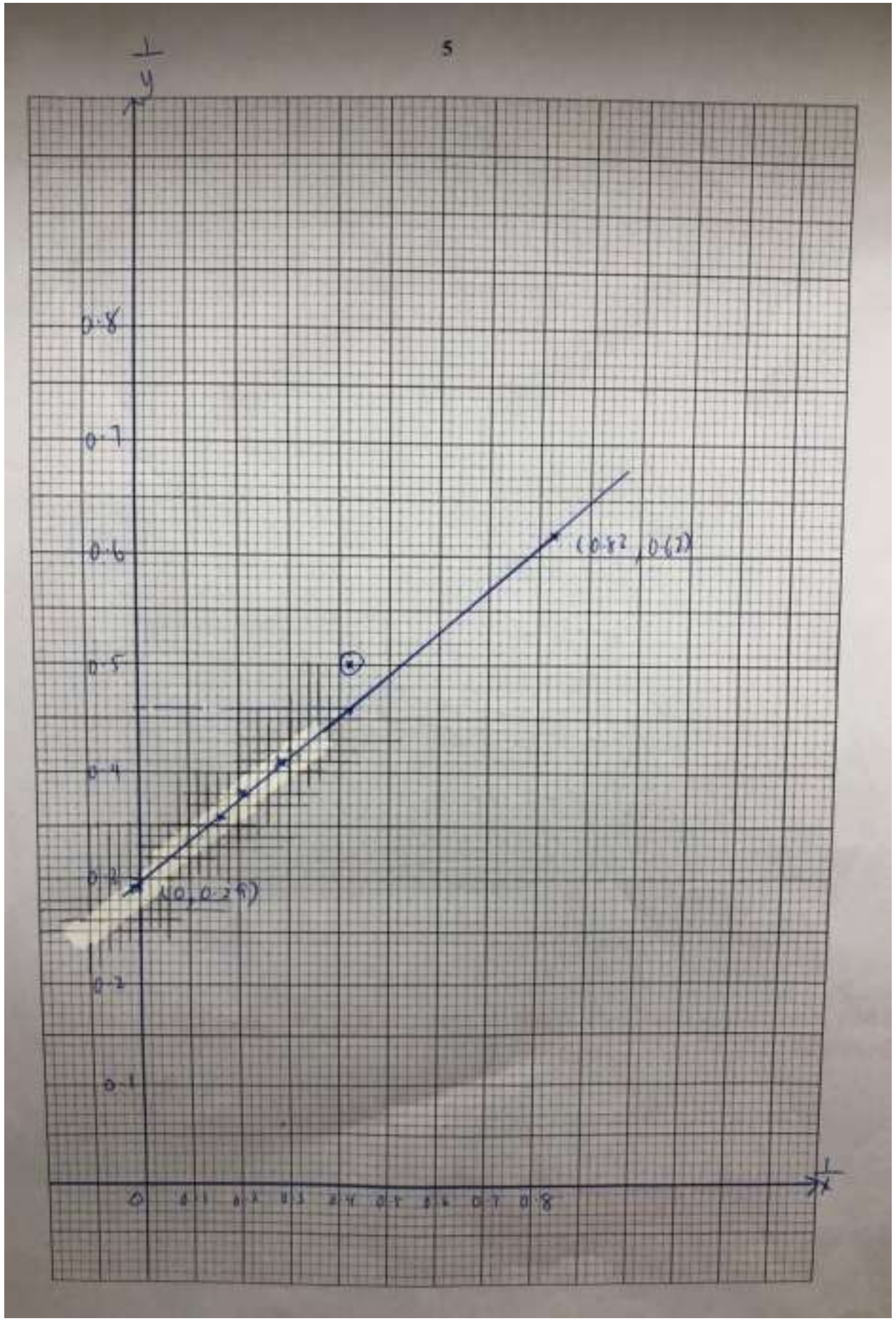
Additional Mathematics
Sec 4 Express MYE Examination 2021 (Paper 2)
Marking Scheme (Setter: Teo Hock Siong)

