PEICAI SECONDARY SCHOOL
SECONDARY 3 EXPRESS
END-OF-YEAR EXAMINATION 2023

CANDIDATE NAME: Solutions

CLASS | | | | REGISTER NUMBER | | |

MATHEMATICS **4052/01**
Paper 1 **2 October 2023**
2 hour 15 minutes

Candidates answer on Question Paper

READ THESE INSTRUCTIONS FIRST
Write your register number, class and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 90.

	ANnotations	**AC**curacy
Marks Deducted	1	1

For Examiner's Use

This document consists of **19** printed pages and **1** blank page.

Setter: Mr Lim Jit Chong

[Turn over

Mathematical Formulae

Compound Interest

$$\text{Total Amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone = πrl

Surface area of a sphere = $4\pi r^2$

Volume of a cone = $\frac{1}{3}\pi r^2 h$

Volume of a sphere = $\frac{4}{3}\pi r^3$

Area of triangle $ABC = \frac{1}{2}ab\sin C$

Arc length = $r\theta$, where θ is in radians

Sector area = $\frac{1}{2}r^2\theta$, where θ is in radians

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

Answer **all** questions

1 A bag contains 7 red marbles, 5 blue marbles and 3 yellow marbles.

(a) A marble is chosen at random and then replaced.
What is the probability that it is a red marble?

Answer $\frac{7}{15}$ B1 [1]

(b) How many more blue marbles must be placed in the bag so that the probability of choosing a blue marble would be $\frac{1}{2}$?

$\frac{5+x}{15+x} = \frac{1}{2}$ M1

Answer 5 A1 or B2 [2]

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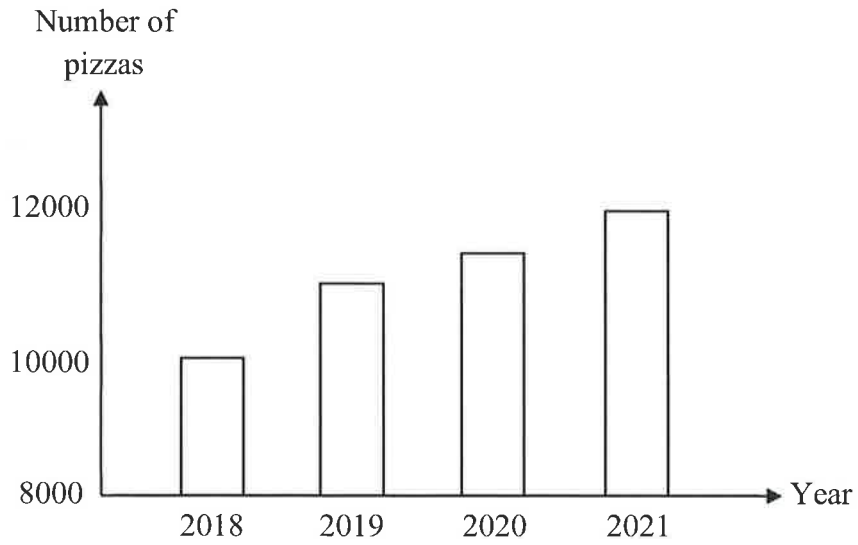
2 The graph below shows the total number of pizzas sold by a pizza restaurant between 2018 to 2021.



Explain how the graph above may be misleading.

Answer: The vertical axis did not start from zero B1

The number of pizzas for year 2021 looks like it is twice of 2018 but it is not true. B1 [2]

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3 The sine of an angle is 0.7654.
Give two possible values for the angle.

Answer 49.9 B1 ° or 130.1 B1 ° [2]

---

4 The speed of a space shuttle is 28 000 km/h. The distance of the Moon from the Earth is approximately 3.84×10^5 km. Calculate the time taken for the space shuttle to reach the Moon from Earth, giving your answer in hours and minutes.

$\frac{3.84 \times 10^5}{28\,000}$ M1

$= 13\frac{5}{7}$ h

Answer 13 h 43 min [2] A1

---

5 $x = 3$ is a solution to the equation $x^2 + kx - 15 = 0$, where k is a constant.

