tilted 11 ° with respect to its rotational axis. Hence, the geographical North does not coincide exactly with the magnetic North.
- The field pattern is similar to that of a bar magnet embedded deep inside the centre of the Earth.
  - The north of a compass needle points towards the Earth's magnetic North pole (because the north end of a compass needle is attracted to the south of a bar magnet).



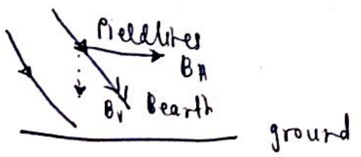


Fig. 4

- From the field pattern, it can be seen that, near the equator, the Earth's magnetic field, B_{Earth}, is almost horizontal.
- At all other positions, the B_{Earth} is inclined at various angles (called the *angle of inclination,* α) from the horizontal.

It is convenient to resolve B_{Earth} into horizontal and vertical components.

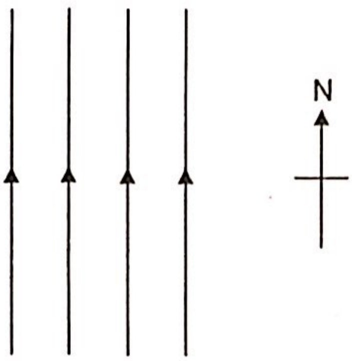
Formula

$$B_H = B_{Earth} \cos \alpha$$
$$B_V = B_{Earth} \sin \alpha$$

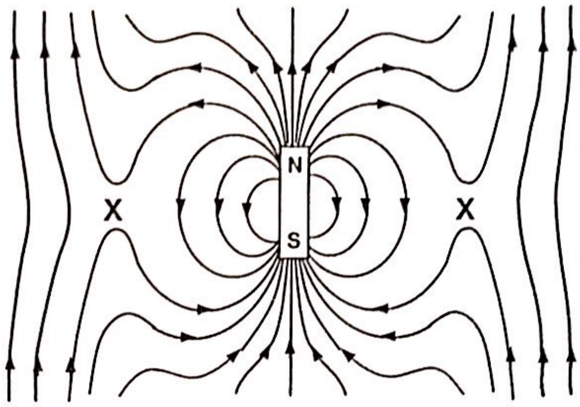
Instruments such as compass needles whose motion is confined in a horizontal plane are affected only by B_H.

Locally, the Earth's magnetic field can be considered as a uniform field; the lines are parallel, equally spaced and point due north.

Fig. 5 (a) shows the Earth's local field in a horizontal plane and Fig. 5 (b) shows the combined field due to the Earth and a bar magnet with its N pole pointing N. The crosses indicate neutral points where the resultant field is zero.



(a) Earth's local field in a horizontal plane



(b) Effect of Earth's field on that of a bar magnet.
X = neutral point

Fig. 5

16.2 Magnetic fields due to currents

The birth of electromagnetism

In 1819, Hans Christian Oersted discovered the magnetic effect of an electric current. His findings saw the birth of electromagnetism.

- It was found that when a compass was aligned on one side of a current-carrying conductor, the needle would be deflected in one way.
- When aligned on the other side of the same conductor, the needle would deflect in the opposite direction.
- By arranging an assortment of compasses around the current-carrying conductor, the pattern of the magnetic field which surrounds it can be seen.

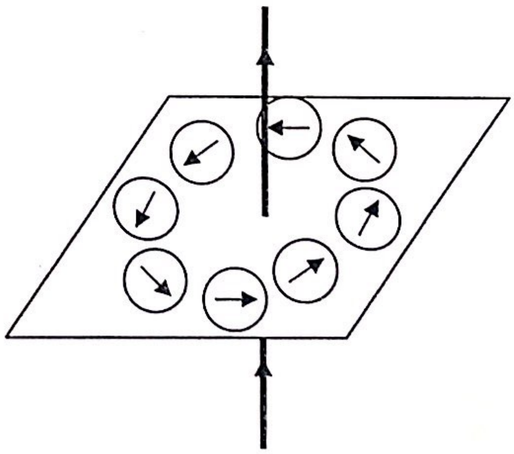


Fig. 6

Following Oersted's discovery, experiments deduced that there is a relationship between the magnetic field of a current carrying conductor and the current which flows through it.

static charge → electric field
moving charge → electric field + magnetic field

Magnetic field due to an infinitely long straight wire

Fig. 7 shows the field pattern around an infinitely long straight conducting wire.

- The magnetic field lines are concentric circles.
- The separation between the field lines increase with distance from the wire. This indicates that the magnetic flux density decreases in magnitude.
- The direction of the field can be determined using Maxwell's *right-hand grip rule*: Grip the wire using the right hand with the thumb pointing in the direction of the current - the fingers then point in the direction of the field.

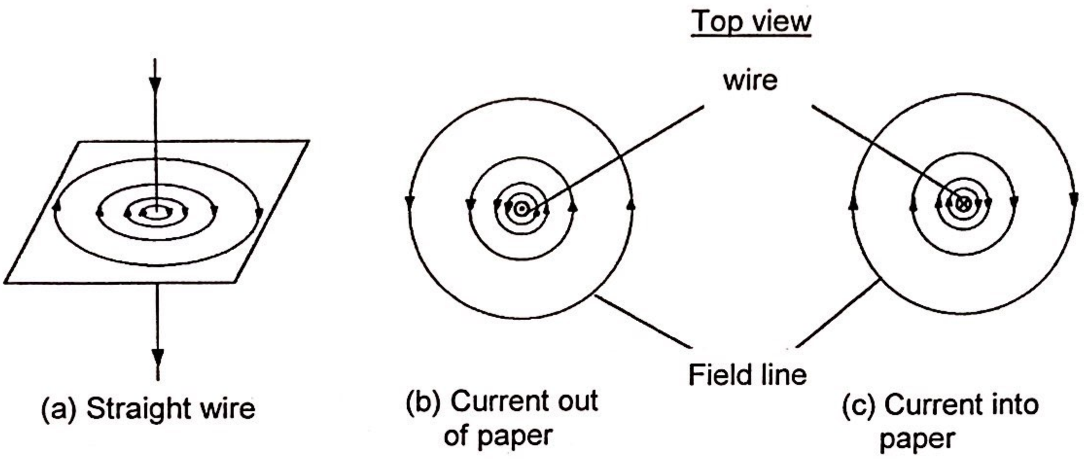


Fig. 7

good to have a compass

Fig. 8 shows an infinitely long straight wire lying in the plane of the paper. At the point P, the magnetic flux density due to the current in the wire is directed into the paper and its magnitude is given by

Formula

$$\boxed{B = \frac{\mu_0 I}{2\pi r}}$$

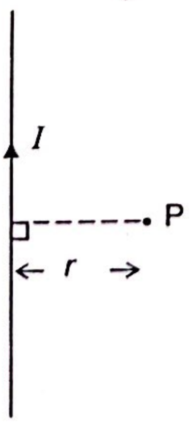


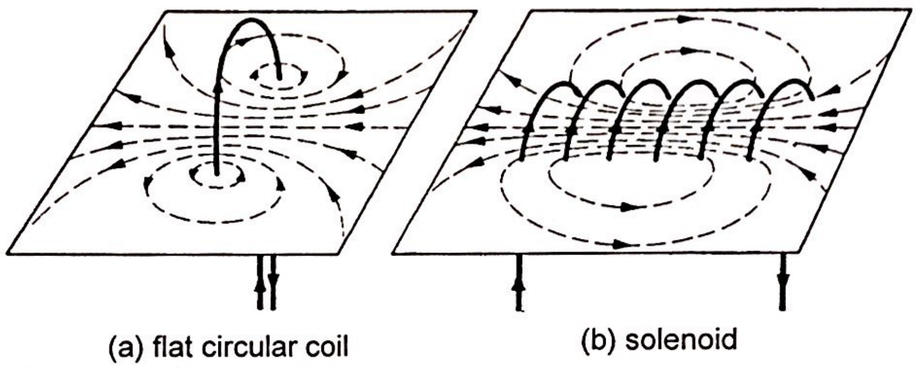
Fig. 8

where
B = magnetic flux density
I = current in the wire
r = perpendicular distance of P from wire
μ_0 = a constant known as the permeability of free space (vacuum).
It is assigned a value of $4\pi \times 10^{-7}$ H m^{-1}.

