2023 TJC_NYJC_VJC H2 FM Prelim Paper 1 [Suggested Solution]

Question 1	
(a)	Using Simpson's Rule with $h=0.5$, $\int_1^3 \frac{k}{4x-1}\,\mathrm{d}x \approx \frac{1}{3}(0.5)\left[\frac{k}{4(1)-1}+\frac{4k}{4(1.5)-1}+\frac{2k}{4(2)-1}+\frac{4k}{4(2.5)-1}+\frac{k}{4(3)-1}\right]$ $\frac{3386}{31185}=\frac{1}{6}\left(\frac{k}{3}+\frac{4k}{5}+\frac{2k}{7}+\frac{4k}{9}+\frac{k}{11}\right)$ $\frac{3386}{31185}=\frac{1}{6}\left(\frac{6772k}{3465}\right)$ $k=\frac{1}{3}$
(b)	Volume required $=\int_1^3 2\pi x\left(\frac{k}{4x^2-x}\right)\mathrm{d}x = 2\pi\int_1^3 \frac{k}{4x-1}\,\mathrm{d}x \approx \frac{1112}{5079}\pi$

Question 2
Let P_n be the proposition that $\sqrt[n]{n} < 2-\frac{1}{n}$, for $n \geq 2$.

When $n=2$, $\sqrt[n]{n}=\sqrt{2}<1.5=2-\frac{1}{2}$

P_2 is true.

Assume P_k is true for $k\geq 2, k\in\mathbb{Z}^+$ i.e. $\sqrt[k]{k}<2-\frac{1}{k}\Rightarrow k<\left(2-\frac{1}{k}\right)^k$

To show P_{k+1} is true. i.e. $\sqrt[k+1]{k+1}<2-\frac{1}{k+1}$.

i.e. $k+1<\left(2-\frac{1}{k+1}\right)^{k+1}$

$$\begin{aligned}\left(2-\frac{1}{k+1}\right)^{k+1} &> \left(2-\frac{1}{k}\right)^{k+1} \quad \text{since } k>2\\ &= \left(2-\frac{1}{k}\right)^{k}\left(2-\frac{1}{k}\right)\\ &> k\left(2-\frac{1}{k}\right) \quad \text{by inductive hypothesis}\\ &= 2k-1\\ &= k+k-1\\ &> k+1 \text{ since } k>2\end{aligned}$$

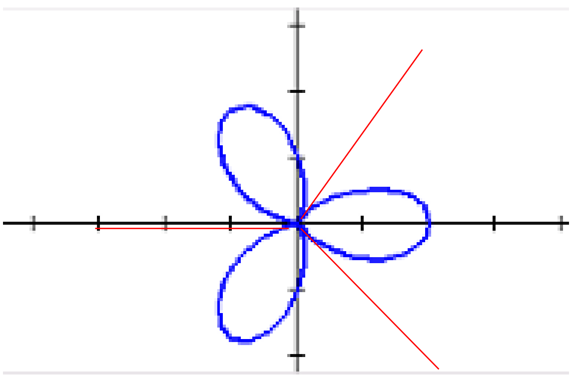
$\therefore k+1<\left(2-\frac{1}{k+1}\right)^{k+1}$

i.e. $\sqrt[k+1]{k+1}<2-\frac{1}{k+1}$.

P_k true $\Rightarrow$ P_{k+1} true.

Since P_2 is true and P_k true $\Rightarrow$ P_{k+1} true by mathematical induction $\sqrt[n]{n}<2-\frac{1}{n}$, for $n\geq 2$.

