Chapter 4

Forces

"The same principle that allows an airplane to rise off the ground by creating lift under its wings is used in reverse in F1 cars to generate an additional 'downward force' to press the race car against the surface of the track. This increases the contact force between the tires and the road surface, allowing the car to turn corners at amazing speeds. F1 cars achieve downward force-to-weight ratio of 1:1 at about 125 km/h. At 190 km/h (118 mph) the ratio is roughly 2:1."

- Anonymous Formula One Expert

TOPIC 4: Forces

H2 Physics Syllabus 9749

Forces	**Learning Outcomes** Students should be able to:
Types of force	(a) recall and apply Hooke's law (*F* = *kx*, where *k* is the force constant) to new situations or to solve related problems (b) describe the forces on a mass, charge and current-carrying conductor in gravitational, electric and magnetic fields, as appropriate *(covered later in relevant chapters)* (c) show a qualitative understanding of normal contact forces, frictional forces and viscous forces including air resistance (no treatment of the coefficients of friction and viscosity is required) *(partly covered in Kinematics)*
Centre of gravity	(d) show an understanding that the weight of a body may be taken as acting at a single point known as its centre of gravity *(covered in Dynamics)*
Turning effects of forces	(e) define and apply the moment of a force and the torque of a couple (f) show an understanding that a couple is a pair of forces which tends to produce rotation only (g) apply the principle of moments to new situations or to solve related problems
Equilibrium of forces	(h) show an understanding that, when there is no resultant force and no resultant torque, a system is in equilibrium (i) use a vector triangle to represent forces in equilibrium
Upthrust	(j) derive, from the definitions of pressure and density, the equation $p = \rho gh$ (k) solve problems using the equation $p = \rho gh$ (l) show an understanding of the origin of the force of upthrust acting on a body in a fluid (m) state that upthrust is equal in magnitude and opposite in direction to the weight of the fluid displaced by a submerged or floating object (n) calculate the upthrust in terms of the weight of the displaced fluid (o) recall and apply the principle that, for an object floating in equilibrium, the upthrust is equal in magnitude and opposite in direction to the weight of the object to new situations or to solve related problems.

Contents

Topic 4: Forces

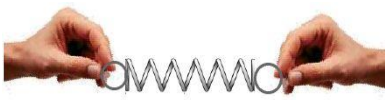
Videos of Lecture Examples can be found at

https://youtube.com/playlist?list=PL_b5cjrUKDlaffEZq6U4qTf1LEZbylgF-

Let us begin by introducing some forces that we will encounter in this chapter.

4.1 Hooke's Law

When we try to extend a spring by pulling it apart or compressing it with both hands, each of our hands will be subjected to an opposing force by the spring. We refer to this force as the tension or compression in the spring. For our purposes, we usually consider light springs of negligible mass. In 1676, Robert Hooke stated an empirical law that allows us to calculate the magnitude of this force.



Hooke's Law states that the **magnitude of force *F* exerted by a spring# on a body** attached to the spring is proportional to the **extension** *x* of the spring from its natural length provided the proportional limit of the spring is not exceeded.

$$F = kx$$

k = constant of proportionality (also referred to as force constant or spring constant or stiffness of spring)

Example 1 (N94/I/23)

A spring, obeying Hooke's Law, has an unstretched length of 50 mm and a spring constant of 400 N m^{-1}. What is the tension in the spring when its overall length is 70 mm?

[Answer: 8.0 N]

Note that, in general, all elastic materials obey Hooke's Law within their elastic limits. The law is not restricted to springs.

4.2 Upthrust or Buoyant Force

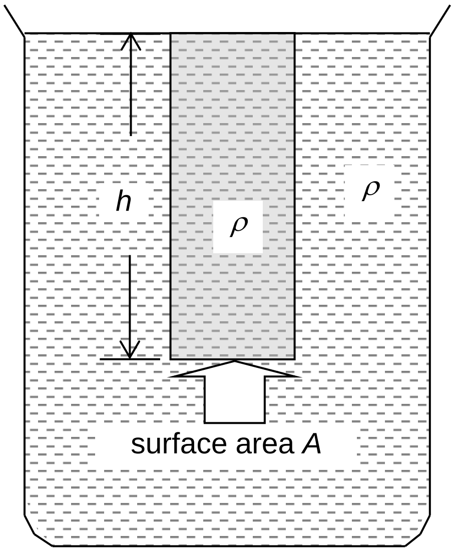
4.2.1 Pressure due to Fluid

If an object is immersed in a fluid, the fluid will press on the surface of the object. The normal force per unit area of the surface is referred to as the pressure due to the fluid. For a cylinder that is filled with water, the water pressure on the base of the cylinder will increase if more water is added and the water level rises. Similarly, if a small sheet of metal is dropped into the cylinder, the pressure on the surface of the metal sheet will increase as it sinks deeper.

Apart from water, we also experience atmospheric pressure due to the air above us. The atmospheric pressure we experience as we climb up a mountain will decrease as we ascend higher.

4.2.2 Derivation of the equation *p* = *ρgh*

Consider a column of fluid as shown in the diagram. This column of fluid is pressed inwards on all sides by the rest of the fluid in the beaker. Let



ρ (Greek letter *rho*) = density of the fluid,
A = surface area of the bottom of the column,
h = depth from the surface of the fluid,
V = volume of the fluid column,
m = mass of the fluid column.

The weight of the fluid is pressing down on the bottom surface *A*. Since *pressure is force over area*, the pressure due to the weight of the fluid column is

$$p = \frac{mg}{A}.$$

Recall that *density is mass divided by volume*, *ρ* = *m* / *V*. Rearranging, we find that *m* = *ρ V*, from which we get

$$p = \frac{\rho V g}{A}.$$

Note further that the volume of the fluid column is equal to the height of the column multiplied by the surface area, *V* = *h A*. Thus, we find that *V* / *A* = *h*, from which

$$p = \rho g h.$$

Hence, the average pressure due to the weight of a fluid column is *p* = *ρgh*.

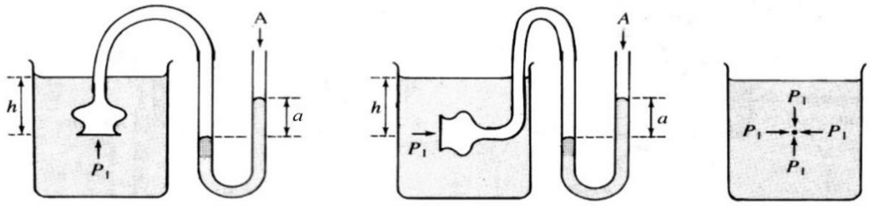
Note that we also have atmospheric pressure on top of the fluid column in the derivation above, pressing the fluid column down as it were. The total pressure at the surface area *A* is therefore equal to the atmospheric pressure added to the pressure due to the fluid:

$$p_{total} = p + p_{atm},$$

where p_{atm} is the atmospheric pressure at the surface of the fluid and *p* is the pressure at the bottom surface of the column due to the weight of the fluid alone.

Note that this expression for the pressure on the surface of the column is independent of the cross-sectional area of the column. Hence, if we consider a column where the cross-sectional area of the column approaches zero, we then obtain the same formula for the *pressure at a point* in the fluid.

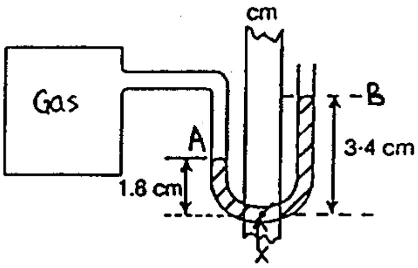
By considering an object that is very small, of negligible volume, we can say that the *pressure at a point in a fluid is the same in all directions*. A simple experiment can be conducted to illustrate this using a manometer. Since the height difference a remains the same as we change the direction that the opening of the tube is facing in the fluid, we can see that the pressure at the opening of the tube in the fluid is the same in all directions.