Magnetic fields due to a circular coil and solenoid

Fig 9 shows the field patterns due to a circular coil and a long solenoid

- The direction of the field lines within the coil and the solenoid can be found using the *right-hand grip rule*: Grip the coil in your right hand with your fingers pointing in the direction of the current, then your thumb gives the direction of the magnetic field.
- When the turns of a solenoid are closely spaced, each can be regarded as a circular coil. Hence, the net field in a solenoid is the effect of many circular turns.
- The magnetic field inside a long solenoid is uniform.



Top view:

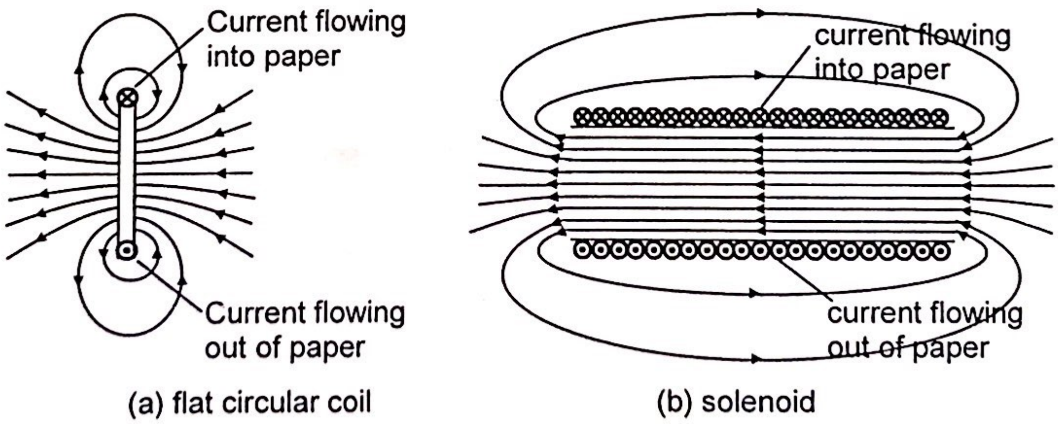


Fig 9

Fig. 10 shows a flat circular coil lying in the plane of the paper. At P, the centre of the coil, the magnetic flux density due to the current in the coil is directed into the paper (by the right-hand grip rule) and its magnitude is given by

Formula

$$\boxed{B = \frac{\mu_0 N I}{2r}}$$

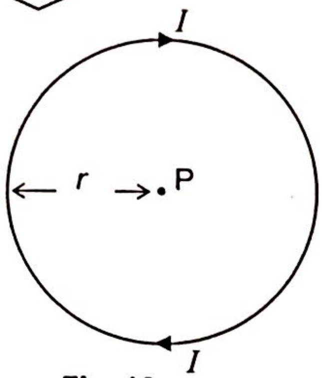


Fig. 10

where B = magnetic flux density

μ_0 = permeability of free space

N = Number of turns

I = current in the coil

r = radius of the coil

The magnetic flux density on the axis of an infinitely long solenoid is directed along the axis and its magnitude is given by

Formula

$$\boxed{B = \mu_0 n I}$$

where B = magnetic flux density

μ_0 = permeability of free space

n = number of turns per unit length of solenoid

I = current in the coil

r = radius of the coil

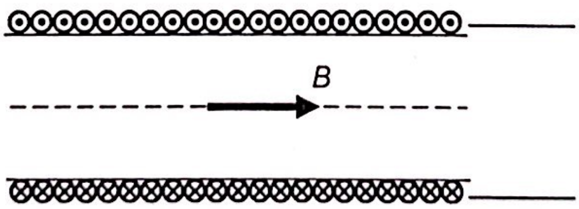


Fig. 11: Cross section view of a Solenoid

At either end of a finite length solenoid, the flux density on the axis is

$$\frac{1}{2}\mu_0 n I .$$

Note

For the A-Level Examination, the above formulae are provided in the formulae list.

wire coil solenoid

Solenoids and electromagnets

Effect of a ferrous core

A bar of iron can be magnetized by placing it inside a solenoid. When a current passes through the solenoid, it produces a magnetic field along its axis and the bar is magnetized accordingly. The resultant magnetic field is the sum of the field due to the current and that due to the iron core so that the magnitude of the resultant field can have a magnitude hundreds to thousands times that due to the current alone.

Uses of electromagnets

An iron core with a magnetizing coil wound around it is called an electromagnet. Its strength can be adjusted to exert large mechanical forces to lift heavy loads. See Fig. 12.

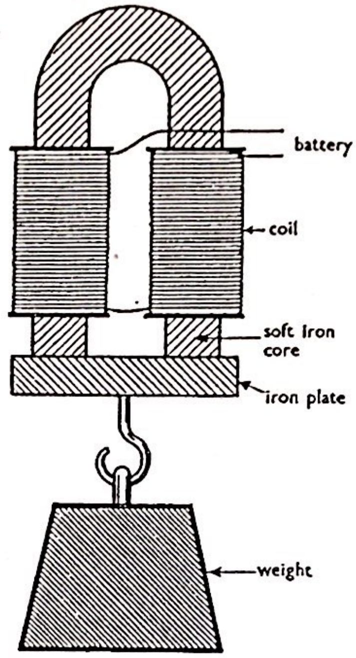


Fig. 12

Relay:
This is a switch worked by an electromagnet. It is useful if we want a small current in one circuit to control another circuit containing a device such as a lamp, electric bell or motor which requires a large current.

The structure of a relay is shown in Fig. 13. When the controlling current flows through the coil, the soft iron core is magnetized and attracts the L–shaped soft iron armature. This rocks on its pivot and closes the electrical contacts in the circuit being controlled.