Question 3



At the pole, $r=0$. $\therefore \cos 3\theta=-1$.

$$3\theta=\pi,3\pi,5\pi$$

$$\therefore \theta=\frac{\pi}{3},\pi,\frac{5\pi}{3}$$

The cartesian forms are $y=\left(\tan\frac{\pi}{3}\right)x$, $y=(\tan\pi)x$ and $y=\left(\tan\frac{5\pi}{3}\right)x$.
Area $=2\times3\times\frac{1}{2}\int_0^{\frac{\pi}{3}} r^2\,\mathrm{d}\theta$ $=3\int_0^{\frac{\pi}{3}}(1+\cos3\theta)^2\,\mathrm{d}\theta$ $=4.712$
Maximum value of r is 2 and it occurs when $\cos3\theta=1$. $\Rightarrow 3\theta=0,2\pi,4\pi$ $\theta=0,\frac{2\pi}{3},\frac{4\pi}{3}$ The points are $(2,0),\left(2,\frac{2\pi}{3}\right),\left(2,\frac{4\pi}{3}\right)$. Using cosine rule, Distance = $\sqrt{2^2+2^2-2(2)(2)\cos\frac{2\pi}{3}}$ $=\sqrt{12}$ $=2\sqrt{3}$ Or Distance between point $\left(2,\frac{2\pi}{3}\right)$ and $\left(2,\frac{4\pi}{3}\right)$. $=2\sin\left(\frac{2\pi}{3}\right)-2\sin\left(\frac{4\pi}{3}\right)$ $=2\left(\frac{\sqrt{3}}{2}\right)-2\left(-\frac{\sqrt{3}}{2}\right)$ $=2\sqrt{3}$

Question 4	
(a)	$H_{n+1} - H_n = 1.025(H_n - H_{n-1})$ $H_{n+1} - 2.025H_n + 1.025H_{n-1} = 0$ $m^2 - 2.025m + 1.025 = 0$ $m = 1$ or $m = 1.025$ $H_n = A + B(1.025)^n$ $H_0 = 50 = A + B$ $H_1 = 54.9 = A + 1.025B$ $B = 196, A = -146$ $H_n = -146 + 196(1.025)^n$
(b)	Let I_n denote the new HPI n years after the start of 2020. $I_0 = H_{10} = 104.897$ $I_n = 0.5I_{n-1}$ $I_n < 10$ When n = 4, $I_n = 6.56 < 10$ Thus in the year 2024.

Question 5

$$(1+\mathrm{i}\tan\theta)^k = \left(\frac{1}{\cos\theta}\right)^k \left(\cos\theta+\mathrm{i}\sin\theta\right)^k$$

$$= \sec^k\theta\left(\cos k\theta+\mathrm{i}\sin k\theta\right)$$

$$\sum_{k=0}^{n-1}\cos k\theta\sec^k\theta=\mathrm{Re}\left[\sum_{k=0}^{n-1}(1+\mathrm{i}\tan\theta)^k\right]$$

$$\sum_{k=0}^{n-1}(1+\mathrm{i}\tan\theta)^k=\frac{\left(1+\mathrm{i}\tan\theta\right)^n-1}{1+\mathrm{i}\tan\theta-1}\quad\text{(sum of GP)}$$

$$=\frac{\left(1+\mathrm{i}\dfrac{\sin\theta}{\cos\theta}\right)^n-1}{\mathrm{i}\tan\theta}$$

$$=\frac{\left(\dfrac{\cos\theta+i\sin\theta}{\cos\theta}\right)^n-1}{\mathrm{i}\tan\theta}$$

$$=\left[\sec^n\theta\left(\cos n\theta+i\sin n\theta\right)-1\right](-\mathrm{i})\cot\theta$$

$$=\sec^n\theta\sin n\theta\cot\theta+\mathrm{i}\left[1-\sec^n\theta\cos n\theta\right]\cot\theta$$

$$\sum_{k=0}^{n-1}\cos k\theta\sec^k\theta=\mathrm{Re}\left[\sum_{k=0}^{n-1}(1+\mathrm{i}\tan\theta)^k\right]$$

$$\sum_{k=0}^{n-1}\cos k\theta\sec^k\theta=\sec^n\theta\sin n\theta\cot\theta$$

Let $\theta=\dfrac{\pi}{3}$,

$$\sum_{k=0}^{n-1}\cos\left(\frac{k\pi}{3}\right)\sec^k\left(\frac{\pi}{3}\right)=\sec^n\left(\frac{\pi}{3}\right)\sin\left(\frac{n\pi}{3}\right)\cot\left(\frac{\pi}{3}\right)$$

$$\sum_{k=0}^{n-1}\cos\left(\frac{k\pi}{3}\right)2^k=2^n\sin\left(\frac{n\pi}{3}\right)\frac{1}{\sqrt{3}}$$

