Candidate Index Number

Anglo - Chinese School

(Independent)

FINAL EXAMINATIONS 2020
YEAR 3 INTEGRATED PROGRAMME
CORE MATHEMATICS
PAPER 1

FRIDAY **2nd OCTOBER 2020** **1 h 30 min**

Candidates answer on the Question Paper.
No additional materials are required.

INSTRUCTIONS TO CANDIDATES

- Write your index number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Answer all questions in the spaces provided.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.
- The maximum mark for this paper is 80.

For Examiner's Use

This paper consists of 17 printed pages and 1 blank page.

[Turn over

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

*Answer **all** the questions in the spaces provided.*

1 *[Maximum mark: 4]*

(a) Evaluate $\left(\sqrt{2}-\sqrt{10002}\right)\left(\sqrt{10002}+\sqrt{2}\right)$ *[2 marks]*

(b) If $\frac{a-b}{a+b}=2$, find the value of $\frac{a}{b}$. *[2 marks]*

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2 *[Maximum mark: 6]*

(a) Express $\frac{2}{m+4}+\frac{3}{m-2}$ as a single fraction. *[2 marks]*

(b) Make y the subject of the formula $x=\frac{y+z}{y-z}$. *[4 marks]*

3 *[Maximum mark: 7]*

(a) Simplify $\left(\frac{x^2-3x}{4x}\right)\left[\frac{5(x+1)}{x-3}\right]$. *[3 marks]*

(b) **(i)** Hence, solve $0 \leq \left(\frac{x^2-3x}{4x}\right)\left[\frac{5(x+1)}{x-3}\right] < 5$. *[3 marks]*

(ii) State the largest integer value of x that satisfy the inequalities. *[1 mark]*

4 *[Maximum mark: 5]*

A cuboid consisting of a rectangular base of breadth $3-\sqrt{5}$ cm and length $\sqrt{5}+1$ cm has a volume of $48-16\sqrt{5}$ cm^3. Find the height of the cuboid in the form $a(\sqrt{5}-1)$, where a is an integer to be determined. *[5 marks]*

5 *[Maximum mark: 9]*

Consider the equation $y = -x^2 - 6x + 15$,

(a) Write the expression $-x^2 - 6x + 15$ in the form $-(x+h)^2 + k$, where h and k are constants to be determined. *[2 marks]*

(b) State the coordinates of the turning point of $y = -x^2 - 6x + 15$. *[1 mark]*

(c) Solve the equation $-x^2 - 6x + 15$, **simplifying** your answers and leaving them in **surd form**. *[3 marks]*

(d) Sketch the graph of $y = -x^2 - 6x + 15$, stating the coordinates of the points of intersection with the axes and turning point clearly. *[3 marks]*

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[Continuation of working space for Question 5]

6 *[Maximum mark: 12]*

Solve the following equations, leaving your answers in **exact form**.

(a) $7^{x+3} = 1^x$ *[2 marks]*

(b) $e^{x+2} = 2^{x-1}$ *[4 marks]*

(c) $3^{x^2} - 18(3^{-x^2}) = 7$ *[6 marks]*

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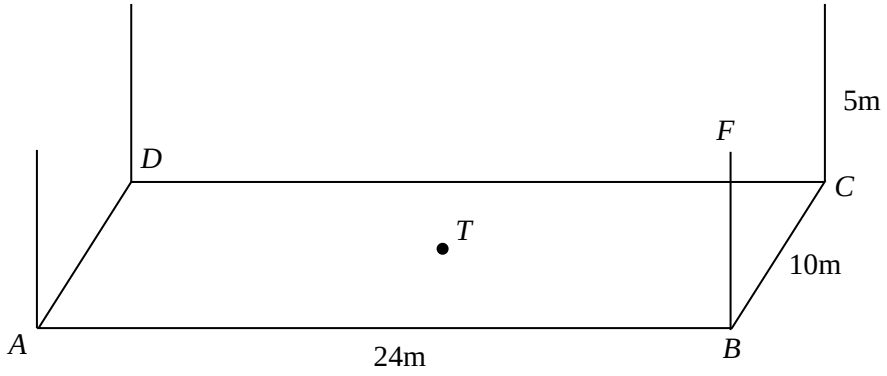
[Continuation of working space for Question 6]……………………………………………………………

7 *[Maximum mark: 7]*

The diagram below shows a small field where *A, B, C* and *D* are points on level ground. Four vertical poles are erected at each corner. The height of each pole is 5m, *T* is the center of the field and *BF* is the pole at *B*. It is known that $AB = 24\text{m}$ and $BC = 10\text{m}$.



(a) Find the length

(i) *TB* *[2 marks]*

(ii) *TF* *[2 marks]*

(b) Hence, find

(i) $\tan \measuredangle FTB$ *[1 mark]*

(ii) $\cos \measuredangle FTD$ *[2 marks]*

Leave your answers in surd form where applicable.

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[Continuation of working space for Question 7]…………………………………………………….……………

8 *[Maximum mark: 9]*

In the diagram below, the points $A(-2, 2)$ and $B(6, 8)$ are two vertices of a quadrilateral *ABCD*. *AD* is parallel to the vertical axis and is equal in length to *AB*.