(a) Show that $k = 2$. [1]

Answer: $(2)^2 + k(2) - 15 = 0$ B1

(b) Find the other solution of x.

$x^2 + 2x - 15 = 0$

$(x-3)(x+5) = 0$ M1

$x = 3$ or $x = -5$

Answer $x =$ −5 A1 [2]

---

6 The stem and leaf diagram below shows the math exam scores of students in Class A.

Stem	Leaf							
5	2	2	5	7	8			
6	2	3	5	5	6	9	9	
7	0	0	0	0	1	4	5	9
8	1	6	7					
9	2	5						

Key: 9 | 2 means 92

Find

(a) the mean score,

$\frac{1753}{25} = 70.12$ (exact)

Answer 70.12 B1 [1]

(b) the median score,

Answer 70 B1 [1]

(c) the modal score.

Answer 70 B1 [1]

7 In Singapore, Mr Lim pays $2.80 for one litre of petrol.
On a visit to America, he paid 8.40 US dollars for one gallon of petrol.

(a) Given that 0.74 US dollar (USD) = 1 Singapore dollars (SGD), find the amount that Mr Lim paid for the petrol in Singapore dollars on his visit to America.

Answer 11.35 B1 SGD [1]

(b) Given that 1 gallon = 3.785 litres, is petrol cheaper in Singapore or America? Justify your answer with workings clearly.

$\frac{11.35135}{3.785}$ M1

$= 3.00$ SGD/l

$3 - 2.80 = 0.20$

Answer: Petrol is cheaper in Singapore by $0.20 A1

.....

..... [2]

8 **(a)** Express 540 as a product of its prime factors.

Answer $2^2 \times 3^3 \times 5$ B1 [1]

(b) Written as a product of its prime factors, $168 = 2^3 \times 3 \times 7$.
Find the highest common factor of 540 and 168.

Answer 12 B1 [1]

(c) Find the lowest common multiple of 540 and 168.
Leave your answer in index notation.

Answer $2^3 \times 3^3 \times 5 \times 7$ [1] B1

(d) Find the smallest positive integer value of n such that $168n$ is a perfect cube.

Answer 441 B1 [1]

9 **(a)** **(i)** Calculate $\frac{\sqrt[3]{234}-1.3}{0.2^4}$ and write down the first 5 digits.

Answer 3038.9 B1 [1]

(ii) Correct your answer in part (a)(i) to 3 significant figures.

Answer 3040 B1 [1]

(b) Arrange the following numbers in order of size, starting with the largest.

$-0.2^{\frac{4}{3}}$, 3, $\sqrt[4]{123}$, -0.1

-0.117 3.33

M1: Any 2 pairings in order

A1: All 4 pairings

Answer $\sqrt[4]{123}$, 3, -0.1, $-0.2^{\frac{4}{3}}$ [2]

---

10 **(a)** Simplify $(2x+5)^2-3(x-2)$.

$4x^2+20x+25-3x+6$ M1

Answer $4x^2+17x+31$ A1 [2]

(b) Factorise $2ax-6a+3bx-9b$

$2a(x-3)+3b(x-3)$ M1

Answer $(2a+3b)(x-3)$ A1 [2]

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11 The first five terms of a sequence are 2, 5, 8, 11 and 14.

(a) Write down the next two terms in the sequence.

Answer 17 , 20 [1]

(b) Find an expression for the nth term of the sequence.

Answer $3n-1$ [2]

(c) Is 61 a term in this sequence? Explain your answer with clear workings.

$3n-1=61$

$3n=62$

$n=20\frac{2}{3}$

Answer: Since the value of n is not an integer, 61 is not a term in this sequence [2]

12 **(a)** Express $x^2 - 6x + 11$ in the form $(x-a)^2 + b$ where a and b are integers.