Let

$x=\cos\theta$

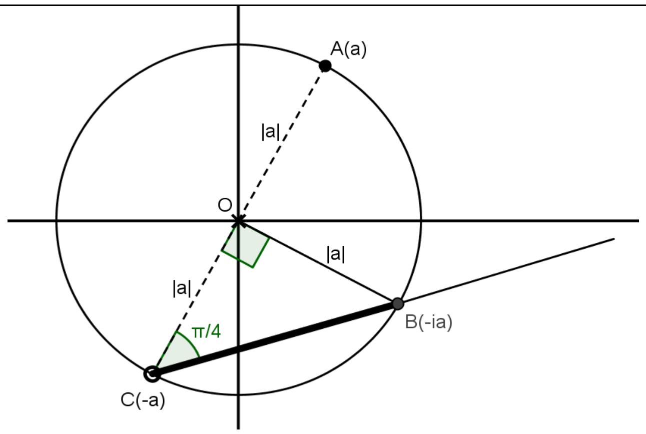
$$\Rightarrow\sec\theta=\frac{1}{x}$$

$$\cot\theta=\frac{x}{\sqrt{1-x^2}}$$

$$\sum_{k=0}^{n-1}\cos k\theta\sec^k\theta=\sec^n\theta\sin n\theta\cot\theta$$

$$\sum_{k=0}^{n-1}\cos\left(k\cos^{-1}x\right)\frac{1}{x^k}=\frac{1}{x^n}\sin\left(n\cos^{-1}x\right)\frac{x}{\sqrt{1-x^2}}$$
$$=\frac{\sin\left(n\cos^{-1}x\right)}{x^{n-1}\sqrt{1-x^2}}$$

Question 6



Minimum value will be the perpendicular distance from origin to the line segment CB. Let the point of intersection of this perpendicular and CB be D. By observing the right angle triangle formed by O, C, and D, OD = $|a|\sin\frac{\pi}{4}=\frac{|a|}{\sqrt{2}}$.

It is clear that the locus of z satisfying both relations is the line segment CB, not including C. The argument of the point represented by B is $\arg(a)-\frac{\pi}{2}$, while the argument of the point represented by C is $\arg(a)-\pi$.

Hence the range of values is

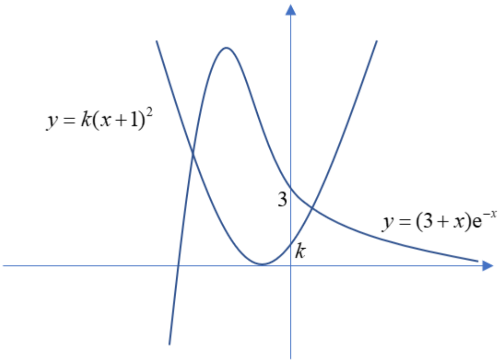
$$\arg(a)-\pi<\arg(z)\le\arg(a)-\frac{\pi}{2}.$$

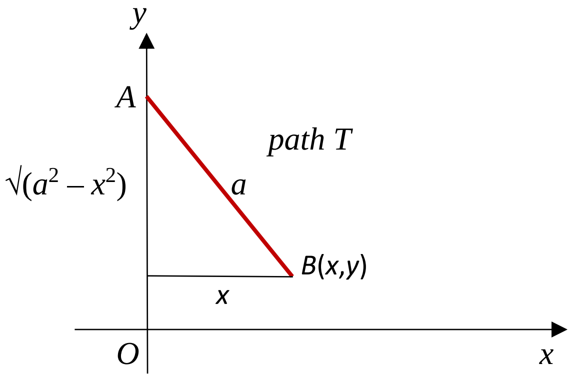
Let $z=r\,e^{i\theta}$, $z^2=r^2\,e^{i2\theta}=r^2(\cos 2\theta+i\sin 2\theta)$

$$\operatorname{Re}(z^2)<0\Rightarrow\cos 2\theta<0\Rightarrow-\frac{3}{2}\pi<2\theta<-\frac{\pi}{2}$$
$$\Rightarrow\ -\frac{3}{4}\pi<\theta<-\frac{\pi}{4}$$