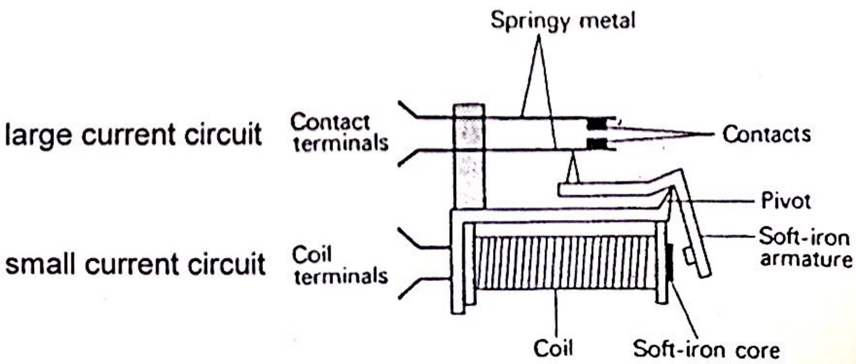
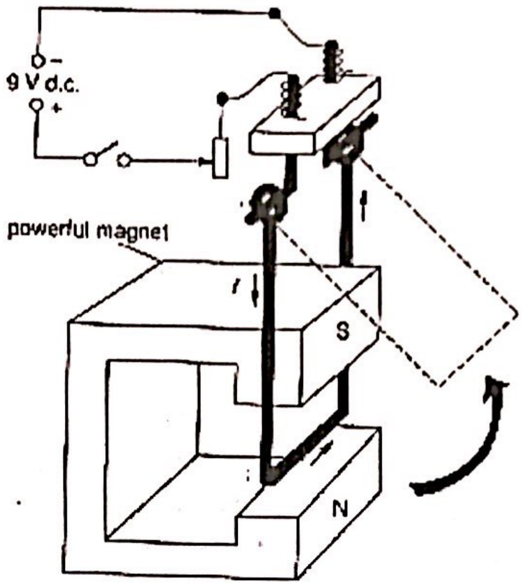


Fig. 13

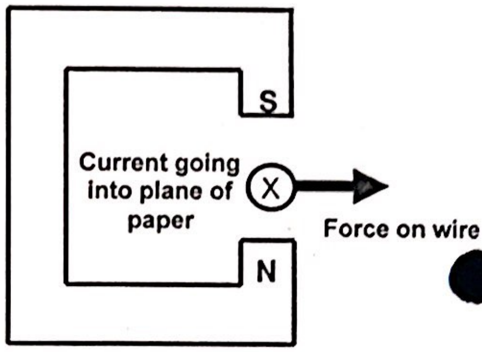
16.3 Force on a current-carrying conductor

Direction of the magnetic force

A current–carrying conductor in a magnetic field experiences a force.



(a) The 'kicking wire' experiment



(b) front view of magnet and wire

Fig. 14

The magnetic force is always perpendicular to the direction of the current and the direction of the magnetic field.

The relative directions of current, field and force are summarized by ***Fleming's Left–Hand Rule***. It states that *if the thumb and first two fingers of the left hand are put mutually at right–angles,*

- *the first finger is pointed in the direction of the field*
- *while the second finger is in the direction of the current,*
- *then the thumb gives the direction of the force or motion.*

common test · only left hand

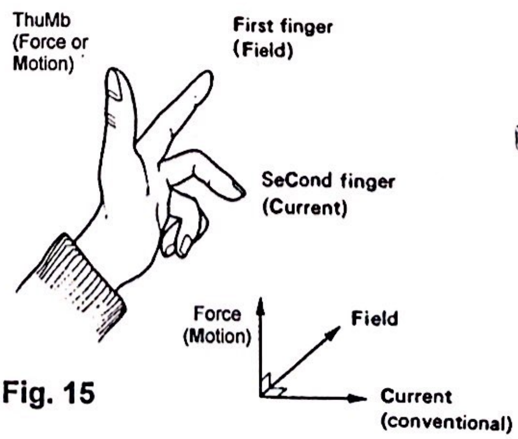


Fig. 15

F B I

father mother child

Magnitude of the magnetic force and definition of magnetic flux density

Experiments with a wire placed at right angles to the magnetic field show that the magnitude of the magnetic force F is directly proportional to

- the **current I** in the wire
- the **length L** of the wire in the magnetic field
- the **magnetic flux density strength B**

This leads to a definition of magnetic flux density:

Definition

> The **magnetic flux density** of a magnetic field is numerically equal to the force per unit length of a long straight conductor carrying a unit current at right angles to a uniform magnetic field.

$$B = \frac{F}{IL} \quad \ldots(1)$$

where B = magnetic flux density
F = magnetic force acting on the wire
I = current flowing through the wire
L = length of wire

If F = 1 N, I = 1 A and L = 1 m, the S.I. unit for flux density is N A^{-1} m^{-1} which is given the special name, the **tesla** (T).

vector quantity

$$1\ \text{T} = 1\ \text{N A}^{-1}\ \text{m}^{-1}$$

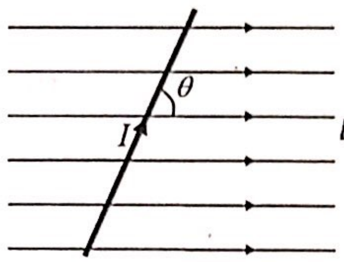
Definition

> One **tesla** is the uniform magnetic flux density which, acting normally to a long straight wire carrying a current of 1 ampere, causes a force per unit length of 1 N m^{-1} on the wire.

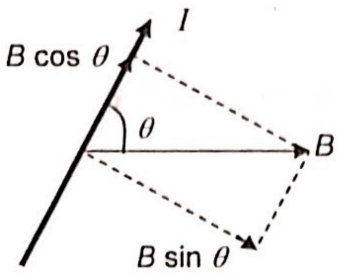
Re–arranging equation (1), the force F acting on a wire of length L when placed at right angles to the field is given by

$$F = BIL$$

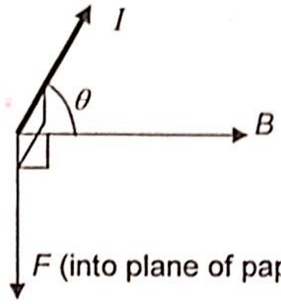
If the wire is placed parallel to the field the force drops to zero. In the general case, if the conductor makes an angle θ with the field (see Fig. 16(a)), the force can be regarded as due to the *component* of the field *perpendicular* to the current *($B \sin \theta$)*; the component parallel to the current produces no force.



(a) Top view of a conductor placed on a horizontal surface



(b) Components of magnetic field parallel and perpendicular to conductor



(c) Direction of force is perpendicular to plane containing B and I

Fig. 16

Formula

B is being resolved

Thus, the force F is given by $\boxed{F = BIL\sin\theta}$

The direction of this force is shown in Fig. 16(c) as predicted by Fleming's Left Hand Rule.

Example 1 A wire, 2.0 m in length, carrying a current of 10 A is placed in a field of flux density 0.15 T. What is the magnitude of the force on the wire if it is placed

(a) at right angles to the field,

(b) at 30° to the field, and

(c) along the field?

Solution:

a) $F = BIL\sin\theta$

$= (0.15)(10)(2.0)$

$= 3.00\text{ N}$

b) $F = BIL\sin 30°$

$= (0.15\sin 30°)(10)(2.0)$

$= 1.50\text{ N}$

c) $F = BIL\,\cancel{\sin 90°}\,\sin 0°$

$= 0\text{ N}$

Measuring magnetic flux density using the current balance

A current balance is an arrangement that can be used to measure the magnetic flux density of a magnetic field. It makes use of the principle of moments.

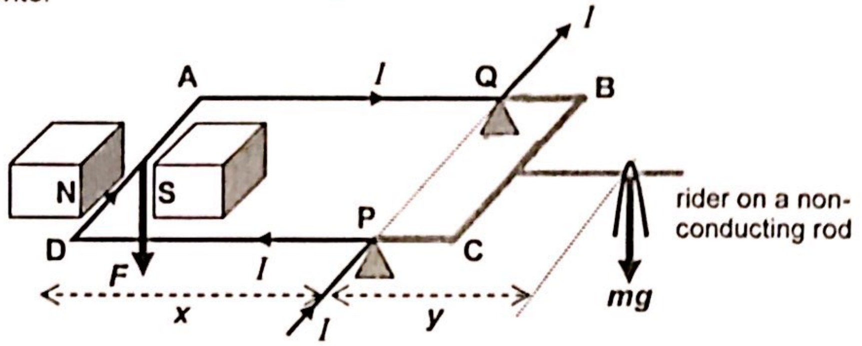


Fig. 17

- A wire frame ABCD is balanced on 2 pivots through which a current I from a d.c. source enters from P and leaves from Q.
- The frame is arranged such that the side AD of the frame (of length L) lies within a magnetic field whose flux density B is to be determined.
- When there is no current, the frame is horizontal.
- When current flows, a magnetic force acts on AD and pushes that side of the frame downwards (Fleming's Left-Hand Rule).
- A mass m (known as a rider) is suspended on the right side to restore the frame to its horizontal position.

