CATHOLIC JUNIOR COLLEGE
General Certificate of Education Advanced Level
Higher 2
JC1 Promotional Examination

CANDIDATE NAME			
CLASS		INDEX NUMBER	

MATHEMATICS

9758/01

Paper 1

7 Oct 2024
3 hours

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF27)

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on the work you hand in.
Write in dark blue or black pen.
You may use a HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers in the spaces provided in the Question Paper.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

Question	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**Total**
Marks													
Total	4	5	5	6	6	7	7	10	12	12	13	13	**100**

This document consists of 28 printed pages, including this cover page.

1 A curve C has equation $y = px + \sqrt{qx} - \ln(rx)$, where p, q and r are constants. Given that C crosses the x-axis at the points where $x = 1$, $x = 2$ and $x = 5$, find the values of p, q and r, giving your answers correct to 3 decimal places. [4]

2 By expressing $\dfrac{3x-7}{(x+3)(x-2)} - 1$ as a single simplified fraction, solve exactly the inequality $\dfrac{3x-7}{(x+3)(x-2)} < 1$. [3]

Hence, solve exactly the inequality $\dfrac{3e^x - 7}{(e^x+3)(e^x-2)} < 1$. [2]

3 It is given that $\displaystyle\sum_{r=1}^{n} \frac{1}{r(r+1)} = 1 - \frac{1}{n+1}$.

(a) Find $\displaystyle\sum_{r=5}^{n+1} \frac{1}{r(r+1)}$. [3]

(b) Give a reason why the series in part **(a)** is convergent and state the value of $\displaystyle\sum_{r=5}^{\infty} \frac{1}{r(r+1)}$. [2]

4 A curve C has equation $y = \dfrac{x^2 - 4x + 4}{x-1}$.

(a) Sketch C, stating clearly the equations of any asymptotes, coordinates of turning points and points of intersection with the axes. [3]

(b) By drawing a suitable graph on the same diagram in part **(a)**, find the range of values of b, where $b > 0$, such that the equation $(x-2)^2 + \dfrac{1}{b^2}\left(\dfrac{x^2-4x+4}{x-1} + 1\right)^2 = 4$ has no real roots. [3]

5 Differentiate each of the following expressions with respect to x.

(a) $\tan^{-1}\sqrt{2+x^3}$ [3]

(b) $\ln\left(\dfrac{x}{\sqrt{1-x}}\right)$ [3]

6 It is given that $y = \sin\left(e^x - 1\right)$.

(a) Show that $\dfrac{d^2y}{dx^2} = \dfrac{dy}{dx} - ye^{2x}$. [3]

(b) Hence, find the Maclaurin series of y, up to and including the term in x^2. [2]

(c) Using the Maclaurin series found in part **(b)**, determine the series expansion of $\sqrt{1+\sin\left(e^x - 1\right)}$, up to and including the term in x^2. [2]

7 **(a)** Three non-zero vectors $\mathbf{p}$, $\mathbf{q}$ and $\mathbf{r}$ are such that $3\mathbf{p}\times\mathbf{q} = \mathbf{r}\times\mathbf{p}$.
Find a linear relationship between $\mathbf{p}$, $\mathbf{q}$ and $\mathbf{r}$. [3]

(b) Referred to the origin O, the points A and B have position vectors $\mathbf{a}$ and $\mathbf{b}$.
It is given that $\mathbf{a}$ and $\mathbf{b}$ are non-parallel unit vectors.
The point B divides AC in the ratio $1:3$.
The point D lies on OB produced such that $OB:OD = 1:m$, where $m \in \mathbb{Z}$, $m \geq 2$.
Given that $\overrightarrow{AB}$ is perpendicular to $\overrightarrow{CD}$, find the numerical value of m. [4]

8 **(a)** An arithmetic sequence has first term a and common difference 0.5, where a is an integer. Find the smallest value of a such that the sum of the first 36 terms is at least 500. [3]

(b) A sequence $u_0, u_1, u_2, \ldots$ is given by

$$u_0 = 400 \text{ and } u_n = 1.01u_{n-1} - x \text{ for } n \geq 1, \text{ where } x \text{ is an integer.}$$

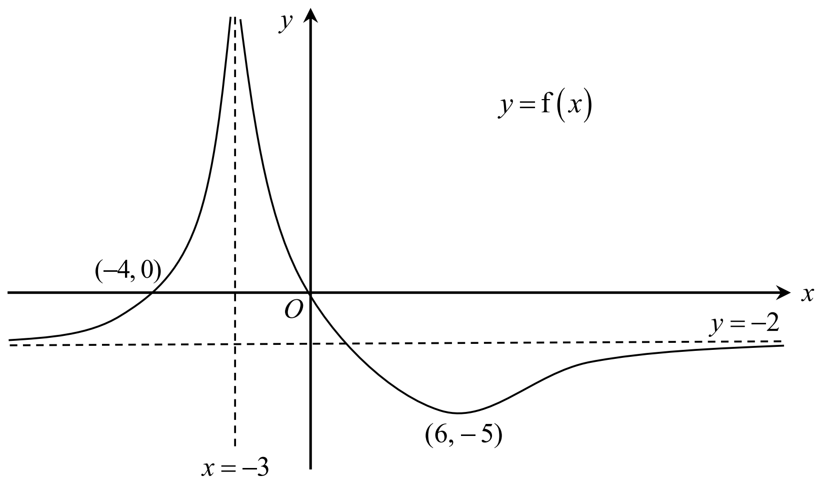
(i) Show that $u_n = 1.01^n(400) - 100x\left(1.01^n - 1\right)$. [3]

