中正中学(义顺)
CHUNG CHENG HIGH SCHOOL (YISHUN)

Mid-Year Examination (2019)
Secondary 4 Express / 5 Normal (Academic)

Candidate	Answer Key		
	Name	Register No	Class

Additional Mathematics Paper 1
4047 / 1

For examiner's use
/ 80

Date : 13th May 2019

Duration : 2 hours

Additional Materials :

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question. The total marks for this paper is 80.

Setter : Poh Eng Hua Terence

This paper consists of **12** printed pages, INCLUDING the cover page.

[Turn over

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$, $\qquad x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \cdots + \binom{n}{r} a^{n-r} b^r + \cdots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \dfrac{n!}{r!(n-r)!} = \dfrac{n(n-1)\cdots(n-r+1)}{r!}$

2. Trigonometry

Identities

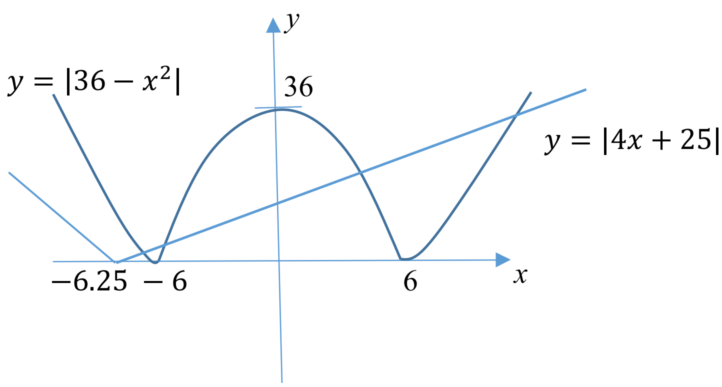
$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\text{area of } \Delta ABC = \frac{1}{2} ab \sin C$$

1(i) $f(x) = 4\|4x-6\| - 2\|9-6x\|$ $= 8\|2x-3\| - 6\|3-2x\|$ $= 8\|2x-3\| - 6\|2x-3\|$ $=2\|2x-3\|$	
1(ii) $2\|2x-3\| = 5$ $2x-3 = \frac{5}{2}$ or $2x-3 = -\frac{5}{2}$ $x = \frac{11}{4}$ $\quad x = \frac{1}{4}$ $= 2\frac{3}{4}$	
2) $2x^2 - 4x + c = 2x + 1$ $2x^2 - 6x + c - 1 = 0$ $b^2 - 4ac > 0$,intersect at two points $36 - 8c + 8 > 0$ $8c < 44$ $c < 5\frac{1}{2}$	
3) $AB = CD = 5$ cm $\frac{AB}{AD} = \tan\frac{\pi}{3}$ $AD = \frac{5}{\sqrt{3}}$ $\frac{\sin\angle AED}{AD} = \frac{\sin\frac{\pi}{3}}{3\sqrt{2}}$ $\sin\angle AED = \frac{\frac{\sqrt{3}}{2}}{3\sqrt{2}} \times \frac{5}{\sqrt{3}}$ $\sin\angle AED = \frac{5}{6\sqrt{2}}$ $\sin\angle AED = \frac{5\sqrt{2}}{12}$ $\angle AED = \sin^{-1}\frac{5\sqrt{2}}{12}$ (proven)	
4) $\frac{7}{\sqrt{7}-\sqrt{2}} - \frac{1}{\sqrt{7}} = \frac{7(\sqrt{7}+\sqrt{2})}{7-2} - \frac{\sqrt{7}}{7}$ $= \frac{7}{5}\sqrt{7} + \frac{7}{5}\sqrt{2} - \frac{1}{7}\sqrt{7}$	