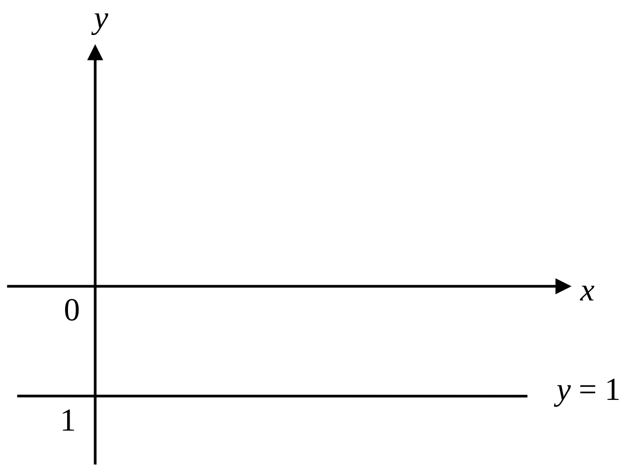
Question	Solution	Marks	
			Total
1	By long division, $$x^3+2x\,\overline{)\,3x^3+6x-8}$$ quotient 3 $-(3x^3+6x)$ -8	M1	
	$$\frac{3x^3+6x-8}{x(x^2+2)}=3-\frac{8}{x(x^2+2)}$$	M1	
	$$\frac{-8}{x(x^2+2)}=\frac{A}{x}+\frac{Bx+C}{x^2+2}$$	M1	
	$8 = A(x^2 + 2) + (Bx + C)(x)$ Using substitution, $x = 0$, - 8 = 2A A = - 4	M1	
	Comparing x-coefficient, $C-0$	M1	
	Comparing x^2-coefficient $0=A+B$ $B=4$	M1	
	Therefore,	A1	7m

	$$\frac{3x^3+6x-8}{x(x^2+2)} = 3-\frac{4}{x}+\frac{4x}{(x^2+2)}$$		
2a	<table><tr><td>$\frac{1}{x}$</td><td>0.83</td><td>0.42</td><td>0.28</td><td>0.21</td><td>0.17</td></tr><tr><td>$\frac{1}{y}$</td><td>0.62</td><td>0.50</td><td>0.41</td><td>0.38</td><td>0.36</td></tr></table>	B1	
	Correct axes and plotted points.	B1	
	Appropriate straight line drawn.	B1	3m



5
1/y
0.8
0.7
0.6
0.5
0.4
0.1
(0.82, 0.62)
(0, 0.29)
0
0.8
1/x

2bi	$\frac{1}{y}=0.46 \quad \therefore y\approx 2.17\ (\pm 0.1)$ (3s.f.)	B1	1m
2bii	$\frac{a}{x}-\frac{b}{y}=1$ $\frac{b}{y}=\frac{a}{x}-1$ $\frac{1}{y}=\frac{a}{b}\left(\frac{1}{x}\right)-\frac{1}{b}$ From the graph, $-\frac{1}{b}=0.29$ $b\approx -3.45(\pm 0.1)$ (3 s.f.) Using 2 points on the graph, (0.83, 0.62) & (0, 0.29) $\frac{a}{b}=\frac{0.62-0.29}{0.83-0}$ $a=\frac{0.62-0.29}{0.83}\times(-3.45)$ $a\approx -1.37(\pm 0.1)$ $(3s.f.)$	M1 A1 M1 A1	4m
2c	$\frac{a}{x}-\frac{b}{y}=1$ $b\left(\frac{x}{y}\right)=a-x$ $\frac{x}{y}=\left(\frac{-1}{b}\right)x+\frac{a}{b}$ By plotting $\frac{x}{y}$ against x, *a* and *b* can be found using	M1	

	gradient = $\frac{-1}{b}$ and $\frac{x}{y}$ - intercept = $\frac{a}{b}$	A1 A1	 3m
3	 		
ai	*x*-intercept of curve	A1	1m