Since $\frac{\pi}{4}<\arg(a)<\frac{\pi}{2}$,

$\arg(a)-\pi<\arg(z)\le\arg(a)-\frac{\pi}{2}$ can satisfy the criteria of $-\frac{3}{4}\pi<\theta<-\frac{\pi}{4}$

Question 7	
(a)	$k(x+1)^2-(3+x)e^{-x}=0$ The roots occur at the point of intersections of $y=(3+x)e^{-x}$ and $y=k(x+1)^2$. Since the y-intercept of $y=k(x+1)^2$ is k and that of $y=(3+x)e^{-x}$ is 3, for the equation to have **(i)** two negative roots is $k>3$, **(ii)** one negative and one positive root is $0<k<3$. 
(b)	If $k=1$, then $f(x)=(x+1)^2-(3+x)e^{-x}$ $f(0)=-2<0$ $f(1)=4-\frac{4}{e}>0$ Thus the positive root lies in $(0,1)$ and so $n=0$. Using linear interpolation, $a_1=\frac{\lvert f(0)\rvert}{\lvert f(0)\rvert+\lvert f(1)\rvert}=\frac{2e}{6e-4}=0.44164\approx 0.44$ $f'(x)=2(x+1)+(3+x)e^{-x}-e^{-x}=2(x+1)+(2+x)e^{-x}>0$ within $(0,1)$ $f''(x)=2-(2+x)e^{-x}+e^{-x}=2-e^{-x}(1+x)>0$ within $(0,1)$ The graph of f is increasing and concave upwards in $(0,1)$ so a_1 is an underestimation.
(c)	$a_2=a_1-\frac{f(a_1)}{f'(a_1)}=0.47186\approx 0.47$
(d)	For $(x+1)^2-(3+x)e^{-x}=0$, $(x+1)^2=(3+x)e^{-x}$ $3+x=(x+1)^2e^x$ $x=(x+1)^2e^x-3$ $\therefore\quad x_{n+1}=(1+x_n)^2e^{x_n}-3$ (shown) Let $F(x)=(x+1)^2e^x-3$ and so $F'(x)=2(x+1)e^x+e^x(x+1)^2=e^x(x^2+4x+3)$ [B1] Since $e^x(x^2+4x+3)>0$ in the interval $[0,1]$, and $\min[F'(x)]=3$, the sequence is not convergent.

Question 8	
(a)	 Since *AB* is tangential to *T*, gradient of *T* at *B*(*x*,*y*) $\frac{\mathrm{d}y}{\mathrm{d}x} = \text{gradient of } AB = -\frac{\sqrt{a^2-x^2}}{x}$.
(b)	$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{\sqrt{a^2-x^2}}{x} \Rightarrow y = -\int \frac{\sqrt{a^2-x^2}}{x}\,\mathrm{d}x$. Using the substitution $x = a\sin\theta$ gives $y = -\int \frac{\sqrt{a^2-a^2\sin^2\theta}}{a\sin\theta}\left(a\cos\theta\right)\mathrm{d}\theta$ $= -a\int \frac{\cos^2\theta}{\sin\theta}\,\mathrm{d}\theta$ $= -a\int \frac{1-\sin^2\theta}{\sin\theta}\,\mathrm{d}\theta$ $= -a\left\{\int \operatorname{cosec}\theta\;\mathrm{d}\theta - \int \sin\theta\;\mathrm{d}\theta\right\}$ $= -a\left\{-\ln\left(\operatorname{cosec}\theta + \cot\theta\right) + \cos\theta\right\} + C$ $= a\ln\left(\operatorname{cosec}\theta + \cot\theta\right) - a\cos\theta + C$ $x = a\sin\theta \Rightarrow \sin\theta = \frac{x}{a} \Rightarrow \cos\theta = \frac{\sqrt{a^2-x^2}}{a}, \cot\theta = \frac{\sqrt{a^2-x^2}}{x}$ Substitute above into the above gives $y = a\ln\left(\operatorname{cosec}\theta + \cot\theta\right) - a\cos\theta + C$ $= a\ln\left(\frac{a}{x} + \frac{\sqrt{a^2-x^2}}{x}\right) - a\left(\frac{\sqrt{a^2-x^2}}{a}\right) + C$ $= a\ln\left(\frac{a+\sqrt{a^2-x^2}}{x}\right) - \sqrt{a^2-x^2} + C$ Since $\left(a,0\right)$ lies on *T*, $0 = a\ln\left(\frac{a+\sqrt{a^2-a^2}}{a}\right) - \sqrt{a^2-a^2} + C \Rightarrow C = 0$.