Example 2

The diagram on the right shows a mercury manometer connected to a gas container. The atmospheric pressure is 760 mm Hg and the point X is at the bottom of the U-tube.

a) What is the pressure of the point X in mm Hg?
b) What is the pressure of the gas in mm Hg?
c) If the manometer is disconnected from the gas container, what will be the vertical heights of the mercury levels from the point X in the left and right limbs of the manometer?



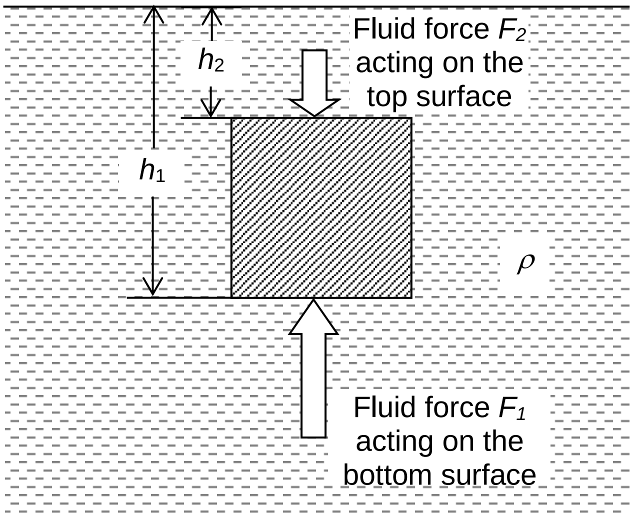
[Answers: 794 mm Hg; 776 mm Hg; 26 mm]

4.2.3 Upthrust

For an object that is immersed in a fluid, the force that the fluid exerts on different parts of its surface differs according to the depth of immersion of that part of the surface. We will now consider the net force that acts on the object due to the fluid.

Upthrust *U* refers to the **net upward** force exerted **by a fluid** on a body **fully or partially** submerged in the fluid.

By considering the fluid forces acting on a cube fully immersed in a fluid as shown in the diagram, we can generalise the result to find the upthrust experienced by any body immersed in a fluid.



Let ρ = density of the fluid,
A = surface area of a face of the cube,
$\Delta h = h_1 - h_2$ = length of a side of the cube.

The fluid forces acting on the side faces of the cube are equal in magnitude and in opposite directions, thus resulting in **zero net horizontal force**.

The fluid force F_1 acting on the **bottom** surface is $F_1 = \rho g h_1 A$.
The fluid force F_2 acting on the **top** surface is $F_2 = \rho g h_2 A$.
Thus, the **net vertical force** is $F_1 - F_2 = \rho g h_1 A - \rho g h_2 A = (h_1 - h_2)\rho g A = \Delta h \rho g A$
$= \rho V g$ (since the volume of the cube is $V = A\,\Delta h$)
= (mass of fluid displaced) g
= weight of fluid displaced

Reference: Why is upthrust always upwards (xmphysics),
https://www.youtube.com/watch?v=yih8xQMa5zw

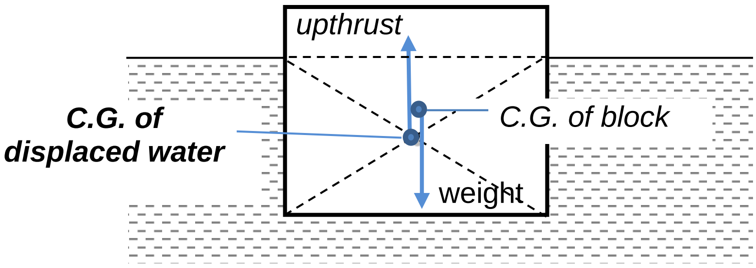
<u>Archimedes' Principle</u>

For a body submerged or floating in a fluid, the upthrust acting on the body **is equal in magnitude and opposite in direction** to the **weight of fluid displaced by the body**.

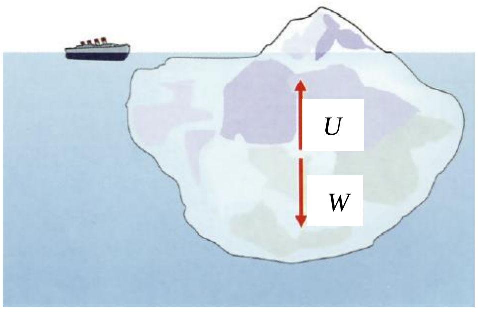
$$U = \rho V g,$$

where U = upthrust,
ρ = density of fluid,
V = volume of fluid displaced by the body,
g = gravitational acceleration.

In free-body diagrams, upthrust must be drawn from the centre of gravity of the fluid displaced.



<u>Example 3</u>
Icebergs are often a danger to ships since most of an ice-berg's volume lies beneath the surface of the water. One can only see the "tip of the iceberg." Find the percentage of the iceberg that is submerged.
(density of ice, ρ_i = 0.917 g cm^{-3}, density of sea-water, ρ_w = 1.025 g cm^{-3})



[Answer: 89.5%]

4.3 Translational Equilibrium

A large object can exhibit different kinds of motion. When we describe the object as rigid, different parts of the object cannot move relative to each other. However, the rigid object as a whole can still rotate about its centre of mass (rotational motion) while the centre of mass can have translational motion. Let us first consider the translational motion of the centre of mass of the object.

The object is said to be in translational equilibrium when it is stationary or moving at constant velocity. According to Newton's First Law, this happens when the net external force acting on the object is zero.

<u>**Condition for Translational Equilibrium:**</u>

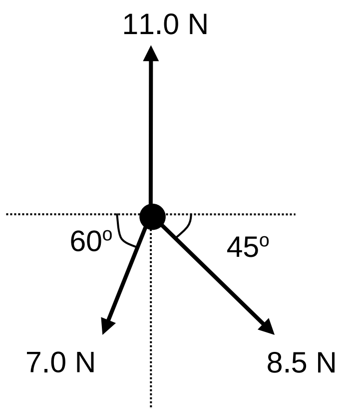
The net external force acting on a body is zero, i.e. $\Sigma F_{ext} = 0$.

There are two common methods to determine whether an object is in equilibrium. The first method is resolving vectors and the second one is the use of a vector triangle.

In the first method, all the vectors acting on an object are resolved into horizontal and vertical components. If the sum of all the horizontal components and the sum of all the vertical components are zero, then the object is in translational equilibrium. For the horizontal and vertical component of a force F, acting at an angle θ, we typically use the following equations to resolve the force vectors: $F_h = F \cos \theta$, $F_v = F \sin \theta$.

For the special case where three coplanar forces (i.e. forces acting on the same plane) are acting on an object and the object is in translational equilibrium, the forces can be represented in magnitude and direction by the adjacent sides of a triangle. In other words, three force vectors that form a triangle for which the directions of the forces are all either clockwise or anticlockwise around the centre of the triangle have **zero resultant**.

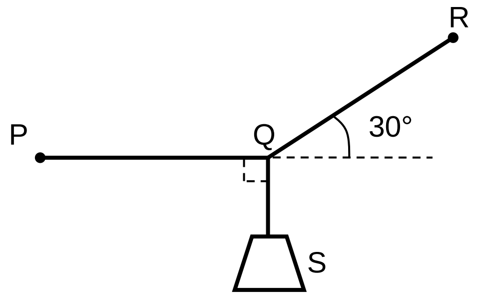
<u>**Example 4 (Resolution of forces into component method)**</u>
Four forces act on a particle such that it is in equilibrium. Three of the forces are shown in the diagram. Find the magnitude and direction of the unknown force F that maintains the equilibrium.



[Answer: 2.73 N, at an angle of 67° anticlockwise from the force of 11.0 N]

Example 5 (J85/I/2) (vector triangle method)

In the diagram below, a body S of weight W hangs vertically by a thread tied at Q to the string PQR. If the system is in equilibrium, what is the tension in the section PQ?