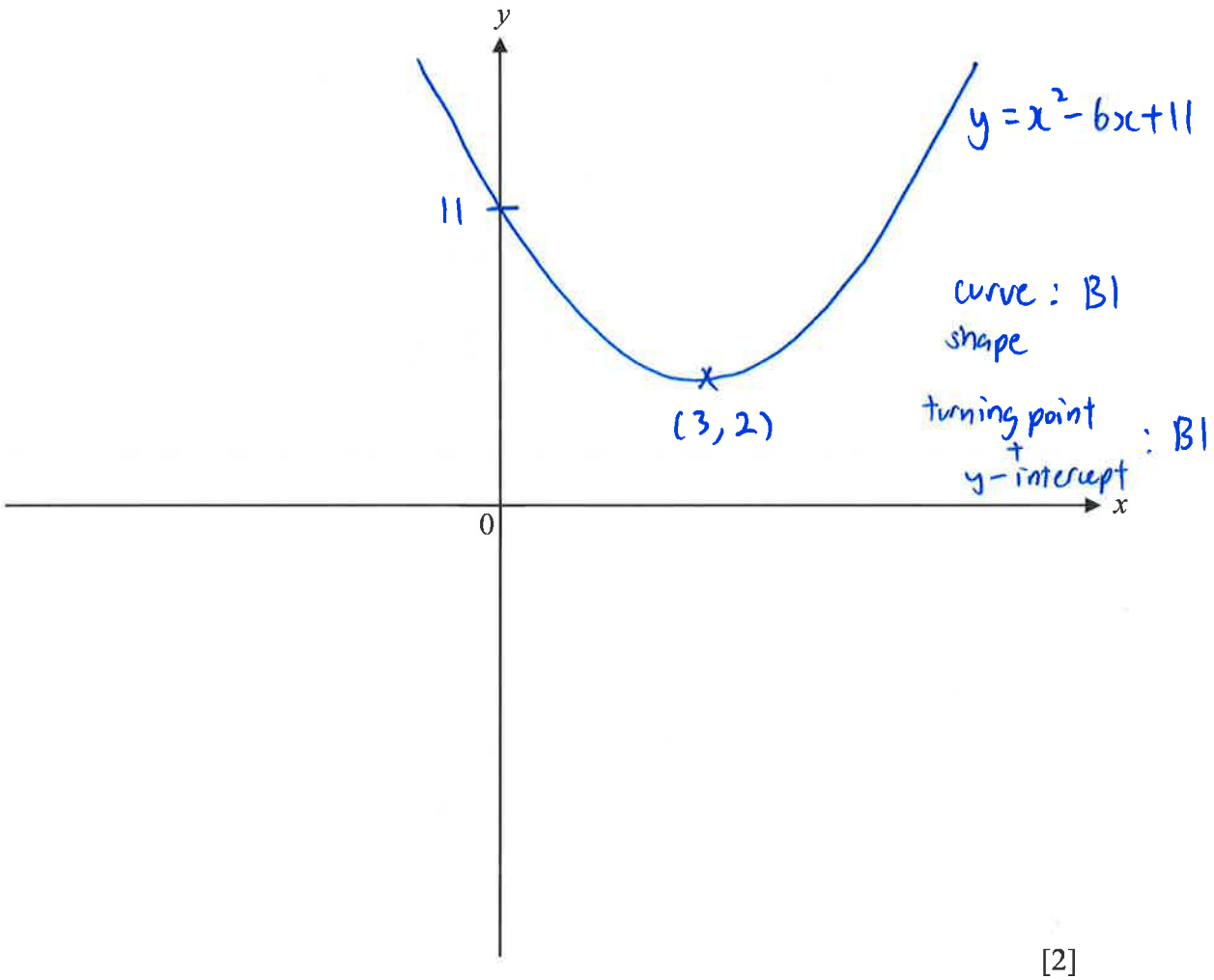
Answer $(x-3)^2 + 2$ [2]

B1 B1

(b) State the minimum value of $x^2 - 6x + 11$.

Answer 2 B1 [1]

(c) Sketch the graph of $y = x^2 - 6x + 11$ on the axes below, indicating clearly the turning point, x-intercept(s) and y-intercept (if any).



[2]

13 **(a)** Solve $2x-1<\frac{x+8}{2}\leq x+3$ and represent your solution on the number line below.