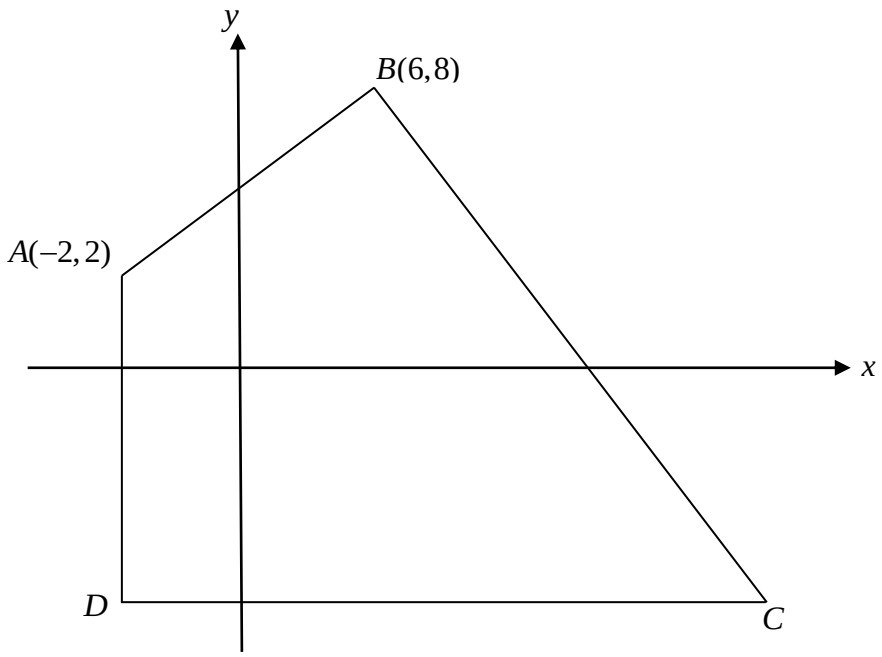
(a) Find the coordinates of point *D*. *[2 marks]*

(b) Show that the equation of the perpendicular bisector of *BD* is $y = -\frac{1}{2}x + 1$. *[4 marks]*

It is further given that *DC* is parallel to the x axis and point *C* lies on the perpendicular bisector of *BD*.

(c) Find the coordinates of *C* *[2 marks]*

(d) Find the area of ΔABC. *[1 mark]*



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[Continuation of working space for Question 8]……………………………………………………………

9 *[Maximum mark: 8]*

(a) Find the range of values of k such that $y = x^2 - kx + 3 - k$ intersects the x-axis at two points. *[3 marks]*

(b) The roots to the equation $x^2 - 4x + 5 = 0$ are p and q. Find the quadratic equation in terms of k, whose roots are kp and kq. *[5 marks]*

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[Continuation of working space for Question 9]……………………………………………………………………

10 *[Maximum mark: 13]*

(a) Find the value of p in the equation $6\log_3(p-5)=12$. *[3 marks]*

(b) Solve the equation $\log_9 x+\log_x 81=3$. *[5 marks]*

(c) Explain, with clear mathematical reasonings, if there are any solutions to the equation $\log_4(x-3)^2+2\log_2\sqrt{x+1}=\log_2(3-x)$. *[5 marks]*

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[Continuation of working space for Question 10]……

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End of Paper 1

Answer Key

1a) -10000

1b) -3

2a) $\dfrac{5m+8}{(m+4)(m-2)}$

2b) $y=\dfrac{z(x+1)}{x-1}$

3a) $\dfrac{5}{4}(x+1)$

3bi) $-1\le x<3$

3bii) 2

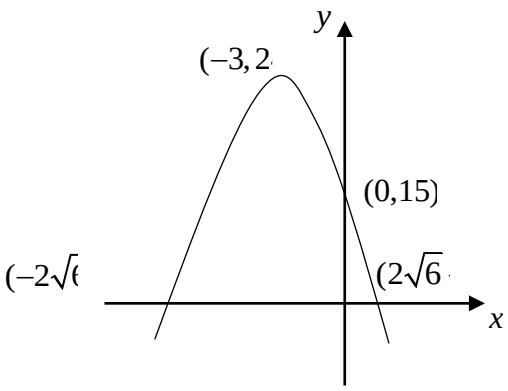
4) $h=4(\sqrt{5}-1)$

5a) $-(x+3)^2+24$

5b) $(-3,24)$

5c) $x=\pm 2\sqrt{6}-3$

5d)



6a) $x=-3$

6b) $x=\dfrac{-2-\ln 2}{1-\ln 2}$

6c) $x=\pm\sqrt{2}$

7ai) 13cm

7aii) $\sqrt{194}$ cm

7bi) $\dfrac{5}{13}$

7bii) $-\dfrac{13}{\sqrt{194}}$

8a) $D=(-2,-8)$

8b) Showing Question

8c) $C=(18,-8)$

8d) 100

9a) $k<-6, k>2$

9b) $x^2-4kx+5k^2=0$

10a) $p=14$.

10b) $x=81, x=9$

10c) $x=0$