[Answer: 1.73W]

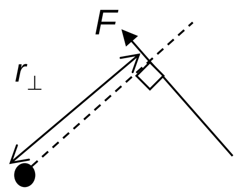
4.4 Moment of a Force

Besides translational motion, objects also exhibit rotational motion. Forces applied on a rigid body can also cause or change its rotational motion. The quantitative measure of the extent to which a force can cause or change the rotational motion of a rigid body is called the **moment of the force**, τ (Greek letter *tau*). The **moment of a force** is also called the **torque**.

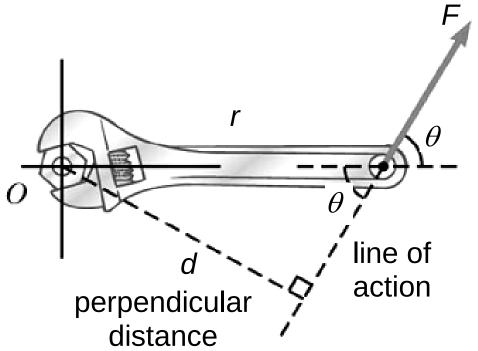
Definition of Moment of a Force

The **moment of a force** τ about a point is the product of the (magnitude of the) force ***F*** and the perpendicular distance $\boldsymbol{r}_\perp$ from the line of action of the force to the point.

$$\tau = F(r_\perp)$$



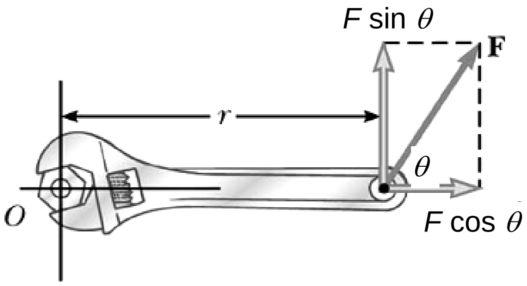
To illustrate the concept of perpendicular distance and line of action, let us consider the example of a spanner being used to loosen a bolt. A force F is applied at an acute angle θ to the length of the spanner and at a distance r away from the point O as shown in the diagram below.



Method 1: Finding the perpendicular distance

The line of action of a force is the line along which the force acts. The perpendicular distance d is the length of the line that joins the point O to the line of action perpendicularly. Using simple trigonometry, it can be deduced that $d = r \sin \theta$. The torque of the force F about the point O can then be calculated as

$$\tau = (F)(r_\perp) = (F)(r \sin\theta) = Fr \sin\theta.$$



Method 2: Finding the perpendicular component of the force

Another method is to resolve the force into two components: perpendicular and parallel to the spanner length. Only the perpendicular component $F \sin\theta$ produces the turning effect. The line of action of the parallel component $F \cos\theta$ passes through the reference point O and produces no turning effect. The torque of the force F about the point O is then given by

$$\tau = (r)(F_{\perp}) = (r)(F\sin\theta) = Fr\sin\theta .$$

Since both approaches lead to the result $\tau = Fr\sin\theta$, one could treat the equation as either

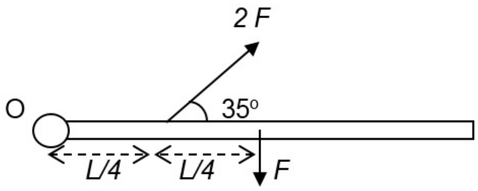
1. magnitude of force × perpendicular distance (between the reference point and the line of action),

or

2. distance (from reference point to point of application of force) × perpendicular component of the force.

Example 6

A uniform rod of weight F and length L is pivoted on one end O. A force $2F$ is applied at a quarter of its length from O. Find the magnitude of the net moment about point O.



[Answer: 0.213*FL*]

4.5 Rotational Equilibrium and Principle of Moments

A rigid object is said to be in rotational equilibrium when the net moment of the object about any point is zero. Note that we may freely choose the reference point to be anywhere (even outside the object) when calculating the net moment of the object. If the object is in rotational equilibrium, regardless of where we choose reference point to be, the net moment about that point will be zero.

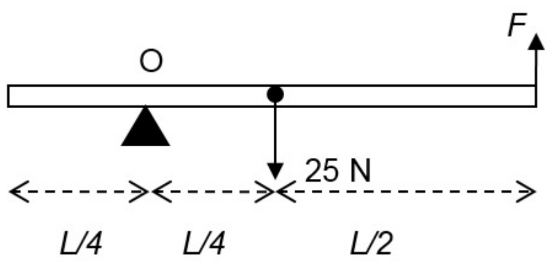
Condition for Rotational Equilibrium:
The net moment on the body ***about any point*** is zero, i.e. $\Sigma\tau = 0$.

When all lines of action of the forces are in the same plane (the forces are *coplanar*), the condition for rotational equilibrium is simplified to the principle of moments.

For rotations confined to a single plane:

Principle of moments states that for a body to be in rotational equilibrium, the sum of clockwise moments about any point equals the sum of anticlockwise moments about that same point.

Example 7
A uniform rod of weight 25 N and length L is balanced on a knife edge at O. Find the unknown force F required to maintain rotational equilibrium.



[Answer: 8.33 N]

4.6 Static Equilibrium for Rigid Extended Bodies

Equilibrium is a concept to describe the state of a body that is either at rest or moving at a constant velocity. The conditions for equilibrium are the same, regardless whether the body is at rest or moving at a constant velocity. However, if the body is at rest, we say that the body is in *static* equilibrium.

Conditions for (Static) Equilibrium of a rigid extended body:
The net external force acting on the body is zero. $\Sigma F_{ext} = 0$.
The net moment on the body *about any point* is zero. $\Sigma \tau = 0$.

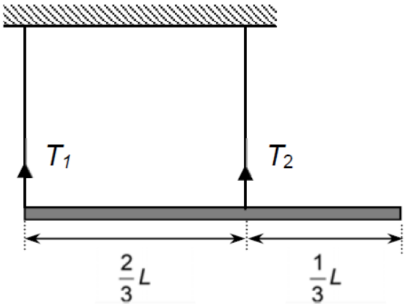
TIPS **Solving Static Equilibrium problems**

1. Isolate the body of interest.
2. Draw the free-body diagram and identify **all** forces acting on the body.
3. Set up the torque equation (condition for rotational equilibrium)
   - Choose a “good” reference point.
   - A “good” reference point is a point which lies on the lines of action of one or more of the unknown forces, thereby reducing the number of unknown terms in the equation.
   - Apply the condition $\sum \tau = 0$,
     sum of clockwise moments = sum of anti-clockwise moments.
4. Set up the two force equations (x and y components) (conditions for translational equilibrium)
   - Choose two perpendicular directions x and y.
   - Resolve all forces into x and y components.
   - Apply the condition $\sum F_x = 0$,
     sum of leftward forces = sum of rightward forces
   - Apply the condition $\sum F_y = 0$,
     sum of upward forces = sum of downward forces
5. Solve the equations to find the unknown forces.

Example 8 (J95/I/5)

A uniform beam of length L and mass M is supported by two vertical cords as shown in the diagram.

What is the ratio $\frac{T_1}{T_2}$ of the tensions in these cords?



[Answer: 0.333]

4.7 Three-Force Systems

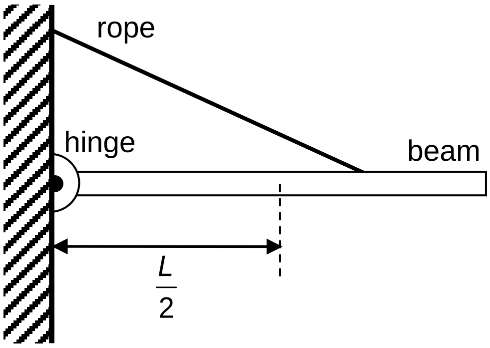
A solid body submitted to three forces whose lines of action are not parallel is in equilibrium, if the following three conditions apply:

1. The lines of action are coplanar (⇒ all forces lie in one and the same plane)
2. The vector sum of the forces is equal to the zero vector (⇒ they can form a closed vector triangle)
3. The lines of action are *convergent* (⇒ their lines of action cross through a single point).