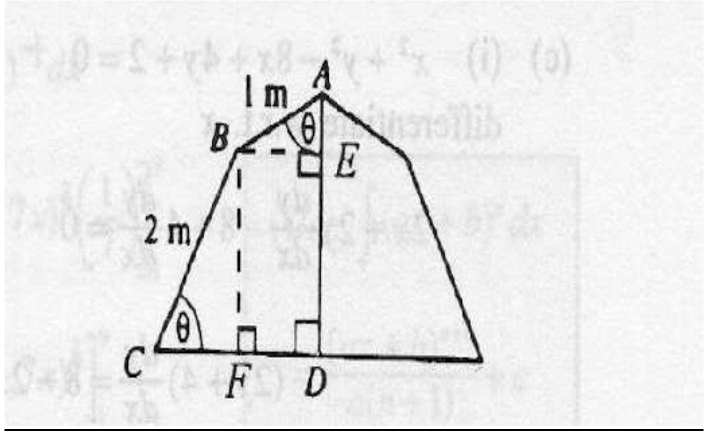
	$\cos\frac{1}{3}x=0$ $x=\frac{3\pi}{2}$		
aii	$2\cos\frac{1}{3}x=-1$ $\cos\frac{1}{3}x=-\frac{1}{2}$ $Basic\angle=\cos^{-1}\frac{1}{2}$ $=\frac{\pi}{3}$ $\frac{1}{3}x=\pi-\frac{\pi}{3}$ $\frac{1}{3}x=\frac{2\pi}{3}$ $x=2\pi$	M1 A1	 2m

3b	*Area* of region A $= \int_0^{\frac{3\pi}{2}} 2\cos\frac{1}{3}x \; dx$ $= \left[\dfrac{2\sin\frac{1}{3}x}{\frac{1}{3}}\right]_0^{\frac{3\pi}{2}}$ $= \left(6\sin\frac{1}{3}x\right)_0^{\frac{3\pi}{2}}$ $= 6$ *Area* of rectangle $= (1)(2\pi)$ $= 6.28319$ *Area* of region B $= -\int_{1.5\pi}^{2\pi} 2\cos\frac{1}{3}x$ $= -\left[\dfrac{2\sin\frac{1}{3}x}{\frac{1}{3}}\right]_{\frac{3\pi}{2}}^{2\pi}$ $= -\left[6\sin\frac{1}{3}x\right]_{\frac{3\pi}{2}}^{2\pi}$ $= -\left[\left(6\sin\frac{1}{3}(2\pi)\right) - \left(6\sin\frac{1}{3}(\frac{3\pi}{2})\right)\right]$ ≈ 0.8038 *Area* of region C $= 6.28319 - 0.8038$ $= 5.480$	M1 M1 M1 M1	

	Required shaded area $= 6 + 5.480$ $= 11.5$ units^2 (3 s.f.)	A1	5m
4a	$x^2 + 2(2x+k)^2 = 8$ $9x^2 + 8kx + 2k^2 - 8 = 0$ Line and Curve intersect at 2 distinct points, $(8k)^2 - 4(9)(2k^2 - 8) > 0$ $64k^2 - 72k^2 + 288 > 0$ $8k^2 - 288 < 0$ $k^2 - 36 < 0$ $(k-6)(k+6) < 0$ $-6 < k < 6$ Largest integer k is 5.	M1 M1 M1 M1 A1	5m
4b	Since $y > 0$,	M1	

	$(2m)^2 - 4(m-5)(m+3) < 0$ $4m^2 - 4m^2 + 8m + 60 < 0$ $8m + 60 < 0$ $m < -\frac{15}{2}$ But $y > 0$ means the x^2 coefficient must be positive, that is $m - 5 > 0 \quad \Rightarrow \quad m > 5$ Since m must be $m < -\frac{15}{2}$ and $m > 5$ for $y > 0$, $\therefore$ there are no values of m for which $y > 0$ (is always positive).	M1 M1 A1	4m
5a	$\int_3^0 f(x)dx + \int_0^3 g(x)dx = \int_0^3 e^{-3x}dx$ $-\int_0^3 f(x)dx + \int_0^3 g(x)dx = \int_0^3 e^{-3x}dx$ $-a + (-2a) = \int_0^3 e^{-3x}dx$ $-3a = \left[\frac{e^{-3x}}{-3}\right]_0^3$ $-3a = \left[\frac{e^{-3(3)}}{-3}\right] - \left[\frac{e^{3(0)}}{-3}\right]$ $-3a = 0.33329$	M1 M1 M1	