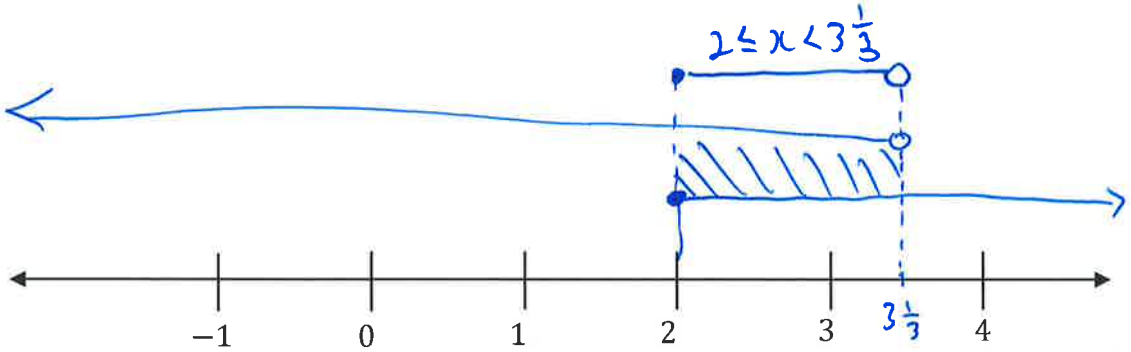
$2x-1<\frac{x+8}{2}$ $\quad\quad$ $\frac{x+8}{2}\leq x+3$

$4x-2<x+8$ $\quad\quad$ $x+8\leq 2x+6$

$3x<10$ $\quad\quad$ $2\leq x$

$x<3\frac{1}{3}$ $\quad\quad$ $x\geq 2$

$\therefore\ 2\leq x<3\frac{1}{3}$



[4]

(b) Hence, state the largest prime number that satisfies the inequalities in part **(a)**.

Answer 3 ..... [1]

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14 Mr Lim is considering between banks *A* and *B* to invest \$50 000.

(a) Based on Bank *A*'s calculations, his money will grow to \$54 636.35 after three years, compounded annually. Find the interest rate at which his money will compound annually in bank *A*.

$54\,636.35 = 50\,000\left(1+\frac{r}{100}\right)^3$ M1

$\frac{54636.35}{50\,000} = \left(1+\frac{r}{100}\right)^3$

$\sqrt[3]{\frac{54636.35}{50\,000}} = 1+\frac{r}{100}$ M1

$r = 100\left(\sqrt[3]{\frac{54636.35}{50\,000}} - 1\right)$

$= 3$ A1

Answer.....3..... % [3]

(b) Bank *B* offers an investment product which pays simple interest at 2.5% per annum for three years. The bank will also offer a sign up bonus of \$1000. Determine which Bank he should invest in, justifying your reasons clearly.

$I = \frac{(50\,000)(2.5)(3)}{100}$

$= 3750$

$1000 + 3750 = 4750$

$4750 - 4636.35 = 113.65$

Answer: Bank ..B..... because ..he will earn \$113.65 more.....

..... [2]

15 4 cm on map A represents an actual distance of 120 m.

(a) Express the scale of the map in terms of 1 : n.

4cm : 12000cm M1

Answer 1 : 3000. A1 [2]

(b) The perimeter of a pond on the map A is 24 cm.
Find the perimeter of the actual pond.
Give your answer in kilometres.

1cm : 0.03km M1

Answer 0.72 A1 km [2]

(c) A building takes up an actual area of 0.108 km^2.
Calculate this area on map A.
Give your answer in cm^2.