$=\frac{44}{35}\sqrt{7}+\frac{7}{5}\sqrt{2}$ $$b=\frac{44}{35}, a=\frac{7}{5}$$	
5) $$2x^2-x-1=(2x+1)(x-1)$$ If $2x^2-x-1$is a factor for $2x^4-7x^3+ax^2+13x+b$, $(x-1)$ and $(2x+1)$ are factors too. $$f(1)=2(1)^4-7(1)^3+a(1)^2+13(1)+b$$ $$0=8+a+b \quad \text{-------} \ (1)$$ $$f\left(-\frac{1}{2}\right)=2(-\frac{1}{2})^4-7\left(-\frac{1}{2}\right)^3+a\left(-\frac{1}{2}\right)^2+13(-\frac{1}{2})+b$$ $$0=-5\frac{1}{2}+\frac{a}{4}+b$$ $$22=a+4b \quad \text{---------} \ (2)$$ Eqn (2) – eqn (1), $$3b=30$$ $$b=10,$$ sub into eqn (1), $$a=-8-b$$ $$a=-18$$	
6)  (i) Shape x and y-intercepts Turning pt	

(ii) Shape
x and y-intercepts

(iii) 4 solutions

7)

(i) Midpoint of AB $\left(\frac{1+4}{2}, \frac{5+1}{2}\right)$

$$\left(2\frac{1}{2}, 3\right)$$

Gradient of $AB = \frac{5-1}{1-4} = 1\frac{1}{3}$

Gradient of the perpendicular bisector $= \frac{-1}{-\frac{4}{3}}$

$= \frac{3}{4}$

Equation of perpendicular bisector,

$$y = \frac{3}{4}x + c$$

Sub $\left(2\frac{1}{2}, 3\right)$ in,

$$3 = \frac{3}{4}\left(2\frac{1}{2}\right) + c$$

$$c = 3 - \frac{15}{8} = \frac{9}{8}$$

$$y = \frac{3}{4}x + \frac{9}{8}$$

(ii)
Let $y = 0$,

$$0 = \frac{3}{4}x + \frac{9}{8}$$

$$x = -\frac{3}{2}$$

$$C\left(-\frac{3}{2}, 0\right)$$

Area of ΔABC,

$= \frac{1}{2}\begin{vmatrix} -1.5 & 4 & 1 & -1.5 \\ 0 & 1 & 5 & 0 \end{vmatrix}$ unit^2

$= \frac{1}{2}\left(-1.5 + 20 - 1 + \frac{15}{2}\right)$ unit^2

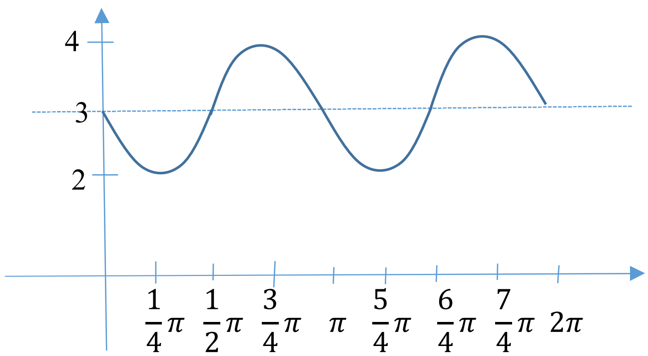
$= 13 \text{ unit}^2$

8) (i)

amplitude = 1,

Period = 2π

(ii)



(iii)

$$2.5 = 3 - \sin 2x$$
$$\sin 2x = 0.5$$
$$\sin\alpha = 0.5$$
$$\alpha = \frac{\pi}{6}$$
$$0 \le 2x \le 4\pi$$

$2x$ is in first and second quadrant,

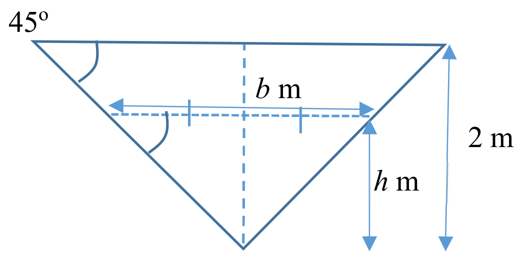
$$2x = \frac{\pi}{6}, \quad \frac{5}{6}\pi, \quad \frac{13}{6}\pi, \quad \frac{17}{6}\pi$$

$$x = \frac{\pi}{12}, \quad \frac{5}{12}\pi, \quad \frac{13}{12}\pi, \quad \frac{17}{12}\pi$$