Reference: The 'Three forces must intersect at the same point' Rule, (xmphysics)
https://www.youtube.com/watch?v=u5bF80D0D2c

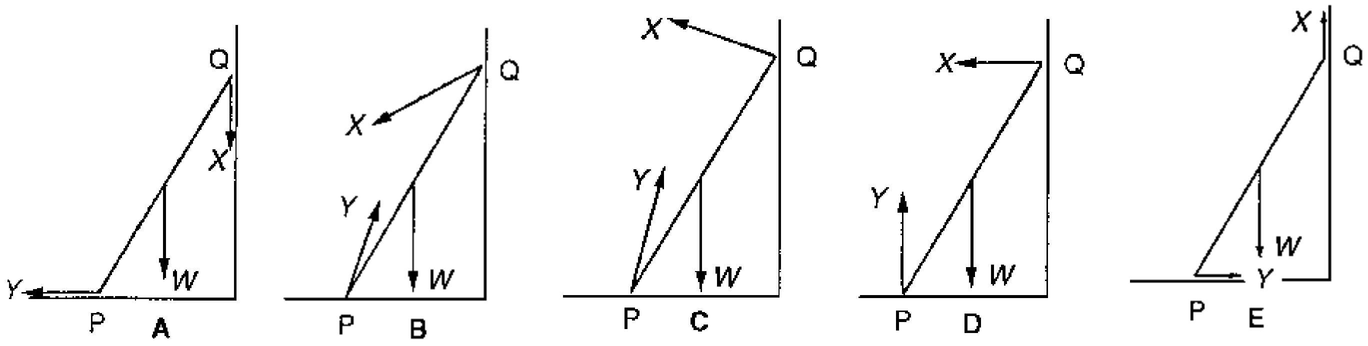
Example 9

A uniform beam balance of length L is hinged to a wall and is supported by a rope as shown. Show the direction of the force exerted by the hinge (attached to the wall) on the beam.



Example 10 (N84/II/1)

A ladder PQ, resting on a rough floor and leaning against a rough wall, is on the point of slipping. It is of weight *W* and the contact forces exerted on the ladder by the wall and the floor are *X* and *Y*, respectively. Which one of the following diagrams correctly shows the directions of these forces?



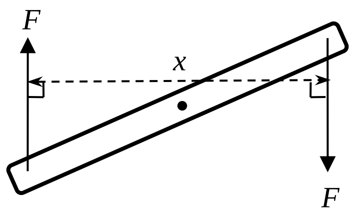
4.8 Couple

Definition of a Couple

A pair of forces, which are equal in magnitude but opposite in direction, whose lines of action do not coincide.

Couples produce **only rotation** but no translation.

An example of a couple is shown in the figure below.



Taking moments about the centre of mass,

the torque due to the couple $=\left(\frac{1}{2}x\right)F + \left(\frac{1}{2}x\right)$

$=xF$ (in the clockwise direction),

where F is the magnitude of the forces and x is the *perpendicular distance* between the lines of action.

In fact, taking moments about **any point** would produce the same result.

Torque of a couple is the product of the perpendicular distance between the lines of action of the forces and the magnitude of one of the forces.

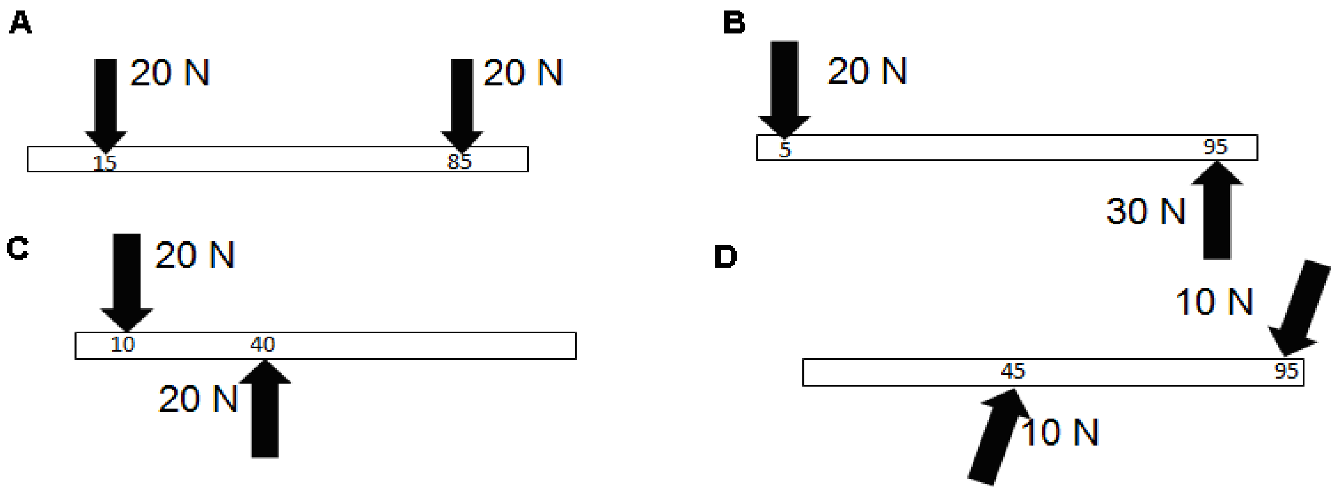
Example 11

The following diagrams show forces acting on uniform metre rules. The rulers are lying on a horizontal smooth table top and shown as seen from above.

Which of the diagrams do not show a couple?

For the other diagrams, what is the torque of the couple?

Assume that for each option, the two forces are parallel to each other.



Appendix I

Contact force and Frictional force

When two bodies are in contact such as a stone at rest on a road, the two rough surfaces actually make close contact only at relatively few places. Where contact is made, the road will exert a force on the stone as shown in Fig. (a). The vector sum of these forces is represented by the single force known as the contact force which the road exerts on the stone as shown in Fig. (b).

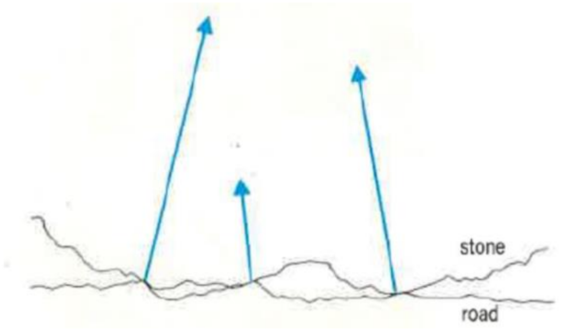


Fig. (a)

Fig. (b)

[on a different scale from fig. (a)]

If the stone slides across the road to the left say, the forces exerted by the road on it are likely to be both upwards and to the right as shown in Fig. (c). The resultant of all these forces is hence tilted to the right (Fig. (d)). The horizontal component of this force is known as the frictional force and the vertical component is known as the normal contact force. Friction which is in the opposite direction to the direction of motion, is usually a component of a contact force. For two surfaces where friction can be neglected, the only force between them will be the normal contact force.

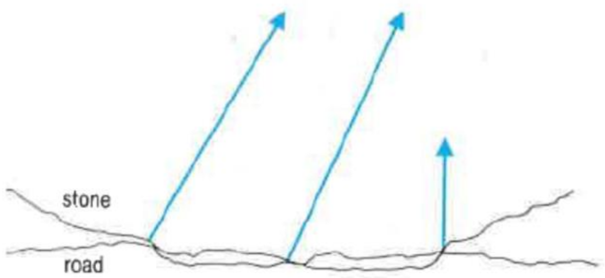


Fig. (c)

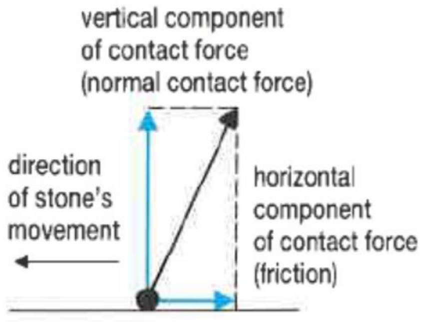


Fig. (d)

[on a different scale from Fig. (c)]

(Ref: Physics, Robert Hutchings)

Appendix II

Viscous Force (air resistance and air drag)

A viscous force refers to the resistance force which a body experiences when it moves through a fluid (a gas or liquid). As the moving body exerts a force on the fluid to push it out of the way, the fluid would in turn exert an equal and opposite force on the body (by Newton's third law).