$1cm^2 : 0.0009km^2$ M1

Answer 120 A1 cm^2 [2]

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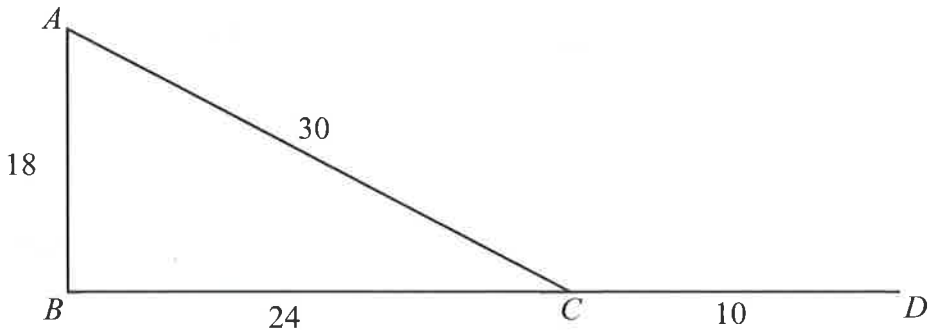
16 In the diagram below, BCD is a straight line.
$AB = 18$ cm, $BC = 24$ cm, $AC = 30$ cm and $CD = 10$ cm.



(a) Show that triangle ABC is a right-angled triangle. [2]

Answer:

$AC^2 = 30^2$ $\quad AB^2 + BC^2 = 18^2 + 24^2$
$= 900$ $\quad = 900$ } M1

Since $AC^2 = AB^2 + BC^2$, by the converse of pythagoras thm, ΔABC is a right-angled Δ. A1

(b) Find the exact value of $\sin \angle ACD$.

Answer $\frac{3}{5}$ B1 [1]

(c) Find the exact value of $\cos \angle ACD$.

Answer $-\frac{4}{5}$ B1 [1]

(d) Calculate the area of triangle ACD.

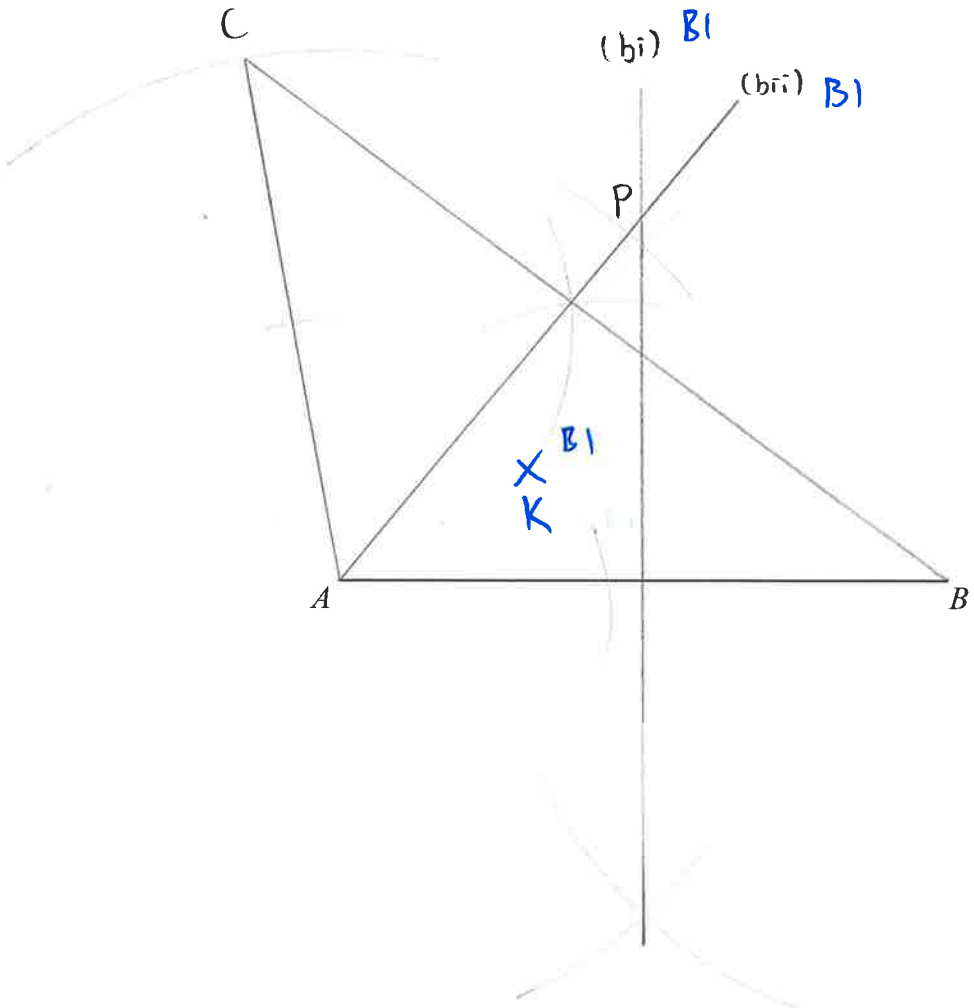
$\frac{1}{2}(30)(10)\sin \angle ACD$

$\frac{1}{2}(30)(10)(\frac{3}{5})$ M1

Answer 90 A1 cm^2 [2]