9) Volume = uniform surface area x length

$= \frac{1}{2} \times b \times h \times 8$ ---------------- (1)

The filled trough cross sectional surface is similar to the trough



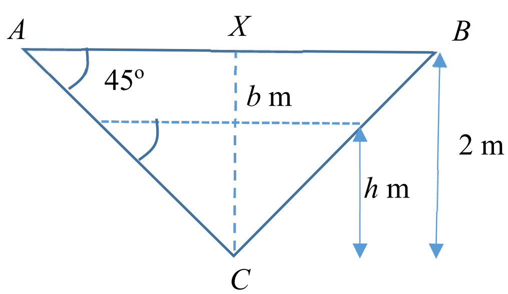
$$\tan 45° = \frac{h}{\frac{b}{2}}$$

$$b = 2h$$

sub into eqn (1)

$$V = \frac{1}{2} \times 2h \times h \times 8$$
$$= 8h^2 \ m^3$$

(i) Alternative,



$$\tan 45° = \frac{2}{AX}$$

$$AX = 2 \text{ m}$$
$$AB = 4 \text{ m}$$

$$\frac{h}{2} = \frac{b}{4}$$

$$b = 2h$$

$$V = \frac{1}{2} \times 2h \times h \times 8$$
$$= 8h^2 \ m^3$$

(ii)

$$\frac{dh}{dt} = \frac{dh}{dv} \times \frac{dv}{dt}$$

$$= \frac{1}{16h} \times (-0.25)$$

$$= -\frac{1}{64h} - - - - - - (1)$$

$$v = 24 \text{ m}^3$$

$$24 = 8h^2$$

$h = \sqrt{3}$ m , $-\sqrt{3}$ m (rej as $h > 0$)

sub into eqn (1)

$$\frac{dh}{dt} = -\frac{1}{64\sqrt{3}}$$

$$= -\frac{\sqrt{3}}{192} \text{ m/s or } (-0.00902) \text{m/s}$$

10)

(a)

$$2 + \log_3(1-x)^2 = \frac{\log_3 256}{\log_3 9}$$

$$\log_3 9 + \log_3(1-x)^2 = \frac{\log_3 256}{\log_3 9}$$

$$\log_3 9(1-x)^2 = \log_3 \sqrt{256}$$

Compare value,

$$9(1-x)^2 = \sqrt{256}$$

$$(1-x)^2 = \frac{16}{9}$$

$$1 - x = \pm\frac{4}{3}$$

$$x = -\frac{1}{3}, 2\frac{1}{3} \text{ (rej, log value} > 0)$$

b)

$$4^x + 2^{x+3} = 33$$

$$2^{2x} + 2^{x+3} = 33$$

let 2^x be u

$$u^2 + 8u - 33 = 0$$

$$(u + 11)(u - 3) = 0$$

$$u = -11\ (rej), \quad u = 3$$

$$2^x = 3$$

$$\lg 2^x = \lg 3$$
$$x = 1.58$$

11)

Using guess and check,

$$\mathrm{f}(1) = 2(1)^3 + 9(1)^2 + 13(1) + 6 = 30$$

$$\mathrm{f}(-1) = 2(-1)^3 + 9(-1)^2 + 13(-1) + 6 = 0$$

$(x+1)$ is a factor of f(x)

$$\begin{array}{r l}
 & \quad 2x^2 + 7x + 6 \\
(x+1) & \overline{\big)\, 2x^3 + 9x^2 + 13x + 6} \\
 & -(2x^3 + 2x^2) \\
 & \qquad\quad 7x^2 + 13x \\
 & \qquad -(7x^2 + 7x\ \ ) \\
 & \qquad\qquad\qquad 6x \quad + 6 \\
 & \qquad\qquad\quad -(6x + 6) \\
 & \qquad\qquad\qquad\qquad 0
\end{array}$$

$$\mathrm{f}(x) = (x+1)(2x^2 + 7x + 6)$$

$$= (x+1)(2x+3)(x+2)$$

(ii) $$\frac{5x^2-3}{2x^3+9x^2+13x+6} = \frac{a}{x+1} + \frac{b}{2x+3} + \frac{c}{x+2}$$