	$a=-0.111$ (3 s.f.)	A1	4m
5bi	$y=\ln\left(\frac{x-3}{x+3}\right)=\ln(x-3)-\ln(x+3)$ $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{x-3}-\frac{1}{x+3}$ $=\frac{(x+3)-(x-3)}{(x-3(x+3)}$ $= \quad =\frac{6}{x^2-9}$ (shown)	M1 M1 A1	 3m
5bii	$\int_4^6\left(\frac{2x+3}{x^2-9}\right)dx=\int_4^6\left(\frac{2x}{x^2-9}\right)dx+\int_4^6\left(\frac{3}{x^2-9}\right)dx$ $=\left[\ln(x^2-9)\right]_4^6+\frac{1}{2}\int_4^6\left(\frac{6}{x^2-9}\right)dx$ $=\left[\ln(x^2-9)+\frac{1}{2}\ln\left(\frac{x-3}{x+3}\right)\right]_4^6$ $=\left(\ln 27+\frac{1}{2}\ln\frac{3}{9}\right)-\left(\ln 7+\frac{1}{2}\ln\frac{1}{7}\right)$ = 1.77 (3 s.f.)	M1 M1 M1 M1 A1	 5m

6			
6a	$AD = AE + ED$ $= AE + BF$ $= \cos\theta + 2\sin\theta$ $(shown)$	M1 A1	 2m
b	$\cos\theta + 2\sin\theta = R\cos(\theta - \alpha)$ $= R\cos\theta\cos\alpha + R\sin\theta\sin\alpha$ Therefore, $R\cos\alpha = 1$ …… (1) $R\sin\alpha = 2$ …… (2)	M1 M1	

	$(1)^2+(2)^2$ $R^2\cos^2\alpha+R^2\sin^2\alpha=1^2+2^2$ $R^2(\cos^2\alpha+\sin^2\alpha)=5$ $R^2=5$ $R=\sqrt{5}$ $(2)\div(1)$ $\frac{R\sin\alpha}{R\cos\alpha}=\frac{2}{1}$ $\tan\alpha=2$ $\alpha=\tan^{-1}2$ $=63.4°$ $AD=\sqrt{5}\cos(\theta-63.4°)$	M1 A1	4m
c	Maximum value of *AD* is $\sqrt{5}$. $\cos(\theta-63.4°)=1$ $\theta-63.4°=\cos^{-1}1$ $\theta=63.4°$	B1 M1 A1	3m
d	$AD=\sqrt{5}\cos(\theta-63.4°)$ $\sqrt{5}\cos(\theta-63.4°)=2.15$ $\cos(\theta-63.4°)=0.9615$ $\theta-63.4°=15.95°$ $\theta=79.4°$ (1 dec pl)	M1 M1 A1	3m
7a	$\frac{dy}{dx}=\frac{2x(x-3)-(x-3)^2}{x^2}$		

	$=\frac{x^2-9}{x^2}$ To find stationary point, set $\frac{x^2-9}{x^2}=0$ $x^2-9=0$ $(x+3)(x-3)=0$ $x =$ -3 , 3 When x = -3 $y=\frac{(-3-3)^2}{-3}$ $=-12$ When x = 3 $y=\frac{(3-3)^2}{3}$ $=0$ The stationary points are (3, 0) and (– 3, – 12)	A1 M1 A1, A1	4m
b	$\frac{d^2y}{dx^2}=\frac{x^2(2x)-(x^2-9)(2x)}{x^4}$ $=\frac{18}{x^3}$	M1	