∠BAC : B1
length AC : B1

17 **(a)** Construct the triangle ABC in which $AB = 8$ cm, $\angle BAC = 100^\circ$ and $AC = 7$ cm. The line AB has been constructed for you.



[2]

(b) On the same diagram, construct

(i) the perpendicular bisector of AB, [1]

(ii) the angle bisector of $\angle BAC$. [1]

(c) Given that the two bisectors meet at P, measure and write down the length of PB.

Answer 6.2 B1 ..... cm [1]

(d) The region Q, within the triangle ABC, is nearer to A than to B and nearer to line AB than to line AC. Mark a possible point, K, such that K lies in the region Q. [1]

18 **(a)** It is given that y is inversely proportional to the square root of x.
It is known that $y = 12$ when $x = 4$.

(i) Find an equation connecting y and x.

$y = \frac{k}{\sqrt{x}}$ M1

$k = 24$

Answer $y = \frac{24}{\sqrt{x}}$ A1 [2]

(ii) Find the value of y when $x = 9$.

Answer $y =$ 8 B1 [1]

(iii) Find the value of x when $y = 6$.

$6 = \frac{24}{\sqrt{x}}$ M1

$\sqrt{x} = 4$

Answer $x =$ 16 A1 [2]

(b) m is directly proportional to n^2.
It is known that $m = 6$ for a particular value of n.
Find the value of m when n is doubled.

$m = kn^2$ $\quad m_{new} = \left(\frac{m}{n^2}\right)(2n)^2$ M1

$= 4m$

Answer $m =$ 24 A1 [2]

19 **(a)** Given that $27^x = 729$, find the value of x.

$27^x = 27^2$

Answer $x =$ 2 [2]

(b) Simplify $\sqrt[3]{8x^6y^{-9}} \times 3x$, giving your answer in positive index.