	Hence $y = a\ln\left(\dfrac{a+\sqrt{a^2-x^2}}{x}\right) - \sqrt{a^2-x^2}$.
(c)	Height of A above the point where the x-coordinate is b is $\sqrt{a^2-b^2}$. Therefore the height of A above O is $= a\ln\left(\dfrac{a+\sqrt{a^2-b^2}}{b}\right) - \sqrt{a^2-b^2} + \sqrt{a^2-b^2}$ $= a\ln\left(\dfrac{a+\sqrt{a^2-b^2}}{b}\right)$
(d)	Length of arc $= \int_b^a \sqrt{1+\left(\dfrac{dy}{dx}\right)^2}\,dx \quad (b<a)$ $= \int_b^a \sqrt{1+\left(-\dfrac{\sqrt{a^2-x^2}}{x}\right)^2}\,dx$ $= \int_b^a \sqrt{1+\dfrac{a^2-x^2}{x^2}}\,dx$ $= \int_b^a \dfrac{a}{x}\,dx$ $= a\left[\ln x\right]_b^a$ $= a\ln\left(\dfrac{a}{b}\right)$

Question 9	
(a)	$1+\tan^2\theta = \sec^2\theta \quad \Rightarrow \quad 1+\left(\dfrac{y}{2\sqrt{2}}\right)^2 = (x-3)^2 \quad \Rightarrow \quad (x-3)^2 - \dfrac{y^2}{8} = 1$ For hyperbola, $c^2 = a^2+b^2 = 1+8 = 9$ $\therefore \quad \alpha = 2c = 6$
(b)	$e = \dfrac{c}{a} = 3$
(c)	Since the coastline is 18 km in total distance, $OP+PF = 18$ Using cosine rule on ΔOPF , we have $PF^2 = OP^2 + OF^2 - 2(OP)(OF)\cos\theta$ $(18-r)^2 = r^2 + 6^2 - 2(r)(6)\cos\theta$ $324 - 36r = 36 - 12r\cos\theta$ $\therefore \quad r = \dfrac{24}{3-\cos\theta}$

(d)	When $x=0$, $(0-3)^2-\frac{y^2}{8}=1 \Rightarrow y=\pm 8$ *check that $r=\frac{24}{3-\cos\theta}=8$ when $\theta=\frac{\pi}{2}$ Cartesian coordinates of A is (0, 8) which means coordinates of B is (6, 8) $\angle BOF=\tan^{-1}\left(\frac{4}{3}\right)$ Arc length $AB=\int_{\tan^{-1}\left(\frac{4}{3}\right)}^{\frac{\pi}{2}}\sqrt{r^2+\left(\frac{\mathrm{d}r}{\mathrm{d}\theta}\right)^2}\,\mathrm{d}\theta$ where $r=\frac{24}{3-\cos\theta}$ and $\frac{\mathrm{d}r}{\mathrm{d}\theta}=-\frac{24\sin\theta}{(3-\cos\theta)^2}$ $=6.104\approx 6.10$ km
(e)	By property of hyperbola, $OQ-QF=OB-BF=10-8=2$ Perimeter of triangle $OPQ=OP+OQ+PQ$ $=OP+OQ+(PF-QF)$ $=(OP+PF)+(OQ-QF)$ $=18+2$ $=20$ The perimeter will always be 20 km.