Two things to note on viscous forces:

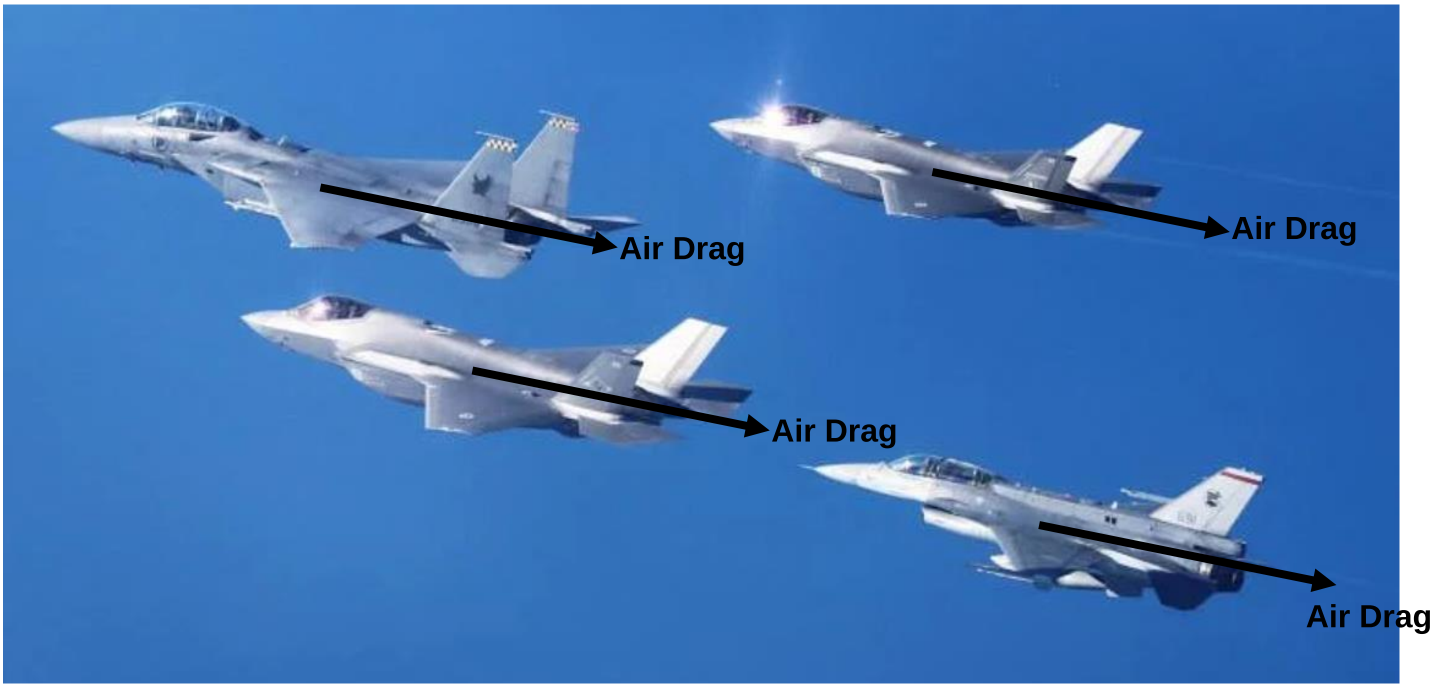
(1) The direction of the fluid resistance acting on a body is always opposite the direction of the body's velocity relative to the fluid.

(2) Magnitude of the fluid resistance f increases with v the speed of the body through the fluid.

(a) For small objects moving at very low speeds, the air resistance is proportional to v, ($f \propto v$)

(b) For larger objects (such as airplanes) moving at very fast speeds (through air), the fluid resistance also known as air drag is proportional to v^2 , ($f \propto v^2$)

[refer to kinematics notes, section 2.6 Viscous Drag Force (pg 17-18) on other factors affecting the viscous force]



(Ref: Sears & Zemansky's University Physics with Modern Physics, Hugh D. Young, Roger A. Freedman)

(image: https://www.straitstimes.com/singapore/rsaf-gain-first-hand-experience-with-australian-air-force-s-f-35as-in-training-exercise , Straits Times, Nov 24, 2022)

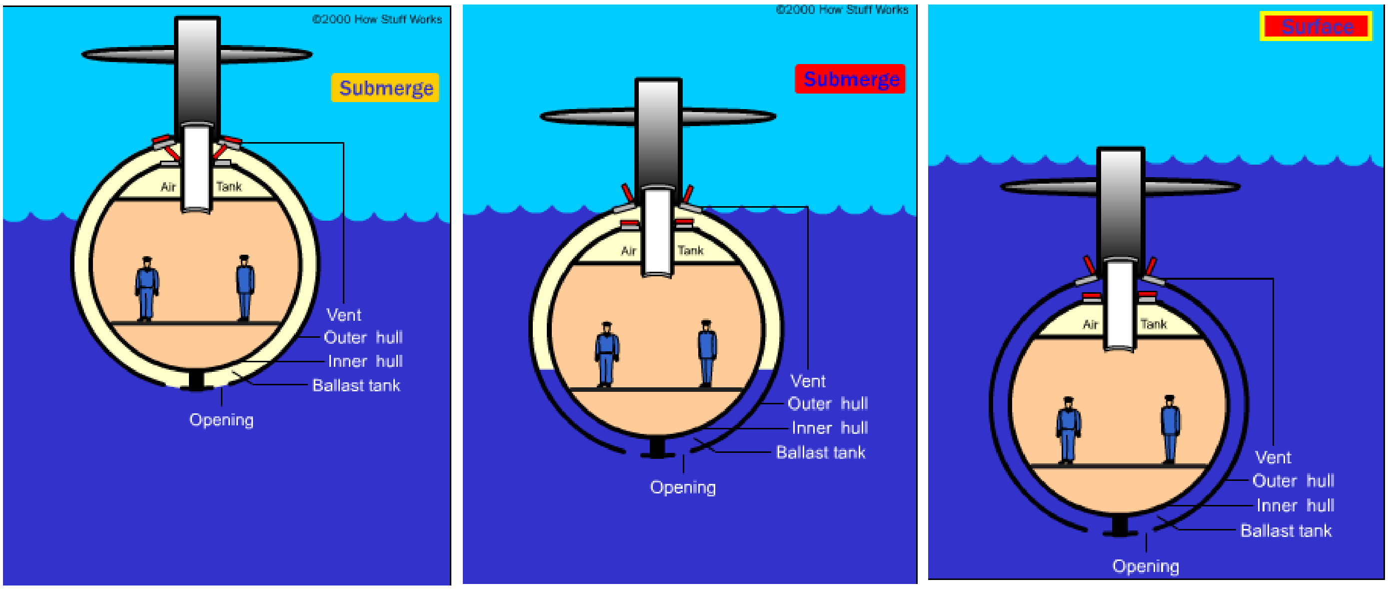
Appendix III

Buoyancy of a submarine

A submarine or a ship can float because the weight of the water it displaces is equal to the weight of the ship. This displacement of water creates an upward force called the buoyancy (upthrust). Unlike a ship, a submarine can control its weight, thus allowing it to sink and surface at will.

To control its weight, the submarine has ballast tanks and auxiliary, or trim tanks, that can be filled with water or air. A supply of compressed air is maintained aboard the submarine in air flasks for life support and for use with the ballast tanks.

When the submarine is on the surface, the ballast tanks are filled with air and the submarine's overall density is less than that of the surrounding water. As the submarine dives, the ballast tanks are flooded with water and the air in the ballast tanks is vented from the submarine until its overall density is greater than that of the surrounding water and the submarine begins to sink. To keep the submarine level at any set depth, the submarine maintains a balance of air and water in the trim tanks so that its overall density is equal to the surrounding water.