	When $x = -3$ $\frac{d^2y}{dx^2} = \frac{18}{(-3)^3}$ $= -\frac{2}{3} < 0$ Therefore, (-3, -12) is a maximum point. When $x - 3$ $\frac{d^2y}{dx^2} = \frac{18}{(3)^3}$ $= \frac{2}{3} > 0$ Therefore, (3, 0) is a minimum point.	M1 A1 M1 A1	5m
8ai	Since the centre lies on the line *y* = *x* + 8, let the centre, *C*, be (*a*, *a* + 8). Using the concept of radius, $AC = BC$ $\sqrt{(a-0)^2+(a+8-9)^2} = \sqrt{(a-(-3))^2+(a+8-0)^2}$ $a^2+(a-1)^2 = (a+3)^2+(a+8)^2$ $a^2+a^2-2a+1 = a^2+6a+9+a^2+16a+64$ $24a = -72$ $a = -3$ $a+8 = 5$ Therefore, centre of the line *C* is $(-3, 5)$. (Shown)	M1 M1 M1 A1	4m

	Or Alternative Method		
8ai	Mid-point of *AB* $=\left(\frac{0+(-3)}{2},\frac{9+0}{2}\right)$ $=\left(-\frac{3}{2},\frac{9}{2}\right)$ Gradient of *AB* $=\frac{9-0}{0-(-3)}$ $=3$ Let equation of perpendicular bisector of *AB* $y=-\frac{1}{3}x+c$ Put $\left(-\frac{3}{2},\frac{9}{2}\right)$, $\frac{9}{2}=-\frac{1}{3}\left(-\frac{3}{2}\right)+c$ $c=4$ $y=-\frac{1}{3}x+4$ Solving $y=-\frac{1}{3}x+4$ and $y=x+8$	M1 M1 M1	

	$-\frac{1}{3}x+4=x+8$ $x=-3$ $y=-3+8$ $=5$ Hence the require centre C is (-3, 5). (shown)	A1	
aii	Since $B(-3,0)$ and $C(-3,5)$ Radius of circle C_1 is $=\sqrt{(-3-(-3))^2+(0-5)^2}=5$ Therefore, equation of circle C_1 is $(x+3)^2+(y-5)^2=5^2$ $(x+3)^2+(y-5)^2=25$	M1 A1	2m
bi	Using the equation of circle $x^2+y^2+2gx+2fy+c=0$, where centre C = $(-g, -f)$, Hence for C(-3, 5), g = 3 and f = -5 Then for equation $x^2+y^2+ax+by-2=0$, $a=2g$ $=2(3)$ $=6$	A1	

	$b = 2f$ $= 2(-5)$ $= -10$	A1	2m
bii	Radius of equation of circle C_2 $= \sqrt{f^2 + g^2 - c}$ $= \sqrt{(-5)^2 + (3)^2 - (-2)}$ $= 6$ Radius of C_2 is 6 units and they have the same centre. Since the radius of C_2 is longer than that of C_1, circle C_2 lies outside the circle C_1.	A1 B2	 3m
9a	$2^{x+1} + 3\left(\frac{1}{2^x}\right) = 7$ When $u = 2^x$, $2(2^x) + 3\left(\frac{1}{2^x}\right) = 7$ $2u + \frac{3}{u} = 7$ $2u^2 - 7u + 3 = 0$ $(2u-1)(u-3) = 0$ $u = \frac{1}{2}$ $2^x = 2^{-1}$ $x = -1$	M1 A1	

	$u = 3$ $2^x = 3$ $\lg 2^x = \lg 3$ $x = \dfrac{\lg 3}{\lg 2}$ ≈ 1.584 ≈ 1.58 $(3s.f.)$	M1 A1	 4m
9b	$9\ \log_x 3 = 6 - \log_3 x$ $9\dfrac{\log_3 3}{\log_3 x} = 6 - \log_3 x$ $9 = 6\log_3 x - (\log_3 x)^2$ let $u = \log_3 x$ $9 = 6u - u^2$ $u^2 - 6u + 9 = 0$ $(u-3)^2 = 0$ $u = 3$ $\log_3 x = 3$ $x = 27$	M1 M1 M1 A1	 4m

9c	$7^{n+1}-4(7^n)-\frac{1}{7}(7^n)$ $=7(7^n)-4(7^n)-\frac{1}{7}(7^n)$ $=7^n(7-4-\frac{1}{7})$ $=\frac{20}{7}(7^n)$ $=20(7^{n-1})$ Since 20 is divisible by 2, $7^{n+1}-4(7^n)-7^{n-1}$ is an even number for all positive values of n .	M1 M1 B1	3m