Question 10
(a) (i) Non-singular $\Rightarrow \det(\mathbf{A})\neq 0$ $\det(\mathbf{A})=d\begin{vmatrix}1&3\\1&4\end{vmatrix}+2\begin{vmatrix}1&1\\2&1\end{vmatrix}$ $=d(4-3)+2(1-2)$ $=d-2$ $d-2\neq 0$ $\therefore d\neq 2.$
(ii) $\left(\begin{array}{ccc\|ccc}1&1&3&1&0&0\\2&1&4&0&1&0\\d&0&2&0&0&1\end{array}\right)\xrightarrow[R_3\to R_3-dR_1]{R_2\to R_2-2R_1}\left(\begin{array}{ccc\|ccc}1&1&3&1&0&0\\0&-1&-2&-2&1&0\\0&-d&2-3d&-d&0&1\end{array}\right)$

$$\xrightarrow{R_2 \to -R_2} \left(\begin{array}{ccc|ccc} 1 & 1 & 3 & 1 & 0 & 0 \\ 0 & 1 & 2 & 2 & -1 & 0 \\ 0 & -d & 2-3d & -d & 0 & 1 \end{array}\right)$$

$$\xrightarrow[R_3 \to R_3 + dR_2]{R_1 \to R_1 - R_2} \left(\begin{array}{ccc|ccc} 1 & 0 & 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 2 & -1 & 0 \\ 0 & 0 & 2-d & d & -d & 1 \end{array}\right)$$

$$\xrightarrow{R_3 \to \frac{1}{2-d} R_3} \left(\begin{array}{ccc|ccc} 1 & 0 & 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 2 & -1 & 0 \\ 0 & 0 & 1 & \frac{d}{2-d} & \frac{-d}{2-d} & \frac{1}{2-d} \end{array}\right)$$

$$\xrightarrow[R_2 \to R_2 - 2R_3]{R_1 \to R_1 - R_3} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{-2}{2-d} & \frac{2}{2-d} & \frac{-1}{2-d} \\ 0 & 1 & 0 & \frac{4-4d}{2-d} & \frac{3d-2}{2-d} & \frac{-2}{2-d} \\ 0 & 0 & 1 & \frac{d}{2-d} & \frac{-d}{2-d} & \frac{1}{2-d} \end{array}\right)$$

$$\therefore A^{-1} = \frac{1}{2-d} \begin{pmatrix} -2 & 2 & -1 \\ 4-4d & 3d-2 & -2 \\ d & -d & 1 \end{pmatrix}$$

(b) (i) Origin lies on π_1. $\therefore$ $\mathbf{0} \in \pi_1$.

Let $\begin{pmatrix} a_1 \\ b_1 \\ c_1 \end{pmatrix}$ and $\begin{pmatrix} a_2 \\ b_2 \\ c_2 \end{pmatrix} \in \pi_1$.

$2a_1 + b_1 + 4c_1 = 0$ and $2a_2 + b_2 + 4c_2 = 0$

$$\begin{pmatrix} a_1 \\ b_1 \\ c_1 \end{pmatrix} + \begin{pmatrix} a_2 \\ b_2 \\ c_2 \end{pmatrix} = \begin{pmatrix} a_1 + a_2 \\ b_1 + b_2 \\ c_1 + c_2 \end{pmatrix}$$

$2(a_1 + a_2) + (b_1 + b_2) + 4(c_1 + c_2)$

$= (2a_1 + b_1 + 4c_1) + (2a_2 + b_2 + 4c_2)$

$= 0 + 0$

$= 0$

$\therefore \pi_1$ is closed under addition.