  By the principle of moments,

  Sum of clockwise moments = Sum of anticlockwise moments

$$mgy = Fx$$
$$= BILx$$
$$B = \frac{mgy}{ILx}$$

Hence, the magnetic flux density of a magnetic field can be determined with the appropriate quantities known.

Hall Probe vs Curent Balance

Another device that is more commonly used to measure magnetic field strength is the Hall probe. The following table highlights some differences between the two devices.

Current Balance	Hall Probe
• Does not require calibration. Deduces B using force, current and length of test wire.	• Requires calibration by a magnetic field of known strength.
• Unable to measure weak fields.	• Can measure very weak fields.
• Apparatus is bulky. Not so practical and portable.	• Practical and convenient due to the small size of the probe.

Torque on a current-carrying coil in a magnetic field

Consider a rectangular coil placed in the plane of a uniform magnetic field as shown in Fig. 18. The plane of the coil is parallel to the field lines.

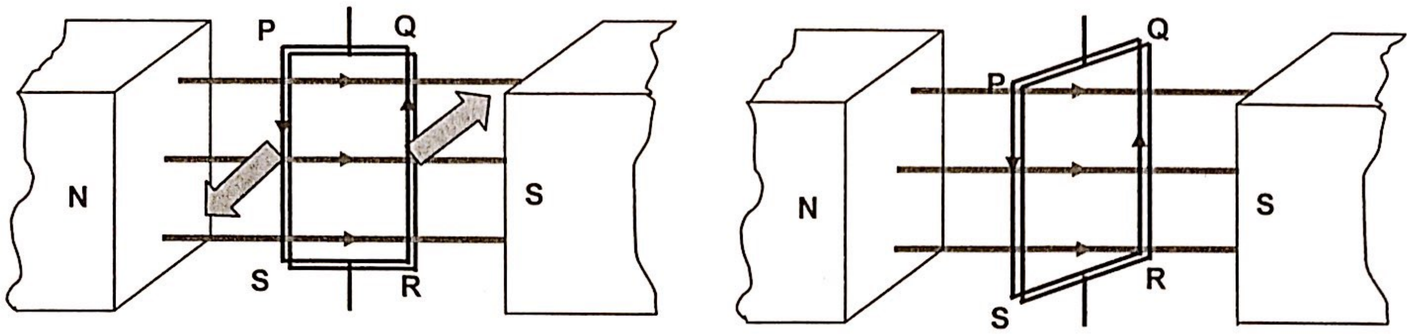
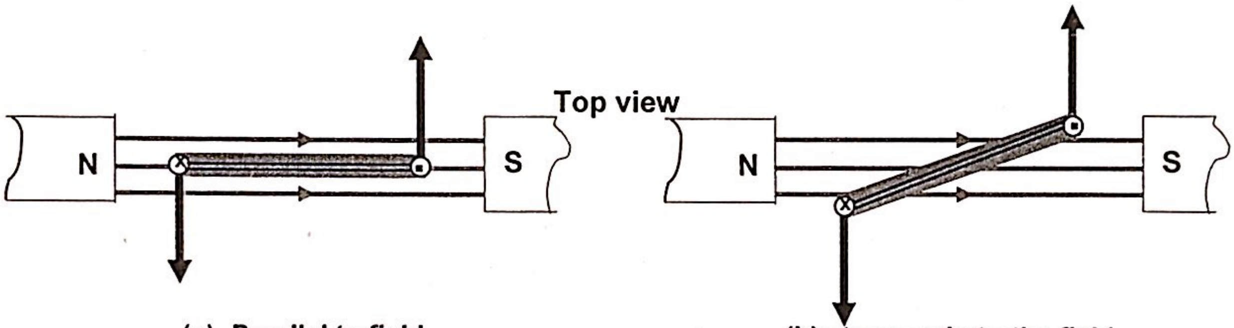


Fig. 18



(a) Parallel to field

(b) at an angle to the field

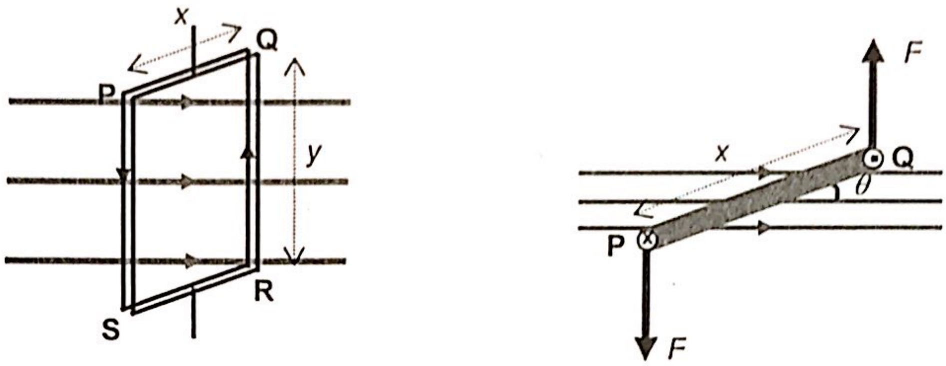
Fig. 19

When a steady current passes through the coil, a magnetic force acts on sides PS and QR, which are at right angles to the field lines.

Using Fleming's Left Hand Rule, the force on PS is opposite to the force on QR. The 2 forces provide a torque (turning effect) on the coil. Hence, the coil is acted upon by a couple due to the magnetic force on both sides as shown in Fig. 19. This is the principle behind moving-coil meters and motors.

Let us take a closer look to determine an expression for the torque on a coil.

Fig. 20 shows the same coil PQRS placed in a uniform magnetic field B such that the vertical sides are 90° to the field while the horizontal sides make an angle θ to the field. Assume the coil has N turns and its horizontal and vertical sides are of length x and y respectively.



(a) side view Fig. 20 (b) top view

- When current flows in the coil, each side of the coil experiences a force.
- The forces on the horizontal sides of the coil (PQ and SR) do not give rise to a turning effect but simply distort the coil.
- The forces on the vertical sides (PS and QR), each of length y, are opposite in direction and equal in magnitude, given by $F = NBIy$.
- Whatever the position of the coil, its vertical sides are at right angles to the magnetic field and so the force F remains constant in magnitude.
- The forces constitute a couple whose torque τ is given by

$$\begin{aligned}\tau &= (\text{one force}) \times (\text{perpendicular distance between lines of action of forces}) \\ &= Fx\cos\theta \\ &= NBIyx\cos\theta \\ &= NBIA\cos\theta\end{aligned}$$

where A = area of the coil = xy

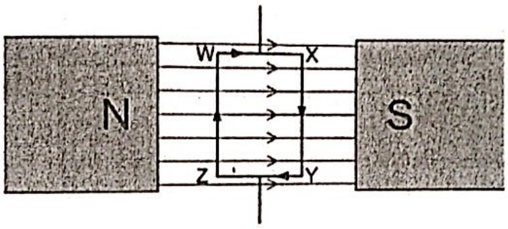
Note

- τ is maximum when $\theta = 0°$, i.e. when the plane of the coil is parallel to B.
- τ is zero when $\theta = 90°$, i.e. when the plane of the coil is perpendicular to B.

coil reverses direction after half cycle – to provide continuous motion in 1 direction, the current in the loop must periodically reverse direction

Example 2

In an electric motor, a rectangular coil WXYZ has 20 turns and is in a uniform magnetic field of flux density 0.83 T.