(ii) Given that $x = 16$ and $y = 1.01u_k$, where $0 < y < 16$, find the value of k and y. [4]

9 **(a)** Express $4x^2-8x+4$ in the form of $A(x-B)^2$ where A and B are constants to be determined. [1]

Hence or otherwise, state a sequence of transformations that would transform the curve with equation $y=3e^{x^2}$ onto the curve with equation $y=3e^{4x^2-8x+4}-10$. [3]

(b)



The diagram shows the curve $y=f(x)$. The curve has a turning point at $(6,-5)$ and crosses the x-axis at $(-4,0)$ and $(0,0)$. The lines $y=-2$ and $x=-3$ are the asymptotes to the curve.

On separate diagrams, sketch the graphs of

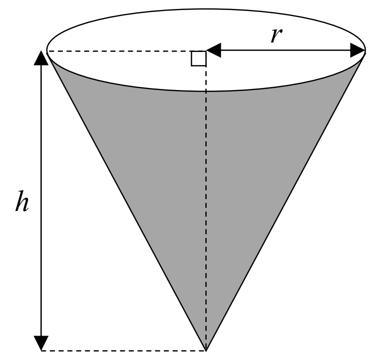
(i) $y=3|f(x)|$, [2]

(ii) $y=\dfrac{1}{f(x)}$, [3]

(iii) $y=f'(x)$, [3]

labelling clearly the equation(s) of any asymptote(s), coordinates of any axial intercept(s) and turning point(s) where applicable.

10



A cone-shaped cup is made of paper of negligible thickness to hold $20\pi\ \text{cm}^3$ of liquid. The open inverted cone has radius r cm and height h cm as shown in the diagram above.

The external surface area of the cup is denoted by $A\ \text{cm}^2$. The manufacturer wants to reduce the cost of production by minimizing the value of A.

(a) Find h in terms of r. [1]

(b) By considering A^2 or otherwise, show that $A\dfrac{\mathrm{d}A}{\mathrm{d}r} = \pi^2\left(2r^3 - \dfrac{3600}{r^3}\right)$. [3]

(c) Find the exact value of r that gives the minimum value of A, proving that A is a minimum. Find also the ratio of the radius to the height, $\dfrac{r}{h}$, giving your answer in terms of $k\sqrt{2}$, where k is a constant to be determined. [5]

The manufacturer decides to make paper cups at minimum external surface area using the ratio $\dfrac{r}{h}$ found in part **(c)**.

(d) The cup is being filled completely with water. However, there is a small hole at the bottom of the cup that causes water to leak out at the rate of $3\ \text{cm}^3$ per second. Find the rate of decrease of the depth of the water at the instant when the depth is 2 cm. [3]

[It is given that the volume of a circular cone with base radius r and height h is $\dfrac{1}{3}\pi r^2 h$ and the curved surface area is $\pi r l$, where l is the slant height of the cone.]

11 A function h is self-inverse if $\mathrm{h}(x)=\mathrm{h}^{-1}(x)$ for all x in the domain of h.

The functions f and g are defined by

$$\mathrm{f}(x)=2+\frac{1}{x-2}, \qquad \text{for } x\in\mathbb{R}, x>2$$

$$\mathrm{g}(x)=\begin{cases}3x-1 & \text{for } x\le 1,\\ (x-1)^2+2 & \text{for } x>1.\end{cases}$$

(a) Sketch the graph of $y=\mathrm{f}(x)$. With the aid of your graph, explain why f has an inverse. [2]

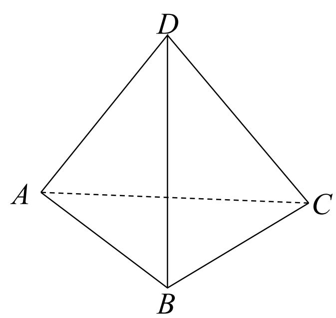
(b) Show that f is self-inverse and find $\mathrm{f}^2(x)$. [4]

(c) Hence, or otherwise, evaluate $\mathrm{f}^{2025}(4)$. [2]

(d) Find an expression for gf and state its domain. [3]

(e) Find the range of gf. [2]

12 The diagram below shows a triangular base pyramid with vertices, A, B, C and D. With reference to the origin O, the points A, B, C and D are $\mathbf{i}+2\mathbf{j}+3\mathbf{k}$, $2\mathbf{i}-\mathbf{j}+4\mathbf{k}$, $4\mathbf{i}+5\mathbf{j}-\mathbf{k}$ and $-2\mathbf{i}-5\mathbf{k}$ respectively. Let the plane containing points A, B and C be represented by Π.



(a) Show that the equation of Π can be expressed as $\mathbf{r}\cdot\begin{pmatrix}9\\7\\12\end{pmatrix}=\alpha$, where α is a constant to be determined. [3]

(b) Find the acute angle between line BD and Π. [2]

(c) Find the position vector of the foot of perpendicular from the point D to Π. [4]

(d) Find the area of triangle ABC. Hence find the volume of the pyramid. [4]

[Volume of pyramid $=\frac{1}{3}\times$ base area $\times$ height]