Let $\lambda \in \mathbb{R}$. $\lambda\begin{pmatrix} a_1 \\ b_1 \\ c_1 \end{pmatrix} = \begin{pmatrix} \lambda a_1 \\ \lambda b_1 \\ \lambda c_1 \end{pmatrix}$ $2(\lambda a_1) + (\lambda b_1) + 4(\lambda c_1)$ $= \lambda(2a_1 + b_1 + 4c_1)$ $= \lambda(0)$ $= 0$ $\therefore \pi_1$ is closed under scalar multiplication. Since $\pi_1 \subset \mathbb{R}^3$, it is a subspace of $\mathbb{R}^3$.
(ii) Let $T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \mathbf{A}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix}$ $\Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = A^{-1}\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \frac{1}{2-d}\begin{pmatrix} -2 & 2 & -1 \\ 4-4d & 3d-2 & -2 \\ d & -d & 1 \end{pmatrix}\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix}$ $x = \frac{1}{2-d}\left[-2x' + 2y' - z'\right]$ $y = \frac{1}{2-d}\left[(4-4d)x' + (3d-2)y' - 2z'\right]$ $z = \frac{1}{2-d}\left[dx' - dy' + z'\right]$ Given $2x + y + 4z = 0$, $\frac{1}{2-d}\left[2(-2x' + 2y' - z') + (4-4d)x' + (3d-2)y' - 2z' + 4(dx' - dy' + z')\right] = 0.$ $(-4 + 4 - 4d + 4d)x' + (4 + 3d - 2 - 4d)y' + (-2 - 2 + 4)z' = 0$ $2(1-d)y' = 0$ $y' = 0$ $\therefore$ the cartesian equation of π_2 is $y = 0$. Alternative

$$T\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}x'\\y'\\z'\end{pmatrix}$$

$$\mathbf{A}\begin{pmatrix}x\\-2x-4z\\z\end{pmatrix}=\begin{pmatrix}x'\\y'\\z'\end{pmatrix}$$

$$x\mathbf{A}\begin{pmatrix}1\\-2\\0\end{pmatrix}+z\mathbf{A}\begin{pmatrix}0\\-4\\1\end{pmatrix}=\begin{pmatrix}x'\\y'\\z'\end{pmatrix}$$

$$x\begin{pmatrix}-1\\0\\d\end{pmatrix}+z\begin{pmatrix}-1\\0\\2\end{pmatrix}=\begin{pmatrix}x'\\y'\\z'\end{pmatrix}$$

π_2 is spanned by $\begin{pmatrix}-1\\0\\d\end{pmatrix}$ and $\begin{pmatrix}-1\\0\\2\end{pmatrix}$ (independent vectors $\because d\neq 2$). This is the x-z plane with cartesian equation $y=0$.

(c)Let $\mathbf{P}=\begin{pmatrix}1&1&3\\2&1&4\\0&0&2\end{pmatrix}$ and $\mathbf{D}=\begin{pmatrix}0&0&0\\0&1&0\\0&0&2\end{pmatrix}$, then

$$\mathbf{M}^n=\mathbf{P}\mathbf{D}^n\mathbf{P}^{-1}$$

$$=\begin{pmatrix}1&1&3\\2&1&4\\0&0&2\end{pmatrix}\begin{pmatrix}0&0&0\\0&1&0\\0&0&2^n\end{pmatrix}\frac{1}{2}\begin{pmatrix}-2&2&-1\\4&-2&-2\\0&0&1\end{pmatrix}$$

$$=\frac{1}{2}\begin{pmatrix}0&1&3(2^n)\\0&1&2^{n+2}\\0&0&2^{n+1}\end{pmatrix}\begin{pmatrix}-2&2&-1\\4&-2&-2\\0&0&1\end{pmatrix}$$

$$=\frac{1}{2}\begin{pmatrix}4&-2&-2+3(2^n)\\4&-2&-2+2^{n+2}\\0&0&2^{n+1}\end{pmatrix}$$

$$=\begin{pmatrix}2&-1&-1+3(2^{n-1})\\2&-1&-1+2^{n+1}\\0&0&2^n\end{pmatrix}$$