The lengths of sides XY and ZW are 0.17 m and of sides WX and YZ are 0.11 m. The current in the coil is 4.5 A.

(a) Calculate maximum torque τ on the coil

(b) State, in terms of τ, the torque on the coil when its plane makes an angle of 30° with the magnetic flux density.

(c) At what angle does the plane of the coil make with the magnetic field when the torque is zero?

(adapted from N2010/1/31)

Solution:

max torque when the plane of the coil is parallel to the B field

a) max τ = NBIA

= (20)(0.83)(4.5)(0.17 × 0.11)

= 1.40 Nm

b) τ (sin30°)

= $\frac{1}{2}\tau$

τ' = F(WX) cos30°

= NBIA cos30°

= 0.866 τ

c) θ = 90°

16.4 Forces between current-carrying conductors

Currents flowing in the same direction

Consider 2 infinitely long parallel vertical wires X and Y carrying currents I_1 and I_2 flowing in the same direction, separated by a distance d.

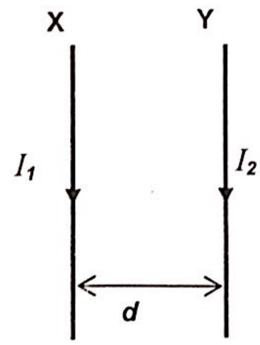


Fig. 21: Front view of wire X and Y

current in same direction attract

x y

The current in X produces a magnetic field directed out of the page at wire Y given by

$$B_X = \frac{\mu_o I_1}{2\pi d}$$

At Y, the direction of the field due to X is perpendicular to Y (see Fig. 22 (a)). Using Fleming's Left-Hand Rule, the magnetic force on a length L of Y would be **towards** X and has a magnitude of

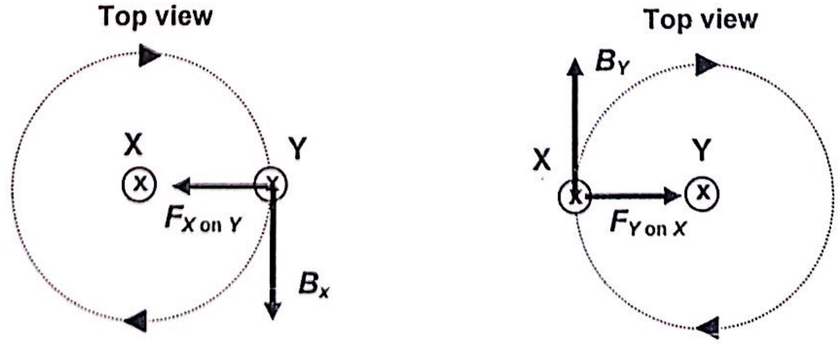
$$F_{X\text{ on }Y} = B_X I_2 L = \left(\frac{\mu_o I_1}{2\pi d}\right) I_2 L$$

Similarly, the magnetic force on a length L of X is **towards** Y and has a magnitude of

$$F_{Y\text{ on }X} = B_Y I_1 L = \left(\frac{\mu_o I_2}{2\pi d}\right) I_1 L$$

By Newton's Third Law, $F_{X\text{ on }Y}$ and $F_{Y\text{ on }X}$ are equal and opposite forces. The 2 wires **attract** one another.

The force per unit length on each wire is given by $\dfrac{F}{L} = \dfrac{\mu_o I_1 I_2}{2\pi d}$



(a) Direction of magnetic field due to X at Y

(b) Direction of magnetic field due to Y at X

Fig. 22

Currents flowing in opposite directions

Consider 2 infinitely long parallel vertical wires X and Y carrying currents I_1 and I_2 flowing in opposite directions, separated by a distance d.

The current in X produces a magnetic field at wire Y given by

$$B_X = \frac{\mu_0 I_1}{2\pi d}$$

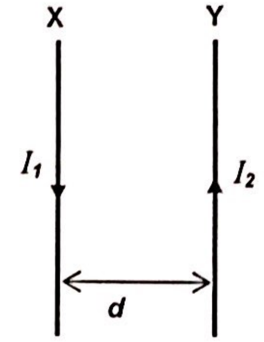


Fig. 23: Front view of wire X and Y

At Y, the direction of the field due to X is perpendicular to Y (see Fig. 24 (a)). Using Fleming's Left-Hand Rule, the magnetic force on a length L of Y would be **away from** X and has a magnitude of

$$F_{X\text{ on }Y} = B_X I_2 L = \left(\frac{\mu_0 I_1}{2\pi d}\right) I_2 L$$

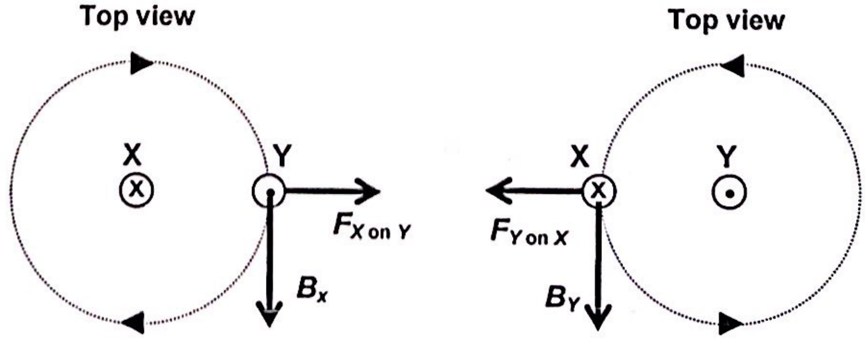
$\frac{\mu_0}{2\pi}$: constant

Similarly, the magnetic force on a length L of X is **away from** Y and has a magnitude of

$$F_{Y\text{ on }X} = B_Y I_1 L = \left(\frac{\mu_0 I_2}{2\pi d}\right) I_1 L$$

By Newton's Third Law, $F_{X\text{ on }Y}$ and $F_{Y\text{ on }X}$ are equal and opposite forces. The 2 wires **repel** one another.