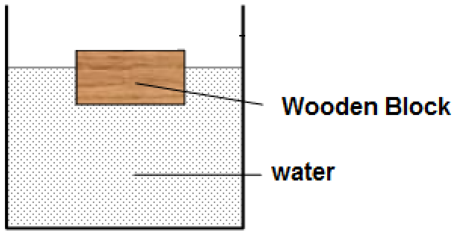
Tutorial 4: Forces

Self-Review Questions

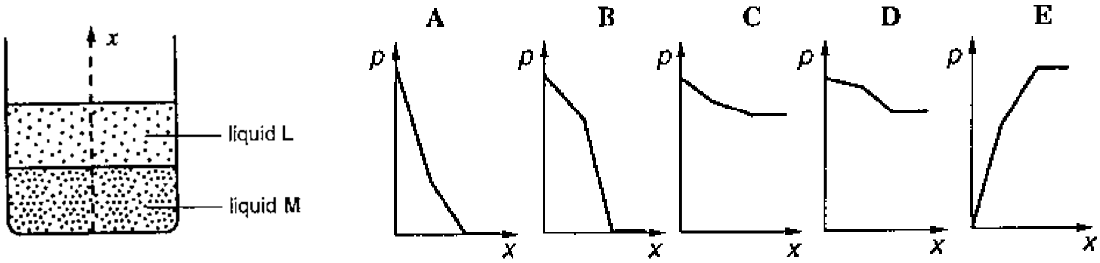
Use these questions to test your familiarity with the concepts. These questions should be sufficiently easy such that you can solve them on your own, with a little bit of thinking, without help from the tutors. The solutions are made available on the cohort Google Classroom for self-check.

S1. A wooden block, with a density of 400 kg m^{-3} and a volume of 0.030 m^3, is floating in water. The density of water is 1000 kg m^{-3}.

(a) What is the net force acting on the wooden block?
(b) What is the weight of wooden block?
(c) What is the upthrust acting on the block?
(d) What is the volume of the block beneath the surface of water?



S2. (J93/P1/Q23) A tall container, which is open to the atmosphere, contains a layer of liquid L, floating on liquid M. Liquid M has a density which is twice as great as that of liquid L. Which graph shows how the pressure p at a point varies with its height x above the base of the container?

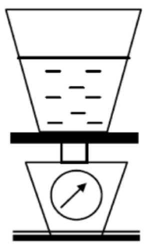


S3. An inextensible string supports a solid iron object of mass 180 g, totally immersed in a liquid of density 800 kg m^{-3}. The density of iron is 8000 kg m^{-3}. Calculate the tension in the string.

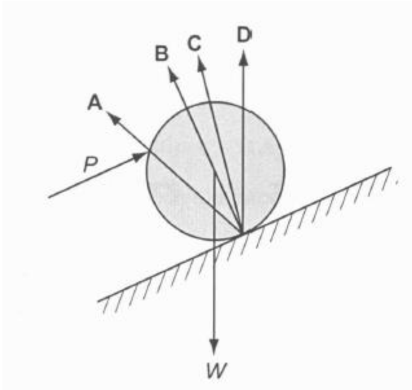
S4. A cup, half-filled with water, is resting on a weighing scale which registers a reading W. When a boy dips his finger in the water without touching the base, the reading of the weighing scale changes to W'.

Which of the following statements is correct?

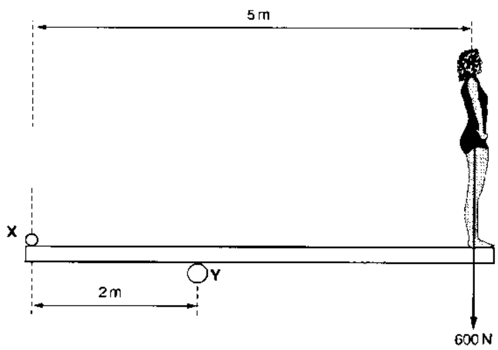
A $W' = W$ because the water has not overflowed from the beaker.
B $W' < W$ because the water exerts an upthrust on the boy's finger.
C $W' > W$ because the water exerts an upthrust on the boy's finger, and thus the boy's finger exerts a force back on the water.
D $W' > W$ because the weight of the boy's finger is added to that of the water in the beaker.



S5. (2010/H1/P1/Q8) A full barrel of weight W is being rolled up a ramp. The force P is required to hold the barrel at rest on the ramp. Friction between the barrel and the ramp stops the barrel from slipping.
Which arrow represents the resultant force the ramp exerts on the barrel?



S6. (J/96/P1/Q5) The diagram shows a light diving-board held in position by two rods X and Y.
Which additional forces do these rods exert on the board when a diver of weight 600 N stands on the right-hand end?



	At X (downwards)	At Y (upwards)
A	400 N	1000 N
B	600 N	1200 N
C	900 N	600 N
D	900 N	1500 N

S7. (2009/H2/P2/Q3) Fig.3.1 shows a force diagram that represents a boat that is being lifted by two ropes so that the boat remains horizontal and travels vertically upwards at a constant speed after leaving the water.

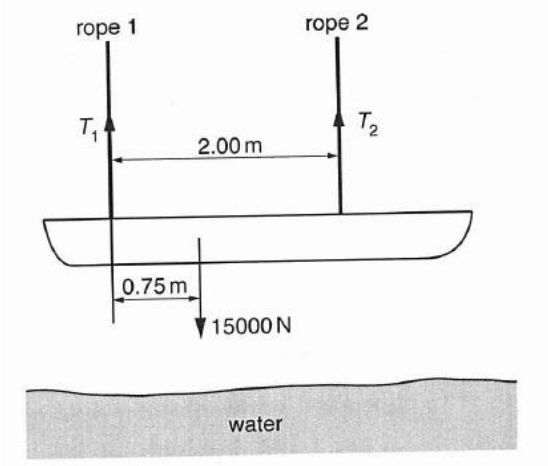


Fig. 3.1

The weight of the boat is 15 000 N and the tensions in the ropes 1 and 2 are T_1 and T_2, respectively.

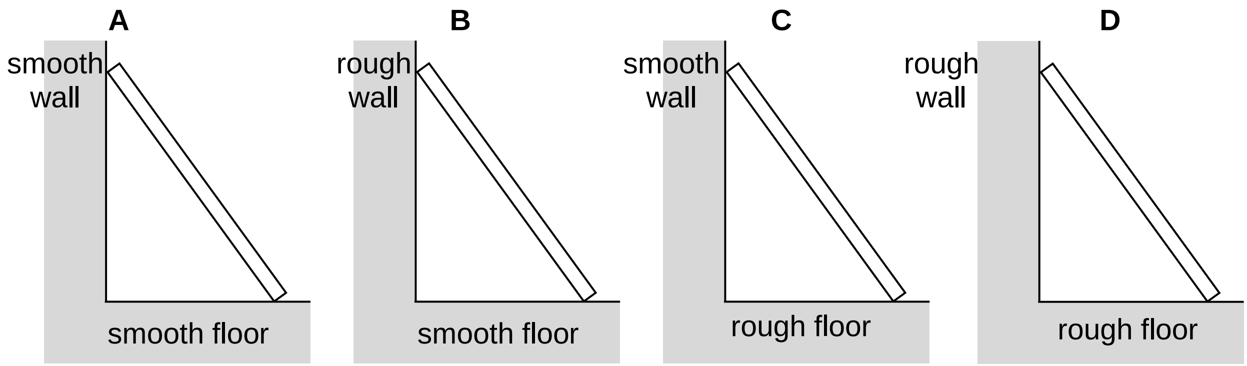
(a) The position of the centre of gravity of the boat is not at its midpoint. Suggest what this implies about the distribution of mass in the boat.

(b) Explain two conditions required for the boat to be in a state of equilibrium while it is moving upwards.