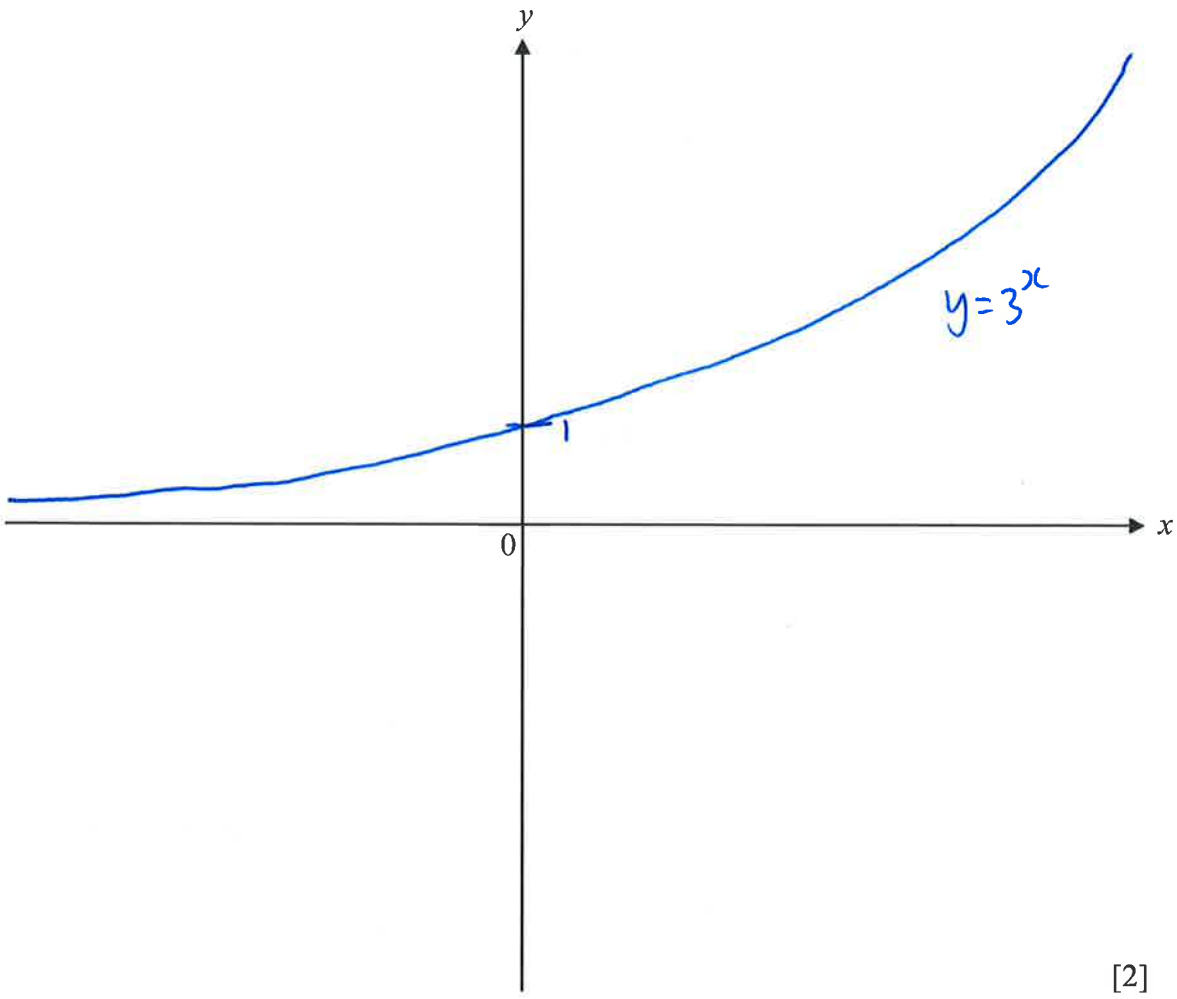
$(8x^6y^{-9})^{\frac{1}{3}} \times 3x$

$= 2x^2y^{-3} \times 3x$

$= 6x^3y^{-3}$

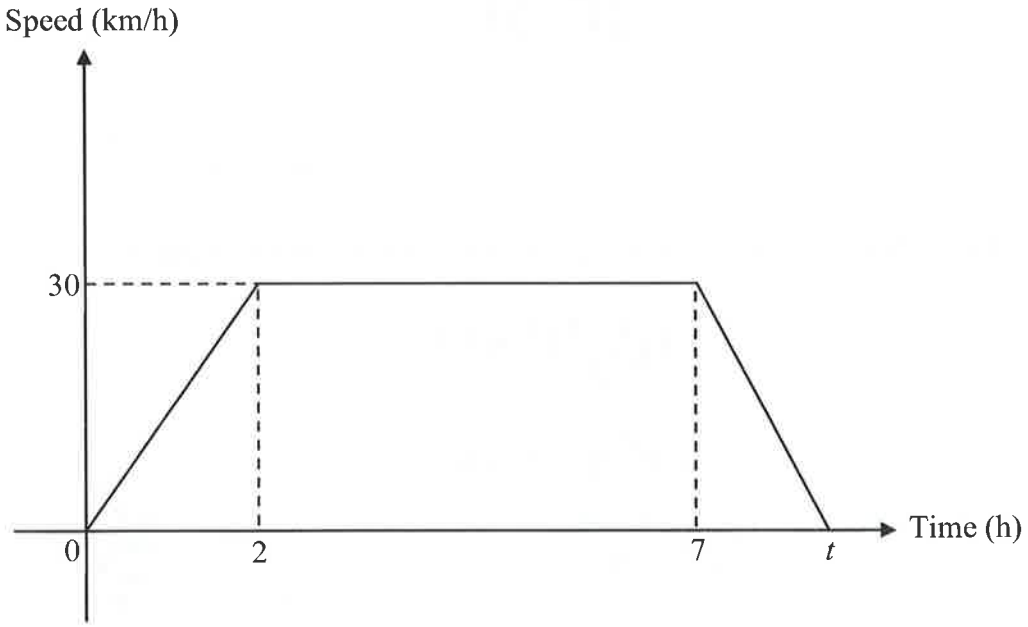
Answer $\frac{6x^3}{y^3}$ [3]