The force per unit length on each wire is given by $\dfrac{F}{L} = \dfrac{\mu_0 I_1 I_2}{2\pi d}$



(a) Direction of magnetic field due to X at Y

(b) Direction of magnetic field due to Y at X

Fig. 24

In conclusion,

For 2 parallel current-carrying conductors,

i) the **force per unit length** on each wire is given by the equation

$$\frac{F}{L} = \frac{\mu_0 I_1 I_2}{2\pi d}$$

ii) Like currents attract, unlike currents repel

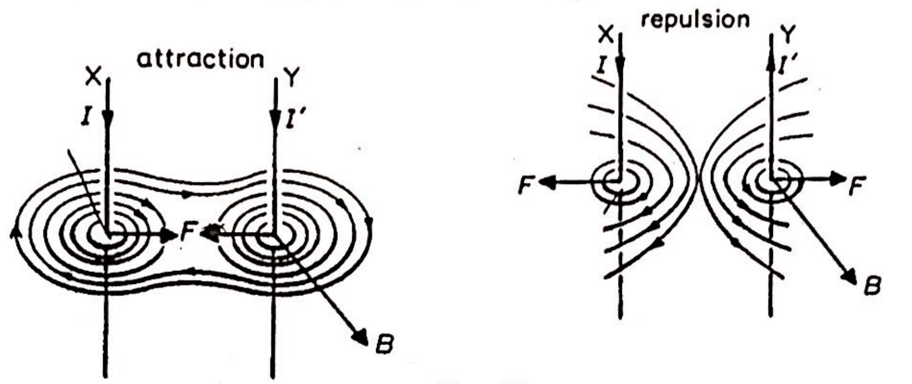


Fig. 25

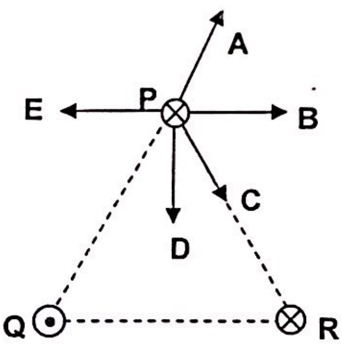
Example 3

Three long vertical wires pass through the corners of an equilateral triangle PQR.

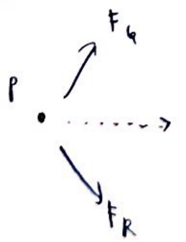
P and R carry currents directed into the plane of the paper and wire Q carries a current directed out of the paper.

All three currents have the same magnitude.

Which arrow shows the direction of the resultant force acting on P?

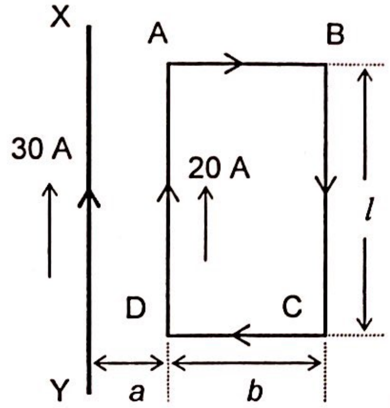


Solution:



B resultant force

Example 4 The figure shows a long wire XY carrying a current of 30 A. The rectangular loop ABCD carries a current of 20 A. Given: a = 1.0 cm, b = 8.0 cm and l = 30 cm.



different distance
• different force

(a) Calculate the magnetic field due to the current in XY along
(i) AD
(ii) BC
(b) Calculate the resultant force acting on the loop.

(The flux density at a perpendicular distance r from a very long straight wire carrying a current I is given by $B = \frac{\mu_0 I}{2\pi r}$.)

Solution:

a) i) $B_{on AD} = \frac{\mu_0 I_{xy}}{2\pi a}$

$= \frac{(4\pi \times 10^{-7})(30)}{2\pi(1.0 \times 10^{-2})}$

$= 6.00 \times 10^{-4}$ T

ii) $B_{on BD} = \frac{\mu_0 I_{xy}}{2\pi (a+b)}$

$= \frac{(4\pi \times 10^{-7})(30)}{2\pi(9.0 \times 10^{-2})}$

$= 6.67 \times 10^{-5}$ T

xy AD same current, attractive
xy BC diff current, repulsive

b) $F_{resultant} = F_{on AB} - F_{on BC}$

$= (B_{on AD} - B_{on BD}) I_{ABCD} l$

$= (6.00 \times 10^{-4} - 6.67 \times 10^{-5})(20)(0.30)$

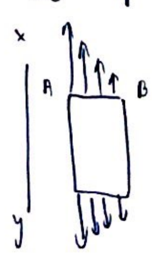
$= 3.20 \times 10^{-3}$ N

towards XY

Think: Is there any force acting on sides AB and CD?

Field going into plane of paper: F_{AB} = upwards

- magnetic force acting on AB is upwards & its magnitude decreases from A to B.
- magnetic force acting on CD is downwards & its magnitude decreases from D to C. forces cancel out.



16.5 Force on a moving charge

Force on a charged particle in a magnetic field

Since a current can experience a force in a magnetic field, **a moving charge should also experiences a force in a magnetic field.**

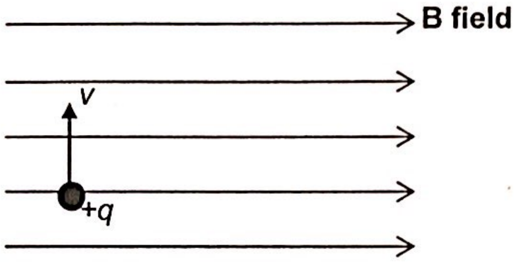


Fig. 26

Consider a positive charge q moving at constant speed v at **right-angles** to a magnetic field of flux density B.

Assume the particle travels a distance L in time t, so its speed is

$$v = \frac{L}{t}$$

The moving charge constitutes a current of $I = \frac{q}{t}$.

Hence, the force on the charge is given by

$$F = BIL = B\left(\frac{q}{t}\right)L = Bq\left(\frac{L}{t}\right) = Bqv$$

If the velocity and field are inclined to each other by an angle θ, then

$$\boxed{F = Bqv\sin\theta}$$

where F = force acting on a current carrying conductor
B = magnetic flux density
q = magnitude of charge
v = speed of charge
θ = angle the velocity makes with the field

Remember that Fleming's Left-Hand Rule considers the direction of conventional current. Hence, for a

a) <u>positive charge</u>, the direction of the current is in the **same** direction of the motion of the charge

b) <u>negative charge</u>, the direction of the current is in the **opposite** direction as the direction of motion of the charge

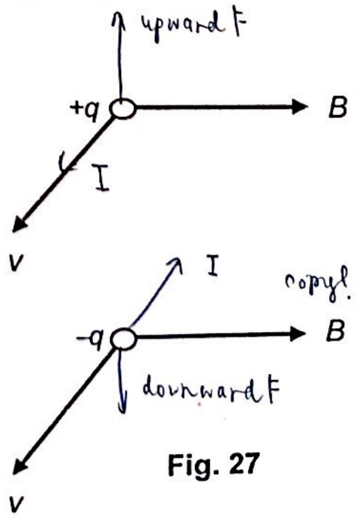


Fig. 27

Motion of a charged particle in a magnetic field

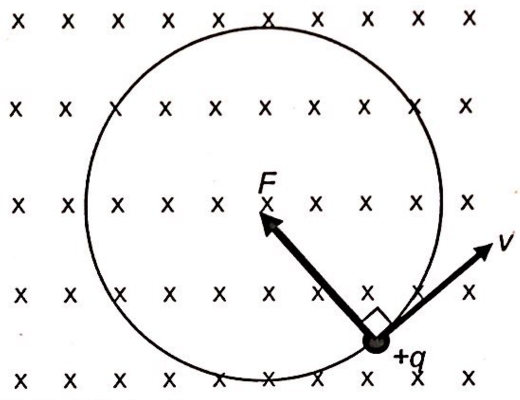
Uniform Circular Motion

- When a charged particle is projected at right angles into a magnetic field, the magnetic force F is always perpendicular to the direction of travel (or velocity) and the distance travelled in the direction of the force is zero, as shown in Fig. 28.

  Therefore the work done on the charged particle is always zero

- This implies that no energy is gained or lost by the particle moving in the magnetic field and the particle's speed is always constant.

- Since the force is of constant magnitude and is always at right angles to the velocity, the conditions are met for **circular motion**.