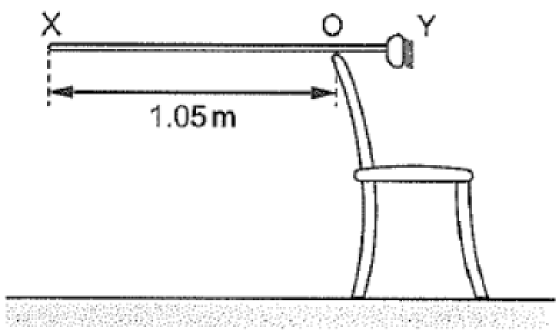
(c) Use the principle of moments to determine the tensions in the two ropes.

Discussion Questions

D1. Consider a uniform ladder leaning against a wall. By drawing and labelling the forces acting on the ladder for each scenario, determine the scenario(s) in which the ladder will definitely slip.



D2. (N/18/P1/Q5) A student balances a long-handled broom XY horizontally across the top of the back of a chair. The top of the chair O is at a distance 1.05 m from the end X of the handle.
A 200 g mass is tied to the handle by a thread at a distance of 0.10 m from the end of X of the handle. The broom now needs to be moved 0.27 m to the right to balance horizontally.
What is the mass of the broom?



A 500 g
B 580 g
C 700 g
D 830 g

D3. (N/16/P2/Q3)
A non-uniform bar AB makes an angle of 60° with a horizontal surface, as shown in Fig. 3.1.

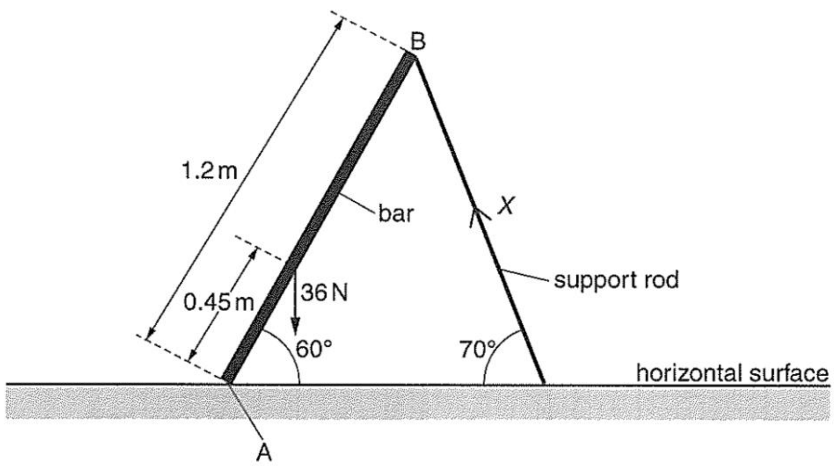


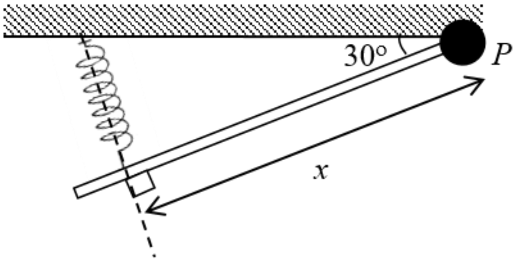
Fig. 3.1

The bar is hinged at A and is supported by a rod at B. The force *X* produced by the rod at B acts at an angle of 70° to the horizontal.
The bar has a length of 1.2 m and a weight of 36 N.
The centre of gravity of the bar is 0.45 m from A.

(a) Use the principle of moments to show that the magnitude of X is 8.8 N.

(b) A force F acts on the bar at A.
(i) Explain why a force is required to act on the bar at A to keep the bar in equilibrium.
(ii) Calculate the magnitude of F.
(iii) On Fig. 3.1, draw an arrow to show the approximate direction of F.

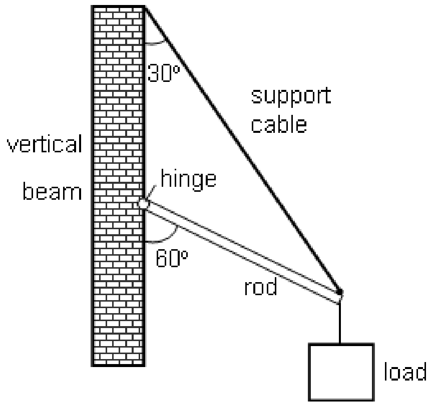
D4. A uniform rod of mass 8.0 kg and length 0.50 m is pivoted about point P. It is held in position by a spring of original length 0.20 m, spring constant 5.0×10^2 N m^{-1}, and negligible mass. Find the length x.



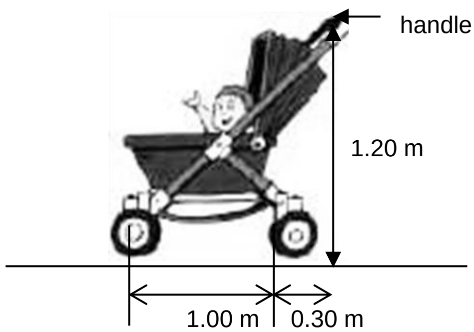
D5. RJC/2009/Prelim/P3/Q1
(a) Define moment of a force.

(b) A uniform rigid rod of weight W_1 = 400 N is attached to a vertical beam by a hinge as shown. The other end of the rod is fastened to a support cable. The structure is used to support a load of weight W_2 = 2000 N.

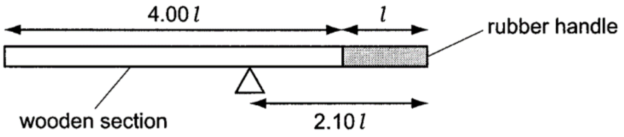
(i) Show that the tension T in the support cable is 3810 N.
(ii) Determine the magnitude and direction of the horizontal and vertical components of the force acting on the rod by the hinge.



D6. (HCI/BT/05/P2/Q3(b))
The diagram shows the side view of a baby stroller. The combined mass of stroller and the boy is 22.0 kg.
(i) Assuming that the centre of gravity of the boy and stroller lies 0.40 m in front of the hind wheels and 0.35 m above the ground, determine the forces experienced by the front wheels F_1 and that experienced by the hind wheels F_2 from the ground.
(ii) As the stroller is not easy to manoeuvre, it is common to see parents hanging their groceries at the handle to free their hands. Determine the maximum load that can be placed at the handle before the stroller topples over.
(iii) It is still extremely dangerous to hang groceries at the handle even though it may be less than the maximum load calculated in (ii). Suggest why this is so.



D7. (2008/P1/Q8) A uniform rod has a wooden section and a solid rubber handle, as shown.



The length of the handle is ℓ and the length of the wooden section is 4.00 ℓ. The rod balances a distance 2.10 ℓ from the rubber end. What is the ratio $\frac{\text{density of rubber}}{\text{density of wood}}$?

A 1.71
B 2.25
C 2.50
D 3.27

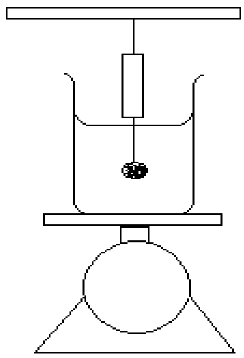
D8. A piece of solid weighs W_1 in air and W_2 when totally immersed in a liquid. Given that the density of the liquid is ρ, determine the volume of the solid in terms of W_1, W_2, ρ and g.

D9. (2012/H2/P1/Q7) A small air bubble in some water is rising to the surface with constant velocity. The volume of the bubble is 2.370×10^{-8} m^3. The density of water is 1000 kg m^3. The density of air is 1.290 kg m^{-3}. What is the magnitude of the viscous force on the bubble?