(c) Sketch the graph $y = 3^x$ on the axes below, indicating clearly the x-intercept(s) and y-intercept (if any).



[2]

20 The diagram below shows the speed-time graph of a car over a period of t hours.



(a) Find the acceleration of the car during the first 2 hours.

$\frac{30-0}{2-0} = 15$

M1 A1

Answer 15 km/h^2 [2]

(b) Given that the deceleration after 7 hours is 20 km/h^2, show that $t = 8.5$ [1]

Answer: $\frac{30}{20} = 1.5$ B1

$7 + 1.5 = 8.5$

(c) Find the average speed of the whole journey.

Dist (0-2) $= \frac{1}{2} \times 30 \times 2$

$= 30$

Dist (2-7) $= 30 \times 5$

$= 150$

Dist (7-8.5) $= \frac{1}{2} \times 30 \times 1.5$

$= 22.5$

Total D $= 30 + 150 + 22.5$

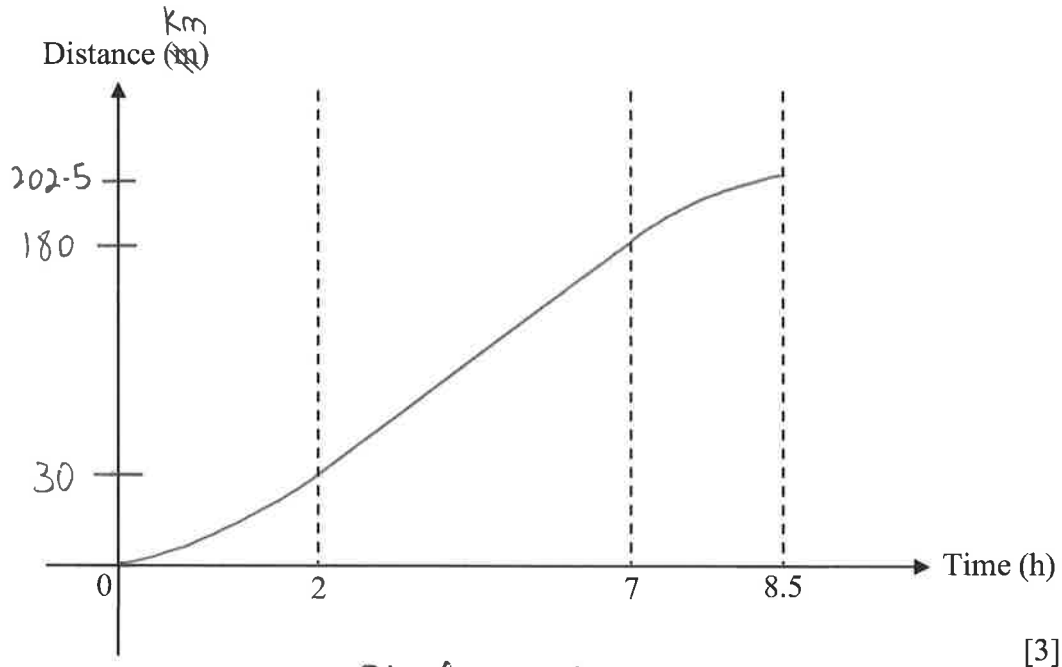
$= 202.5$ M1

Av sp $= \frac{202.5}{8.5}$

$= 23.8 / 23\frac{14}{17}$ A1

Answer ..... km/h [2]

(d) On the axes provided below, sketch the distance-time graph of the car for the first 8.5 hours of the journey, indicating the distance travelled on the vertical axis clearly.



[3]

B1 for each part

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