B field into the paper

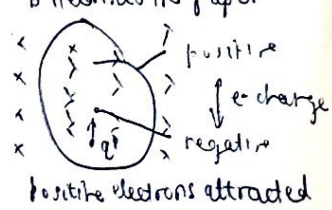
Fig. 28

Hence, the magnetic force on a moving charge provides a *centripetal* force.

$$F = Bqv = \frac{mv^2}{r}$$

$$B = \frac{mv}{rq}$$

where m is the mass of the moving charge and r is the radius of orbit.



Helix

For a charged particle entering a magnetic field at an angle θ such that $0° < \theta < 90°$, the particle would describe a **helix** (see Fig. 29).

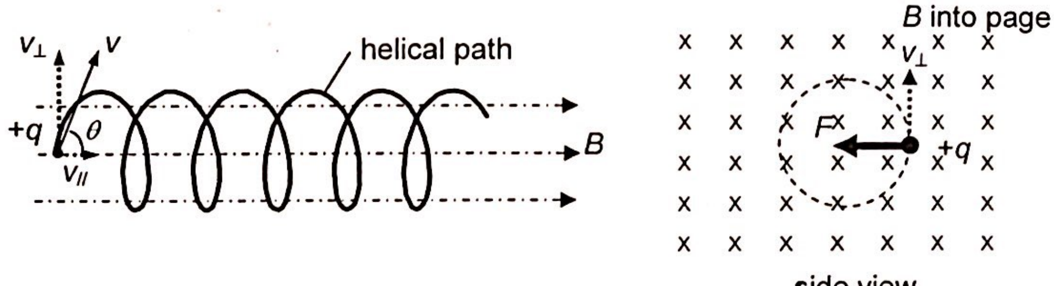


Fig. 29

Explanation:
Resolve v into 2 components:

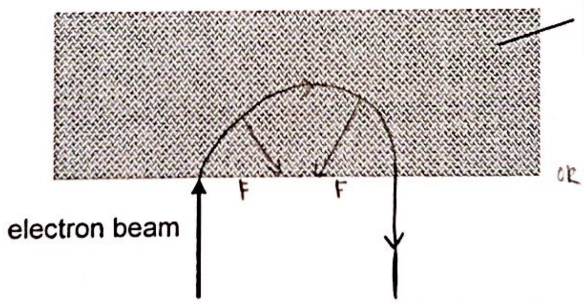
change
no magnetic force - just goes straight

$$v_{//} = v\cos\theta$$
$$v_\perp = v\sin\theta$$

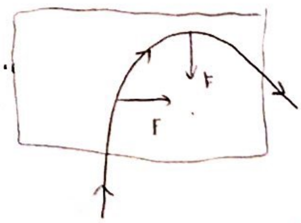
Motion of the charged particle is the result of superposing
- (i) a uniform circular motion in which it has speed $v_\perp$ in a plane perpendicular to the direction of B.
- (ii) a steady axial speed $v_{//}$ along the direction of B.

Example 5

A beam of electrons travelling with a velocity of 3.2×10^7 m s^{-1} enters a magnetic field of 0.47 mT. The electrons are travelling at right angles to the field. ($e = 1.60 \times 10^{-19}$ C; $m_e = 9.11 \times 10^{-31}$ kg.)



or



(i) Calculate the force on each electron within the field.

(ii) Calculate the radius of curvature of each electron's path while in the field.

(iii) Sketch the path travelled by an electron within and beyond the field. Indicate clearly the direction of the electron's path, the field and the force.

Solution:

iii)

i) $F = Bqv$

$= (0.47\times10^{-3})(1.60\times10^{-19})(3.2\times10^{7})$

$= 2.41\times10^{-11}$ N

while: the magnetic force provides the centripetal force $F_m = F_c$

ii) $\frac{mv^2}{r} = Bqv = F$

$r = \frac{mv^2}{F}$

$= \frac{(9.11\times10^{-31})(3.2\times10^{7})^2}{2.41\times10^{-15}}$

$= 0.387$ m //

Example 6

An α–particle, of mass 6.7×10^{-27} kg and charge $+2e$, was injected at right angles into a uniform magnetic field of flux density 1.2 T. It travels in a circular path of radius 0.45 m. Calculate

(i) its speed,

(ii) its period of revolution,

(iii) its kinetic energy, and

(iv) the potential difference through which it would have to be accelerated from rest to achieve this energy.

Solution:

i) magnetic force provides centripetal force

$F_B = F_c$

$B = \frac{mv}{rq}$

$v = \frac{Brq}{m} = \frac{(1.2)(0.45)(+2e)}{6.7\times10^{-27}}$ $(1.60\times10^{-19})(2)$

$= 2.58\times10^{7}\ \text{ms}^{-1}$

ii) $v = \frac{2\pi r}{T}$

$T = \frac{2\pi r}{v} = \frac{2\pi(0.45)}{2.58\times10^{7}}$

$= 1.10\times10^{-7}\ \text{s}$

iii) $KE = \frac{1}{2}mv^2$

$= \frac{1}{2}(6.7\times10^{-27})(2.58\times10^{7})^2$

$= 2.23\times10^{-12}\ \text{J}$

iv) gain in KE = loss in PE

$2.23\times10^{-12} = qV$ (charge x potential difference)

$V = \frac{2.23\times10^{-12}}{2\times1.60\times10^{-19}}$

$= 6.97\times10^{6}\ \text{V}$

Applications involving motion of charged particles in uniform electric and magnetic fields.

Velocity selector

A velocity selector consists of a magnetic field and an electric field applied over the same region, allowing only charged particles of a particular velocity to pass through undeflected.

The two fields are applied perpendicular to each other in an orientation such that the electric force and the magnetic force on the particle act in opposite directions.

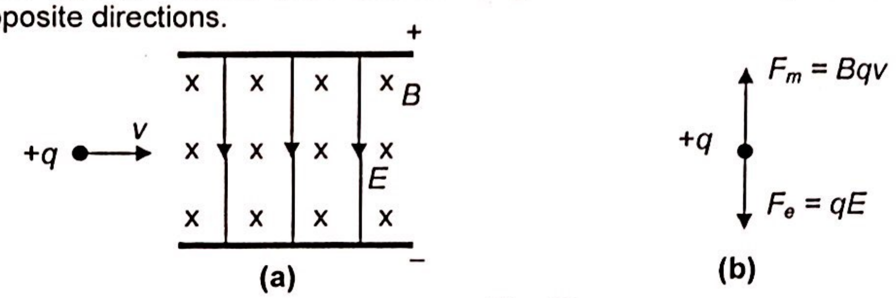


Fig. 30

Suppose a positive charge $+q$ enters the region where the two crossed fields are applied, with its velocity v perpendicular to both fields (see Fig. 30(a)).

It experiences an upward magnetic force $F_m = Bqv$ and a downward electric force $F_e = qE$ (see Fig. 31(b)).

[Note that the gravitational force acting on such charged particles is very small compared to F_m and F_e and hence can be ignored.]

If $F_m > F_e$, the particle will deflect upward.
If $F_m < F_e$, the particle will deflect downward.