$$5x^2 - 3 = a(2x+3)(x+2) + b(x+1)(x+2) + c(x+1)(2x+3)$$

Let $x = -2$,

$$5(-2)^2 - 3 = c(-1)(-1)$$
$$17 = c(-1)(-1)$$

$$c = 17$$

Let $x = -\frac{3}{2}$,

$$5\left(-\frac{3}{2}\right)^2 - 3 = b\left(-\frac{1}{2}\right)\left(\frac{1}{2}\right)$$

$$\frac{33}{4} = -\frac{b}{4}$$

$$b = -33$$

Let $x = -1$,

$$5(-1)^2 - 3 = a(1)(-1)$$
$$a = 2$$

$$\frac{5x^2 - 3}{2x^3 + 9x^2 + 13x + 6} = \frac{2}{x+1} - \frac{33}{2x+3} + \frac{17}{x+2}$$

(iii)

$$\int \frac{5x^2 - 3}{2x^3 + 9x^2 + 13x + 6} dx$$

$$= \int (\frac{2}{x+1} - \frac{33}{2x+3} + \frac{17}{x+2}) dx$$

$$= 2\ln(x+1) - \frac{33}{2}\ln(2x+3) + 17\ln(x+2) + c$$

iv)

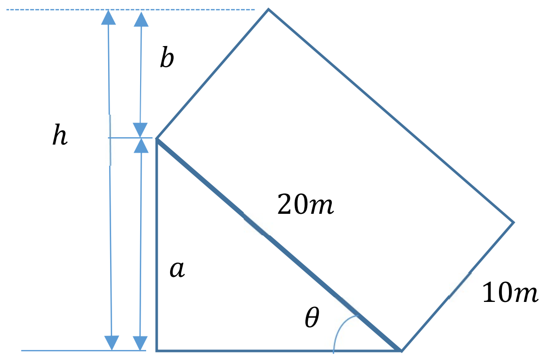
$$\int_0^1 \frac{5x^2 - 3}{2x^3 + 9x^2 + 13x + 6} dx$$

$$= \left[2\ln(x+1) - \frac{33}{2}\ln(2x+3) + 17\ln(x+2)\right]_0^1$$

$$= 2\ln 2 - \frac{33}{2}\ln 5 + 17\ln 3 - 2\ln 1 + \frac{33}{2}\ln 3 - 17\ln 2$$

$$= \quad -\frac{33}{2}\ln 5 + 33\frac{1}{2}\ln 3 - 15\ln 2$$

12)



(i) $h = a + b$

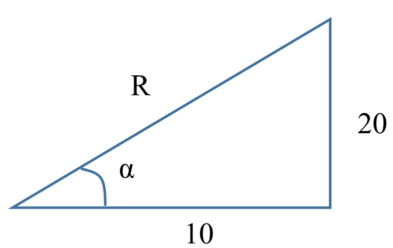
$$\sin\theta = \frac{a}{20}$$

$$20\sin\theta = a$$

$$\cos\theta = \frac{b}{10}$$
$$b = 10\cos\theta$$

$h = 20\sin\theta + 10\cos\theta$ (proven)

(ii) $h = 20\sin\theta + 10\cos\theta$
$= R\cos(\theta - \alpha)$



$$R = \sqrt{20 + 10} = 10\sqrt{5}$$

$$\tan\alpha = 2$$
$$\alpha = 63.4349°$$

$$h = 10\sqrt{5}\cos(\theta - 63.43°)$$

(iii) $h = 10\sqrt{5}cos(\theta - 63.4°)$

$$21 = 10\sqrt{5}cos(\theta - 63.4349°)$$

$$cos(\theta - 63.4349°) = \frac{21}{10\sqrt{5}}$$

$$cos \propto = \frac{21}{10\sqrt{5}}$$
$$\propto = 20.0909°$$

$$(\theta - 63.4349°) = 20.0909°, -20.0909°$$
$$\theta = 83.5258°, 43.344°$$
$$\theta = 83.5°, 43.344°(1dp)$$

iv) nope, since max $cos(\theta - 63.4°)$ = 1, the max height of the figure is = $10\sqrt{5}$ m which is less than 25m.

- The End -