A 2.367×10^{-5} N
B 2.373×10^{-5} N
C 2.322×10^{-4} N
D 2.328×10^{-4} N

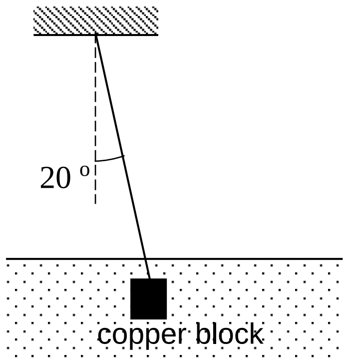
D10. A cube of wood 20 cm on a side and having a density of 0.65×10^3 kg m^{-3} floats on water. Take the density of water to be 1.0×10^3 kg m^{-3}.
(a) What is the distance from the top surface of the cube to the water level?
(b) Find the mass of lead that has to be placed on top of the cube so that the cube's top is just level with the water level.

D11. The weight indicated on a weighing machine is X when a beaker of water is placed on it. A solid object has weight Y in air and displaces weight Z of water when completely immersed. The figure shows the object suspended from a spring balance and completely immersed in the beaker of water. What are the readings on the spring balance and the weighing machine in the given arrangement?



	Spring balance	Weighing machine
A	$Y - Z$	X
B	$Y + Z$	$X + Y - Z$
C	$Y + Z$	$X + Y$
D	$Y - Z$	$X + Z$

D12. A copper block of mass 0.50 kg is hung from the end of a light, inextensible thread and immersed in a fast-flowing river, as shown. The other end of the thread is suspended from a fixed point above water. The river currents caused the copper block to be displaced to the right such that the thread makes an angle of 20 ° with the vertical. The density of copper is 9.00 x 10^3 kg m^{-3} and the density of the river water is 1.00 x 10^3 kg m^{-3}.



Calculate the magnitude of the horizontal force exerted on the copper block by the river currents.

D13. (HCI/BT/08/P2B/Q1)
A spherical steel ball bearing of diameter 1.40 cm is placed on a wooden block of mass 50.0 g, such that the block and ball bearing are floating in a cylinder of oil, as shown in Figure 1.

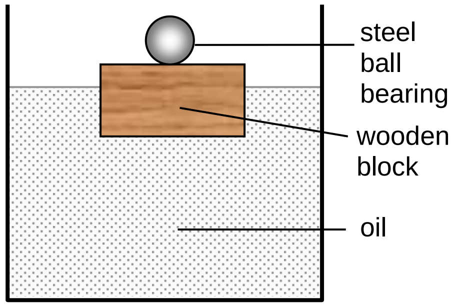


Figure 1

In the following parts of the question, you may assume that:

density of steel, ρ_{steel} = 7.85 g cm^{-3}
density of oil, ρ_{oil} = 0.83 g cm^{-3}

i. Consider the ball bearing and the wooden block as a single system, as in Figure 2. Draw labelled arrows to indicate the external forces acting on this system.
ii. Determine the *upthrust* acting on the system in (i).
iii. Hence, calculate the volume of oil displaced by the wooden block to keep the system afloat.
iv. The ball bearing now rolls off the piece of wooden block and falls into the oil. Explain how the oil level in the cylinder will change.

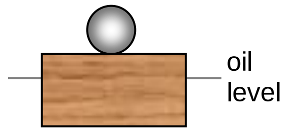


Figure 2

<u>Numerical Answers to Tutorial 4: Forces</u>

D3	(b) (ii) 27.9 N
D4	0.471 m
D5	(b) (ii) 900 N & 1910 N
D6	(b) (i) F_1 = 86 N, F_2 = 129 N; (ii) 288 N
D8	$(W_1 – W_2) / (\rho g)$
D10	(a) 7 cm; (b) 2.8 kg
D12	1.59 N
D13	(ii) 0.601 N; (iii) 73.8 cm^3

FORCES SUMMARY

Types of Forces

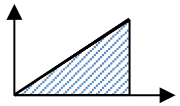
exerted between bodies in physical contact → **Contact Forces**

action at a distance → **Non-Contact Forces**

Spring/Elastic force

For springs that **obey Hooke's Law:**

- External force F_{ext} pulling on extending or compressing spring,
  $F_{ext} = kx$
  where
  k is the spring constant;
  x is the extension or compression



- Restoring force in spring
  $F_{spring} = kx$

Elastic potential energy stored in spring,
U = Area under $F - x$ graph

$$= \frac{1}{2}Fx = \frac{1}{2}kx^2$$

Pressure due to Fluid

(a fluid is any liquid or gas)

- is the **force acting per unit area** by the fluid on a body submerged at a depth in the fluid.
- **Hydrostatic Pressure** or the *pressure due to a fluid at a depth* h is given by:
  $p = h\rho g$
  where
  h is depth
  p is the hydrostatic pressure at this depth
  ρ is the density of the fluid
  g is the gravitational acceleration

Upthrust or Buoyant Force

- is the **net upward force exerted by a fluid** on a body floating/submerged in the fluid.
- **Archimedes' Principle**
  (used to compute *the magnitude* of the upthrust):
  Upthrust (U) on object = Weight of fluid displaced by object
  $U = m_f g = \rho_f V_{dis} g$
  where
  U is the upthrust acting on the object
  m_f is the mass of the fluid displaced
  g is the gravitational acceleration
  V_{dis} is the volume of the fluid displaced
  ρ_f is the density of the displaced fluid

For an **object floating in equilibrium** in a fluid (**Principle of Flotation**):

Weight of object = Upthrust on object

Non-Contact Forces

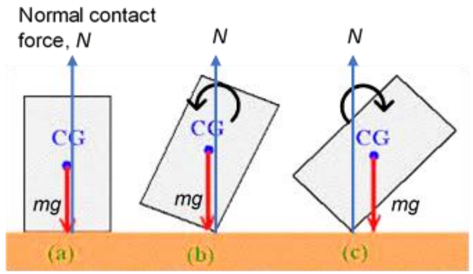
- Gravitational force between masses
- Electric force between charges
- Magnetic force between currents

Drag/Viscous Force

- Force resisting an object **moving relative to a fluid**. (e.g. air resistance)
- Always **opposes** motion (hence, no motion, no force)
- Magnitude is **dependent on the velocity** of the object **relative to** the fluid, e.g.,
  laminar (streamline) flow: $F_D = kv$
  turbulent flow: $F_D = kv^2$
  where
  F_D is the drag force acting on the object
  k is a constant dependent on the dimensions of the object the type of fluid
  v is the velocity of the object relative to the fluid
  (relationship usually provided in exam questions)

Centre of gravity (c.g.) of a body

Centre of gravity (c.g.) of a body is the point at which the weight of the body appears to act. E.g.



Lower c.g. increases stability of object.

Equilibrium of Forces

2 conditions for a rigid body to be in static equilibrium:

1) **Translational equilibrium: net external** force acting on the body is zero.

$$\sum F = 0$$

$$\Rightarrow \sum F_x = 0 \;\&\; \sum F_y = 0 \text{ (\& in fact, } \sum F_z = 0)$$

2) **Rotational equilibrium: net torque** on the body about **any point** is zero.

$$\sum \tau = 0$$

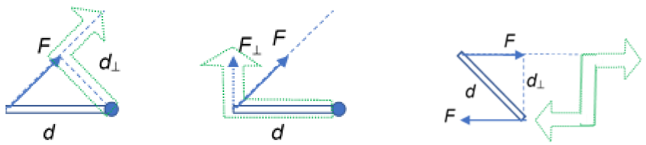
$\Rightarrow$ sum of clockwise moments = sum of anticlockwise moments

Note:

- For 3-forces system in static equilibrium, the 3 forces form ***a closed vector triangle***.
- acting on an ***extended body in static equilibrium***, the line of action of the 3 forces must ***intersect at a common point*** unless the 3 forces are //.

Turning Effects of Forces

The ***moment of a force*** about a point is the **product** of the **magnitude of the force** F and the $\perp$ **distance** $d_\perp$ of the **line of action** of the force to the point. $\tau = Fd_\perp$



A ***couple*** consists of 2 parallel forces which are equal in magnitude and opposite in direction

The ***torque of a couple*** is the **product** of the **magnitude of one of the forces** of the couple and the **perpendicular distance** between the forces