Particles whose velocities are such that $F_m = F_e$ will pass through undeflected.

$$Bqv = qE$$

↳ $F_{resultant} = 0$
straight line, undeflected

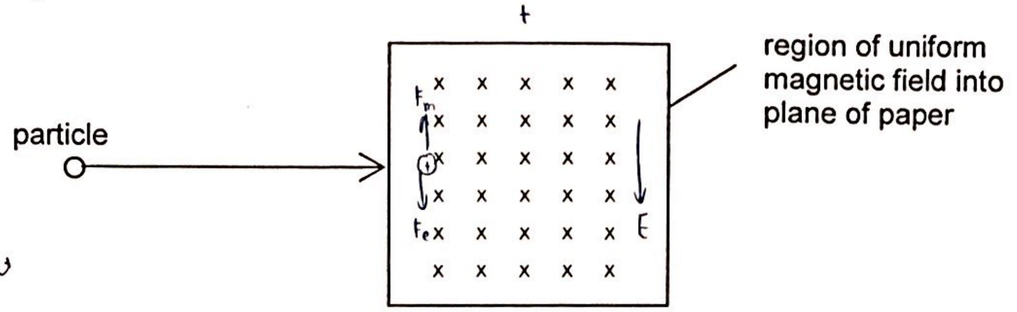
$$v = \frac{E}{B}$$

Hence only particles having this speed v will pass through undeflected.

The concept of charged particles moving through combined electric and magnetic fields has many applications and some of these are discused in the appendix.

Example 7 A particle of charge $+q$ entering a uniform magnetic field B that is directed into the plane of the paper. The particle is travelling with velocity v at right angles to the magnetic field.

direction of e field
∴ direction of current



A uniform electric field is applied in the same region as the magnetic field so that the particle passes through undeflected.

(a) On the figure above, mark, with an arrow labelled E, the direction of the electric field.

(b) State and explain the effect, if any, on a particle entering the region of the fields if the particle has
(i) charge $-q$ and speed v,
(ii) charge $+q$ and speed $2v$.

Solution:

a) use LHR to.. F_m, F_e in opp direction

b) i) the particle remains undeflected
magnetic force F_m is now downwards while electric force F_e is upwards.
hence, there is no resultant force on the particle.

ii) F_m increases with speed. ∴ $F_e < F_m$. particle is deflected upwards.

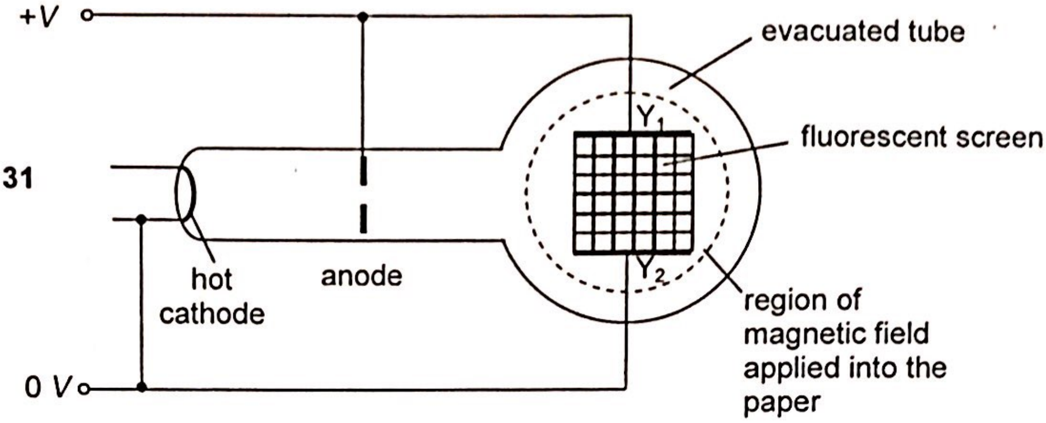
Determination of specific charge of electrons

The specific charge of a particle is the ratio of its charge to its mass:

$$\text{specific charge} = \frac{q}{m}$$

An experiment significant to the discovery of the electron is the measurement of the specific charge of cathode rays by J.J. Thomson in 1897. Fig. 31 shows the set-up of an experiment to find the specific charge of an electron. It is similar in principle to J.J. Thomson's experiment.

Fig. 31



Electrons produced by the hot cathode are accelerated in a vacuum towards an anode held at a potential $+V$ with respect to the cathode.

Assuming that the electrons are emitted with negligible speed from the cathode, they will leave the anode with a speed v which can be found as shown below:

Gain in K.E. of each electron = Loss in P.E. of the electron

$$\frac{1}{2}m_e v^2 = eV$$

$$v = \sqrt{\frac{2eV}{m_e}} \quad \text{.......... (1)}$$

where e and m_e are the charge and the mass of an electron respectively.

The beam of electrons emerging from the anode produces a narrow luminous trace when it hits a vertical fluorescent screen supported between two parallel plates Y_1 and Y_2. Y_1 is held at a potential $+V$ with respect to Y_2, thus creating an electric field E between the plates.

A uniform magnetic field B is applied perpendicular to the electric field as shown in Fig. 31 and the magnitudes of the two fields are adjusted so that the beam passes through undeflected. Hence,

$$Bev = eE$$

$$v = \frac{E}{B} \quad \text{........(2)}$$

Equating equations (1) and (2),

$$\frac{E}{B}=\sqrt{\frac{2eV}{m_e}}$$

$$\frac{e}{m_e}=\frac{E^2}{2B^2V}$$

If the separation between plates Y_1 and Y_2 is d, then $E=\frac{V}{d}$ and the specific charge of an electron will be given by

$$\frac{e}{m_e}=\frac{V}{2B^2d^2}$$

Hence the specific charge of an electron can be found if V, B and d are known.

Mass spectrometer

A mass spectrometer separates ions according to their mass-to-charge ratio. Fig. 32 shows the schematic diagram of a Bainbridge Mass Spectrometer.

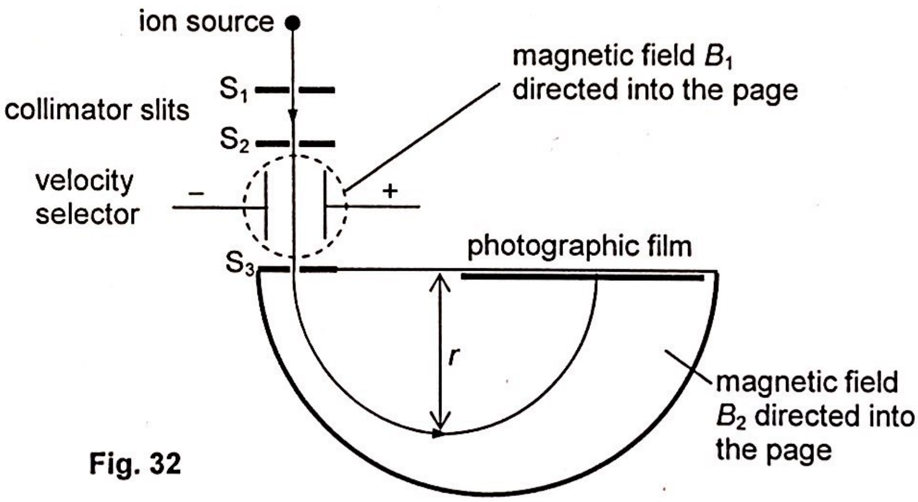


Fig. 32

Ions of charge q and mass m with velocity $v=\frac{E}{B_1}$ pass undeflected through the velocity selector and are then deflected into a semi-circular path with radius r in the magnetic field B_2.

Since the ions experience a centripetal force due to the magnetic force in B_2,

$$F_C=F_B$$

$$\frac{mv^2}{r}=B_2qv$$

$$r=\frac{mv}{B_2q}=\frac{mE}{B_1B_2q}$$

Since B_1, B_2 and E are constant, $r\propto\frac{m}{q}$